A Winning Score-based Evolutionary Process for Multi- and Many-objective Peptide Optimization

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Abstract: Target identification as part of drug design is a long process with high laboratory evaluation costs since optimal candidate leads have to be identified in an iterative process including the determination of diverse physiochemical properties, which have to be optimized simultaneously. MOEAs have become an established optimization method in \textit{in silico}-aided drug design processes. Since target identification becomes more complex, the dimension of molecular optimization problems increases. Less work has been done so far to evolve an evolutionary process efficiently solving both, multi- and many-objective molecular optimization problems while considering application-specific conditions of molecule optimization. This work presents the enhancement of a MOEA especially evolved for molecular optimization. The proposed algorithm is applicable to multi- and many-objective molecular optimization problems identifying a selected number of qualified candidate peptides within a very low number of iterations. It has a simple framework structure and optionally uses two types of winning-score ranking method as survival selection. Default parameters are provided in the components to enable a non-expert use. This algorithm is benchmarked to the recently proposed and promising AnD (ANgle-based selection and shift-based Density estimation strategy) on molecular optimization problems up to 6 objectives. Furthermore, the selection principles are exemplarily compared and discussed.

1 INTRODUCTION

Computer-assisted techniques gain importance in the area of drug discovery and development. The success of molecule design depends on simultaneous optimization on often conflicting biological and physiochemical properties. Multi-objective Evolutionary Algorithms (MOEAs) have proven to enhance the potential of improving promising drug candidates (Nicolotti et al., 2011). The increase of complexity in pharmaceutical research results in the challenge of the design of a Many-objective Evolutionary Algorithm (MaOEA) especially for molecular optimization. To the best of our knowledge, less work has been done so far regarding this issue. The design of a MOEA as well as MaOEA for molecular optimization has to take account of several application-specific conditions: the objective of target identification usually requires expensive and time-consuming laboratory work since the numerical approximation of peptide properties is challenging. Therefore, the objective function evaluations have to be limited to save resources. Furthermore, the algorithm provides default parameter settings to enable the non-expert use and does not utilize weight vectors or reference points, which are commonly unknown in real-world applications but have an impact on the algorithm performance.

In addressing these issues, a single-objective evolutionary algorithm especially evolved for molecular optimization has been introduced in (Rückendorf and Borschbach, 2012), (Krause et al., 2018) providing an exponential fitness improvement within the very low number of 10 iterations and a standard population size. This approach has been enhanced to a MOEA with similar properties. This MOEA is termed COMponent-Specific Evolutionary Algorithm for Molecular Optimization (COSEA-MO) and is presented in (Rosenthal and Borschbach, 2017b). COSEA-MO identifies a selected number of highly qualified candidate peptides with a wide range of genetic diversity within 10 iterations. Nevertheless,
the increase of the problem dimension reveals well-known challenges of Pareto definition-based MOEAs in solving Many-objective Optimization Problems (MaOPs) comprising more than three objectives: the inability of adequately differentiate higher dimensional solutions and the loss of selection pressure.

This work presents an enhancement of COSEA-MO to solve multi- and many-objective molecular optimization problems by the traditional Winning Score (WS) technique (Maneeratano et al., 2006) or optionally a new difference-based WS selection strategy. Furthermore, the selection principles of these WS techniques are exemplarily analyzed and discussed. The performance of the enhanced versions of COSEA-MO are compared to a recently proposed and promising MaOEA termed AnD (ANgle-based selection and shift-based Density estimation strategy) (Lee et al., 2018) on a three-dimensional up to a six-dimensional molecular optimization problem.

The outline of this work is as follows: Section 2 gives an overview of the related work, the proposed approach with WS-based selection strategies is introduced in section 3 with the discussion of the selection principles. Section 4 presents the experiments, section 5 concludes this work and gives an outlook on future work.

2 RELATED WORK

MOEAs are classifiable into three categories according to their selection strategies: Pareto-based, decomposition-based and indicator-based algorithms. The effectiveness of MOEAs on Many-objective Optimization Problems (MaOPs) is significantly decreased with the problem dimension (Ishibuchi et al., 2011). In the case of Pareto-based MOEAs such as NSGA-II (Deb et al., 2002) and SPEA2 (Zitzler et al., 2002), the number of non-dominated solutions increases significantly with the problem dimension since the Pareto dominance principle of individuals (Laumanns et al., 2002). L-dominance is introduced selecting individuals with objectives of similar importance regarding the objective value improvement (Zou et al., 2008). Fuzzy dominance methods are presented using ranking schemes to select promising individuals (He et al., 2014). Moreover, a grid-based method is published adjusting the grid size to control the proportion of Pareto-optimal solutions (Yang et al., 2013).

