Analysis of a Business Environment using Burstiness Parameter: The Case of a Grocery Shop

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Abstract: Nowadays, bursty business processes are part of our everyday life. Bursty business processes include such processes as selling and buying, too. One of the contemporary challenges business environment has to deal with is monitoring and controlling of burstiness in business processes. Monitoring and controlling of burstiness in business processes often leads to the optimization of business processes. Validation of the model for analysing buyers' burstiness in business processes revealed the need in optimisation of the proposed model, as the elaborated model based on gap processes is complex for implementation, as well as for parameter estimation. For optimization of the model for analysing buyers' burstiness in business process, different levels of burstiness in the process of buying are studied in this work. Different approaches to modelling buyers' behaviours are presented and evaluated in this work, too. The novel contribution of this work is based on the estimation of burstiness. With the proposed solution the level of burstiness can be estimated by taking the mean value and the standard deviation of a gap sequence into account, which always exists for a given sequence. As a practical application, the cash register of a medium size grocery shop in Lithuania is analysed. The novelty of this paper is given by the comparison of different approaches to measuring burstiness in real process data. Directions of further research are proposed.

1 INTRODUCTION

Nowadays, bursty business processes or, in other words, business environment are part of our everyday life. Traffic flow in cities is bursty, data traffic implies to be of a bursty nature, customers' flow in shops are not static, too, as shown in Fig. 1.

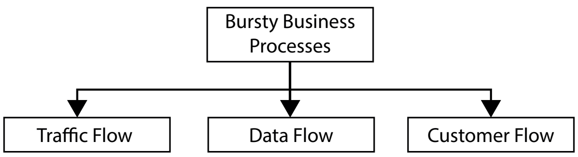

Figure 1: A range of bursty business processes in everyday life.

Bursty business processes include such processes as selling and buying, as shown in Fig. 2. One of the contemporary challenges business environment has to deal with is monitoring and controlling of burstiness in business processes. Monitoring and controlling of burstiness in business processes often leads to the optimization of business processes. Burstiness

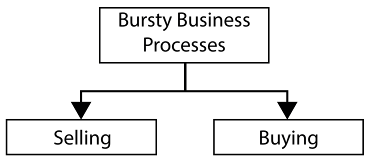

Figure 2: The inter-relationship between bursty business processes as well as selling and buying.

has attracted a lot of attention starting with the modelling of bursts or bundles of bit-errors in telecommunications. Such investigations have led to intensive research for simulation models which are able to take the bursty characteristic of bit-errors into account such as (Gilbert, 1960) or (Elliott, 1963). Similar dependencies can be found in data networks regarding the characteristics of the traffic (e. g. the temporal intervals between consecutive data packets) (Kessler

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Table 1: Burstiness in different scientific fields.

Scientific field	Phenomenon of burstiness
Telecommunications	Burstiness of bit-errors in data transmission
Economics	Burstiness of crises
Natural sciences	Burstiness of disasters or earthquakes
Logistics	Burstiness of traffic
Social media	Burstiness of hot topic, keyword or event
Business	Burstiness of workload
Business	Burstiness of buyers

et al., 2003; Feldmann, 2000).

Furthermore, the phenomenon of burstiness was revealed in a range of scientific fields such as economics, natural sciences, logistics and business. Tab. 1 demonstrates the phenomenon of burstiness in a range of scientific fields.

In business, burstiness is based on visitor-buyer relationship as illustrated in Fig. 3. The visitor-buyer relationship implies binary customer behaviour such as buying or not buying. A visitor becomes with the probability p_e a buyer (also referred as buyer probability) and remains with the probability $(1 - p_e)$ a visitor.

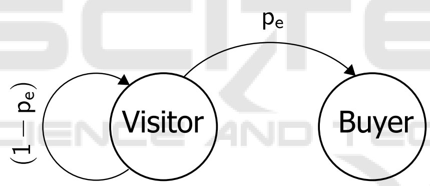

Figure 3: Visitor-Buyer Relationship.

However, these models do not take the concentration of buyers into account as highlighted Fig. 4.

```
xx-xxx--xxxxxx-xx------------
-------xxxx-x--xx--xx--------
-----------------------------
-----------------------------
-----------------------------
-----------------------------
-----------------------------
-------------------------xxxx
-----------------------------
-----------------------------
-----------------------------
-----------------xx----------
-----------------------------
--------------xxxx-----x--xx-
-----------------------------
-----------------------------
----xxx----xxx---------------
-----------------------------
-------xxx---------------xx--
-----------------------------
xxxxx-----------xx--xxxxxx-x-
-----------------------------
---xxxxxx--------------------
```

Figure 4: Buyers' burstiness (represented by "x") within a sequence of shop visitor (represented by "-").

In (Ahrens and Zaščerinska, 2017) a model for analysing buyers' burstiness in business processes has been presented. The model shows that the process of buying can be described by the buyers' probability. However, in order to be able to describe the bursty nature of buyers a second parameter such as the buyers' concentration is needed.

The optimization of bursty business processes requires, on the one hand, appropriate simulation models and, on the other hand, algorithms for estimating burstiness in business processes in order to be able to optimize process systems such as queuing at the cash register in a shop.

Optimization of business processes also depends on a level of burstiness. Hence, an issue is the measurement of burstiness. A couple of approaches to the measurement of burstiness exist. The F (Fano) factor as well as burstiness factor $(1 - \alpha)$ (also referred as parameter in the present research) are widely used to estimate a level of burstiness (Ahrens et al., 2019b).

In this work the level of burstiness in the process of buying is studied. As a practical application, the cash register of a medium size grocery shop in Lithuania is analysed. The proposed solution of burstiness estimation takes the mean value and the standard deviation of real data into account and avoids the complex estimation of distribution or density functions.

The novelty of this paper is given by the comparison of different approaches to measuring burstiness in real process data.

The remaining part of this paper is organized as follows: Section 2