Several enhanced variants of MOEA/D for MaOPs have been published in the past. Most recent algorithms are MOEA/D-AM2M which adaptively allocates the search effort (Liu et al., 2017), MOEA/D-DU which exploits the perpendicular distance from the individuals to the weight vectors (Yuan et al., 2016) and MOEA/D-PaS which uses a Pareto adaptive scalarization method (Wang et al., 2016).

The hypervolume is the mostly used indicator in indicator-based MOEAs such as in IBEA and SMS-EMOA. Its major disadvantage is the experimental increase of the computational complexity with the dimension increase. To overcome this disadvantage, other indicator-based methods have been introduced recently. The Inverse Generational Distance Plus (IGD+) indicator is used in IGD+-EMOA to address MaOPs up to 8 objectives (Lopez and Coello, 2016). Furthermore, the collaboration of different indicators of low computational complexity has been proven to be a promising solution for solving MaOPs (Lee et al., 2018).

Beneath these approaches, the widely used NSGA-II has been improved to NSGA-III (Deb and Jain, 2014) for MaOPs by the use of a set of predefined well-distributed reference points. Non-dominated solutions close to this set are prioritized. An appropriate design of this set is challenging, especially in the case of real-world applications. A recent promising MaOEA has been proposed termed AnD (Lee et al., 2018). It has a simple framework structure and selects promising individuals from the union of parent and child population for the next iteration with a diversity-first-and-convergence-second principle. AnD combines the well-known vector angle and shift-based density estimation in the selection process. Angle-based selection is used to identify two individuals with minimal angle. This is motivated by the idea that these individuals represent the search in the same direction and waste computational resources if both individuals survive. The individual with lower shift-based density estimation is deleted in order to ensure convergence. AnD is compared to seven state-
of-the-art MaOEA on a variety of benchmark problems with 5, 10 and 15 objectives and reveals highly competitive performance (Lee et al., 2018). AnD is chosen for experimental comparison in this work as it is the only algorithm apart from COSEA-MO that has a simple framework structure, provides optimized default parameters for the non-expert use and is independent of weight vectors or reference points, which usually have a strong impact on the performance and are usually unknown in real-world applications. Moreover, the framework structure of AnD is similar to those of COSEA-MO.

The traditional WS technique has been introduced with the Compressed–objective Genetic Algorithm (COGA) (Maneeratano et al., 2006). Two conflicting preference objectives, WS and a vicinity method, are used to assign different preference levels to non-dominated solutions to bound the increasing set of non-dominated solutions in MaOPs. A rank is assigned to each non-dominated solution according to the preference objectives to select high preferred non-dominated solutions in survival selection and the truncation method to maintain the archive size. COGA has been enhanced to the Improved Compressed–objective Genetic Algorithm (COGA-II) (Boolong et al., 2010). A WS-based ranking mechanism is applied instead of the two preference objectives of COGA. The WS value of a non-dominated solution is determined by the weighted sum of competitive scores from all objectives to the remaining non-dominated solutions.

3 PROPOSED APPROACH

This section proposes an enhanced version of COSEA-MO to solve MaMOPs. Its characteristics are a simple framework structure, optimized default parameter settings for the non-expert use, deterministic dynamic variation operators and WS-based selection mechanism. Two alternative WS-based mechanism rank the population and select individuals for the next generation according to the scores. Score values are assigned to each individual based on the number of superior or inferior objectives in the case of the traditional WS and additionally based on the quantity of superiority or inferiority in the case of Difference-based Winning Score (dWS) to the remaining individuals in the population. The framework of COSEA-MO with WS-based selection is referred to as WS-COSEA-MO, the version with dWS is termed dWS-COSEA-MO. The framework of both is given in Algorithm 1.

Algorithm 1: Framework of (d)WS-COSEA-MO.

Input: Population $P_t$, population size $N$, Archive $A_t = \{\}$, number of optimal solutions $m$, total number of generations $T$

Output: Next generation $P_{t+1}$ and archive update $A_{t+1}$

1: Random initialization of $P_0$;
2: while $t < T$ do
   $Q_t \leftarrow$ RandomMatingAndVariation($P_t$);
   $U_t \leftarrow P_t \cup Q_t$;
   $P_{t+1} \leftarrow$ (d)WS-Selection($U_t$);
   $A_{t+1} \leftarrow$ add $m$ fittest individuals of $P_{t+1}$ acc. to (d)WS;
   $t \leftarrow t + 1$;
end

Firstly, the start population $P_0$ of size $N$ is randomly initialized. The individuals represent peptides in form of character strings. During the evolution process, an offspring generation $Q_t$ of size $N$ is determined by randomly selecting three parents of $P_t$ for variation. The specific number of parents is motivated to ensure a high genetic diversity of the genetic material. The variation operators are motivated by a suitable balance of global and local search. Deterministic dynamic variation operators are suitable operators to achieve this purpose. A linear dynamic recombination operator and an adapted version of the deterministic dynamic mutation operator of Bäck and Schütz (Bäck and Schütz, 1996) is used to generate offspring (RandomMatingAndVariation). The variation rates are adapted dynamically by predefined decreasing functions with the iteration progress: the recombination operator varies the number of recombination points by a linearly decreasing function

$$x_R(t) = \frac{l}{4} - \frac{1}{4} \cdot \frac{t}{T}$$

where $l$ is the peptide length, $T$ the total number of the generations and $t$ the index of the current generation. The adapted mutation operator determines the mutation probabilities via

$$p_{\text{NS}} = (a + l - 2 \cdot \frac{t}{T - 1})^{-1}$$

with $a = 5$. The mutation rates of the traditional operator are reduced by a higher value for $a$. After that, $P_t$ and $Q_t$ are combined to a population $U_t$ of size $2N$. Finally, the WS- or dWS-based selection mechanism is performed to select $N$ individuals of $U_t$ for the next generation $P_{t+1}$ ((d)WS-Selection) and the archive is updated by adding the $m$-optimal individuals of $P_{t+1}$ according to the scoring points. Optimal solutions detected in previous generations are not added twice into the archive.
The motivation and the decisive characteristics of WS- and dWS-based selection are described and discussed in the sequel.

### 3.1 WS-based Selection Mechanisms

In (Benedetti et al., 2006), three reasons are summarized about the unsatisfactoriness of Pareto dominance definition in the case of a large number of objectives:

- The number of improved objective function values are not taken into account.
- The (normalized) relevance of improvement is not taken into account.
- No preference among the objectives is considered.

In the present biochemical optimization problems, all objectives are equally important and therefore, the last issue is negligible. The traditional WS method meets the first issue in an intuitive way, it describes the difference between the number of superior and inferior objectives between two individuals: let \( sup_{ij} \) be the number of objectives in a solution \( i \) that is superior to the corresponding objectives in a solution \( j \) while \( inf_{ij} \) is the number of objectives in \( i \) that is inferior to \( j \). The WS-values of the \( i \)-th solution in a population of size \( N \) is given by (Maneeratano et al., 2006):

\[
WS(i) = \sum_{j=1}^{N} w_{ij} \quad \text{with} \quad w_{ij} = sup_{ij} - inf_{ij}
\]

Obviously, it is \( w_{ij} = -w_{ji} \) and \( w_{ii} = 0 \). This assignment ensures that solutions with high WS-values are close to the true Pareto front.

We assume the following expression of a MaOP:

\[
\text{minimize } F(x) = (f_1(x), f_2(x), \ldots, f_K(x)) \quad \text{with } x \in \Omega,
\]

where \( x \) is the decision variable in the search space of all feasible peptides (\( \Omega \)), \( F(x) \) the objective vector and \( K \) the number of objectives. To address the second issue additionally to the first one, \( dWS \) is used:

\[
dWS(x_i) = \sum_{j=1}^{N} \sum_{k=1}^{K} dW_{ijk} \quad \text{with}
\]

\[
dW_{ijk} = \begin{cases}
(f_k(x_i) - f_k(x_j))^2, & \text{if } f_k(x_i) < f_k(x_j), \\
0, & \text{if } f_k(x_i) = f_k(x_j), \\
-(f_k(x_i) - f_k(x_j))^2, & \text{if } f_k(x_i) > f_k(x_j),
\end{cases}
\]

where \( x_i \) and \( x_j \) are two individuals of the population, \( N \) the population size and \( K \) the number of objectives. \( dWS \) has especially been evolved to rank the solutions according to their exact objective value differences. With this approach, the amount of objective improvement and worsening is considered, not only the number of superior and inferior objectives. The square of the differences ensures that higher differences have a stronger impact on the scores.

The aim of WS- or dWS-based selection is to find \( N \) approximately optimal individuals from \( U_i \) for the next generation. Furthermore, the update of the archive with the \( m \) fittest individuals is also based on the ranking according to the scores. Therefore, a score is assigned to each individual \( x_i \) of the current population relative to the remaining population members \( x_j \). WS, or respectively \( dWS(x_i) \) of an individual \( x_i \), reflect the quality of \( x_i \) relative to the remaining members of the current population. A high positive score value indicates superior quality of this individual compared to the others, whereas high negative values indicates a low qualified solution. In contrast to COGA and COGA2, the scores are standalone selection criteria and are assigned to every member of the population with the aim of ranking, they are not only used to differentiate a pair of non-dominated solutions.

After this scoring point assignment to each individual, the population is ranked according to the scores. The selection process of the \( N \) individuals from \( U_i \) performs as described in Algorithm 2. \( R_t \) is the ranked set of \( U_i \) and \( R_t(i) \) is the \( i \)-th front set of \( R_t \).

<table>
<thead>
<tr>
<th>Algorithm 2: Selection process.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input:</strong> Ranked population ( R_t ) with (</td>
</tr>
<tr>
<td><strong>Output:</strong> Next generation ( P_{t+1} )</td>
</tr>
<tr>
<td>1: while (</td>
</tr>
<tr>
<td>( P_{t+1} \leftarrow P_{t+1} \cup R_t(i); )</td>
</tr>
<tr>
<td>( i++; )</td>
</tr>
<tr>
<td>end</td>
</tr>
<tr>
<td>2: while (</td>
</tr>
<tr>
<td>binary tournament selection</td>
</tr>
<tr>
<td>( {x_i, x_j} \in R_t \setminus P_{t+1}; )</td>
</tr>
<tr>
<td>if ( \text{VolumeDominance}(x_i) &lt; \text{VolumeDominance}(x_j) ) then</td>
</tr>
<tr>
<td>( P_{t+1} \leftarrow P_{t+1} \cup {x_i}; )</td>
</tr>
<tr>
<td>end</td>
</tr>
<tr>
<td>else</td>
</tr>
<tr>
<td>( P_{t+1} \leftarrow P_{t+1} \cup {x_j}; )</td>
</tr>
<tr>
<td>end</td>
</tr>
<tr>
<td>end</td>
</tr>
</tbody>
</table>

The population of the next iteration is filled by each rank subsequently until the population size exceeds \( N \). In the case that adding the individual set of rank \( R_t(i) \) exceeds \( N \), the remaining individuals for \( P_{t+1} \) are selected by binary tournament selection of two individuals from the remaining ranks \( R_t \setminus P_{t+1} \) according to the better Volume Dominance (VD) value. VD is simply the spanned space of an individual to the zero point (VolumeDominance). In the case of a min-
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Figure 1: Selection principle of AnD, NSGA-II and (d)WS-COSEA-MO.

In the case of WS-COSEA-MO and dWS-COSEA-MO, A, D, E and F are selected into the next generation. The difference between WS-COSEA-MO and dWS-COSEA-MO is the ranking of the solutions A and D. In the case of WS-COSEA-MO, D has a higher winning score than A, this is vice versa in the case of dWS-COSEA-MO. Summarizing, the selection principles of the winnings score-based algorithms are comparable to the one of NSGA-III. Point B is only selected by AnD. This point is potentially important to maintain the diversity in the population, but the experiments in this work show that this selection mechanism lacks of a suitable convergence within a very low number of generations.

Furthermore, a three-dimensional example is used to illustrate the characteristics of WS- and dWS-based selection according to the characteristics mentioned above: Eleven points are given, \( A(0.15, 0.1, 0.08) \), \( B(0.2, 0.3, 0.15) \), \( C(0.02, 0.3, 0.7) \), \( D(0.25, 0.75, 0.32) \), \( E(0.32, 0.27, 0.81) \), \( F(0.3, 0.4, 0.25) \), \( G(0.28, 0.35, 0.31) \), \( H(0.32, 0.43, 0.28) \), \( I(0.15, 0.1, 0.78) \), \( K(0.17, 0.68, 0.15) \) and \( L(0.18, 0.17, 0.73) \). The winning scores as well as the difference-based winnings scores are determined and given in Table 1. Point A has the highest score value in both cases followed by point B. Points C, I, K and L are boundary points and achieve an evolutionary advantage by good score values in the case of WS. In the case of dWS, the scores of these points are positioned in the lower half of the ranking. F an G are positioned very close to each other and achieve very similar scores in both cases. D and E are worse individuals and have the lowest scores in both cases. Summarizing, the evolutionary advantage of the boundary points in the case of WS is the main difference between the winning score alternatives.

4 EXPERIMENTAL SETUP

The performance of the proposed WS-COSEA-MO and dWS-COSEA-MO are compared to the recently published AnD on four differently dimensional molecular optimization problems according to the convergence behavior and diversity. All experiments are implemented in the open source jMetal library 4.5. (Nebro and Durillo, 2018) and uses the open source BioJava framework 4.2.0 (Prilic et al., 2018). Each experiment is run 20 times on each molecular optimization problem with 10 iterations and a population size of 100. The individuals are 20-mer peptides composed of the 20 canonical amino acids. Short peptides of length 20 are of specific interest because of their fa-
Table 1: Winning Score values of WS-COSEA-MO and dWS-COSEA-MO.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>K</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>WS</td>
<td>26</td>
<td>8</td>
<td>7</td>
<td>-14</td>
<td>-15</td>
<td>-6</td>
<td>-6</td>
<td>-13</td>
<td>8</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>dWS</td>
<td>3.3</td>
<td>1.8</td>
<td>-0.9</td>
<td>-1.7</td>
<td>-2.3</td>
<td>0.6</td>
<td>0.8</td>
<td>0.4</td>
<td>-1.0</td>
<td>-0.03</td>
<td>-0.9</td>
</tr>
</tbody>
</table>

favorable properties as drugs.

4.1 Molecular Optimization Problems

Four molecular optimization problems with 3 to 6 objective functions predicting physiochemical properties are used as experimental studies. Table 2 presents the composed physiochemical optimization problems with the used abbreviations: Needleman Wunsch Algorithm (NMW), Molecular Weight (MW), Average Hydrophilicity (Hydro), pI (PI), Instability Index (InstInd), Isolelectric Point (pH) and Aliphatic Index (aI). These molecular functions are provided by the BioJava library (Prlic et al., 2018). The physiochemical functions are shortly described in the following, a description of the functions is given here (Prlic et al., 2018): NMW is a well known and used method for the global sequence alignment of a solution to a pre-defined reference individual. This algorithm refers to the common hypothesis that a high similarity between molecules refers to similar molecular properties.

MW is an important peptide property as a minimized MW ensures a good cell permeability. MW of a peptide sequence \( a \) of length \( l \) is calculated summarizing the mass of each amino acid \( a_i \) plus a water molecule:

\[
MW(a) = \sum_{i=1}^{l} \text{mass}(a_i) + 17.0073 \cdot \text{OH} + 1.0079 \cdot \text{H},
\]

where \( \text{O} \) (oxygen) and \( \text{H} \) (hydrogen) are the elements of the periodic system.

A common challenge of drug peptides is the solubility in aqueous solutions, especially peptides with stretches of hydrophobic amino acids. Therefore, Hydro is calculated by the hydrophilicity scale of Hopp and Woods (Hopp and Woods, 1983) with a window size equal to the peptide length \( l \). An average hydrophilicity value is assigned to each candidate peptide \( a \) using the scales for each amino acid \( a_i \):

\[
\text{Hydro}(a) = \frac{1}{l} \cdot \left( \sum_{i=1}^{l} \text{hydro}(a_i) \right).
\]

The use of molecules as therapeutic agents is potentially restricted by their instability and their potential degradation by enzymes in systemic application. The stability is addressed by the InstInd as stability is a very important feature of drug components. InstInd is determined by the Dipeptide Instability Weight Values (DIWV) of each two consecutive amino acids in the peptide sequence. DIWV are provided by the GRP-Matrix (Guruprasad et al., 1990). These values are summarized and the final sum is normalized by the peptide length \( l \):

\[
\text{InstInd}(a) = \frac{10}{l} \sum_{i=1}^{l-1} \text{DIWV}(a_i, a_{i+1}).
\]

pI of a peptide is characterized as the pH-value at which a peptide has a net charge of zero. A peptide has its lowest solubility at its pI. Therefore, the charge of a peptide influence the solubility in aqueous solutions. The pI value is calculated as follows: Firstly, the net charge for \( \text{pH} = 7.0 \) is determined. If this charge is positive, the pH at \( 7 + 3.5 \) is calculated; otherwise the pH at \( 7 − 3.5 \) is determined. This process is repeated until the modules of the charge is less or equal 0.0001.

\( aI \) of a peptide is characterized as the relative volume occupied by aliphatic side chains consisting of the amino acids alanine (Ala), valine (Val), isoleucine (Ile) and leucine (Leu). \( aI \) is regarded as a positive factor for the increase of thermostability. \( aI \) is calculated according to the formula:

\[
aI = X(\text{Ala}) + d \cdot X(\text{Val}) + e \cdot (X(\text{Ile}) + X(\text{Leu})),
\]

where \( X(\text{Ala}), X(\text{Val}), X(\text{Ile}) \) und \( X(\text{Leu}) \) are more percent of the amino acids. The coefficients \( d \) and \( e \) are the relative volume at the valine side chain \( (d = 2.9) \) and Ile side chains \( (e = 3.9) \) to the side chain Ala.

These six objective functions comparatively act to reflect the similarity of a particular peptide and a pre-defined reference peptide: \( f(Candidate\text{Pept.}) := \left| f(Candidate\text{Pept.}) - f(Reference\text{Pept.}) \right| \). Therefore, the four objective functions have to be minimized and the optimization problems are minimization problems. Furthermore, the objective values are normalized by the theoretical maximal value of each objective: \( f_k(x_k) = \frac{f_k(x_k)}{\text{Max}_k} \) for the k-objectives.

4.2 Performance Metrics

Two statistical metrics are chosen to evaluate the convergence and diversity performance. These metrics are applied on 10% approximately optimal individuals in each iteration for all algorithms. These optimal individuals are determined by WS in all test cases. The value of 10% optimized individuals is a parameter motivated by the number of peptides selected for subsequent laboratory analysis and therefore motivated by the practical application. Furthermore, the
Table 2: Physiochemical functions of the different optimization problems.

<table>
<thead>
<tr>
<th>dim.</th>
<th>abbr.</th>
<th>objective functions</th>
</tr>
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<tbody>
<tr>
<td>3D</td>
<td>3D-MOP</td>
<td>NMW, MW, Hydro</td>
</tr>
<tr>
<td>4D</td>
<td>4D-MaOP</td>
<td>NMW, MW, Hydro, InstInd</td>
</tr>
<tr>
<td>5D</td>
<td>5D-MaOP</td>
<td>NMW, MW, Hydro, InstInd, pI</td>
</tr>
<tr>
<td>6D</td>
<td>6D-MaOP</td>
<td>NMW, MW, Hydro, InstInd, pl, al</td>
</tr>
</tbody>
</table>

The Average Cuboid Volume (ACV) is used to measure the convergence behavior (Rosenthal and Borschbach, 2017a). ACV calculates the averaged spanned space of each solution to an ideal reference point, which is usually known in real-world applications. The ACV indicator is given by

\[ ACV = \frac{1}{n} \sum_{j=1}^{n} \left( \prod_{j=1}^{k} (x_{ij} - r_j) \right), \] (1)
where $n$ is the number of individuals that are evaluated, $k$ the number of objectives and $r_j$ the ideal point. The lower the ACV values, the better the convergence behavior since the molecular optimization problems have to be minimized. ACV as a simple statistical measure is preferred over traditional convergence metrics since it is independent of Pareto optimal solution sets which are usually unknown in real-world applications, of low computation cost, independent of the problem dimension and relative to the number of solutions allowing a comparison of differently sized archive sets.

A simple statistical evaluation method is used to compare the diversity performance. The diversity is determined by the standard deviation of the solution set to the gravity point of this set.

### 4.3 Experimental Results

The performance results of WS-COSEA-MO, dWS-COSEA-MO and AnD on 3D-MOP are depicted in Figure 2, 4 and 6, the results of 4D-MaOP are shown in Figure 3, 5 and 7, the results of 5D-MaOP are presented in Figure 8, 10 and 12 and those of 6D-MaOP in Figure 9, 11 and 13. Generally, WS-COSEA-MO and dWS-COSEA-MO provide a continuous con-
vergence improvement within 10 iterations, whereas AnD does not provide any convergence behavior in this low number of iterations in these molecular optimization problems except for a slight improvement in 6D-MaOP, though these ACV values are worse compared to those of the winning score-based algorithms. Especially in the case of WS-COSEA-MO, an exponential convergence improvement is observable. As a consequence of the missing convergence, AnD provides the highest diversity values in all test cases. Moreover, AnD has the highest standard deviation values of ACV and diversity indicating an highly varying performance. Comparing dWS-COSEA-MO and WS-COSEA-MO, it is observable that the traditional winning score-based algorithm provides a better convergence behavior and the diversity is of a higher level as well in all test cases.

The archive sizes of WS-COSEA-MO and dWS-COSEA-MO after 10 iterations in each test case have also been examined. Generally, the archive sizes of both algorithms are in the same range of a 95% confidence interval between 32 and 47. The mean of the archive sizes of both algorithms is the same, $\text{avg} = 39$. Figure 14 depicts the archive performance of WS-COSEA-MO and dWS-COSEA-MO after 10 iterations. As is has been expected from the previous results, WS-COSEA-MO provides higher qualified solutions than dWS-COSEA-MO, whose diversity is better compared to dWS-COSEA-MO as well.

Since AnD does not converge within this low number of 10 iterations, further experiments have been applied with a higher iteration number of 100. In the case of 3D-MOP to 5D-MaOP, no convergence is observable within the 100 iterations, whereas convergence behavior is observable in the case of 6D-MaOP. As AnD has not been applied to optimization problems with less than 5 objectives so far, this allows the hypothesis that AnD is only suitable for optimization problems with a higher dimension number.

### 5 CONCLUSION

This work presents an enhancement of COSEA-MO that is especially evolved for multi-objective molecular optimization addressing the application-specific condition of identifying highly qualified candidate peptides while limiting the number of objective function evaluations to save resources. COSEA-MO is enhanced by WS-based selection mechanism. Two types of winning scores are used to and differentiate the individuals of a population effectively in multi- and many-objective molecular optimization problems. The performance of WS-COSEA-MO and dWS-COSEA-MO is compared to the recently proposed and promising AnD in terms of convergence, diversity and exemplary analysis of the selection principles. AnD is chosen for benchmarking as it has a similar properties compared to COSEA-MO and is the only MaOEA apart from COSEA-MO that has a simple framework structure, provides optimized default parameters for the non-expert use and is independent of weight vectors or reference points, which usually have a strong impact on the performance and are usually unknown in real-world applications. WS-COSEA-MO reveals superior performance in terms of convergence and diversity in all test cases. It complies the problem-specific requirement of allocating an evolutionary advantage to boundary solutions. WS-COSEA-MO provides exponential convergence improvement within the very low number of 10 iterations. Otherwise, AnD does not reveal any convergence behavior within this low number of iterations, since diversity is the preference objective of this evolutionary process at the cost of convergence.

In future research, the selection process is analyzed regarding to the important biochemical objective of genetic dissimilarity among the candidate peptides. The improvement of dWS as part of the selection mechanism is in the focus for the specification of the search process. Furthermore, the prioritization of different objectives in the evolutionary process is in the focus and suitable method for this purpose will be addressed. Moreover, since descriptor-based fitness functions are missing for diverse but important molecular properties and the evaluation time of these molecular properties have to be reduced, the proposed approach will be revised by surrogate-assisted principles (Diaz-Manriquez et al., 2016) as pre-screening techniques in advance of the expensive laboratory analysis for further improvement.
REFERENCES


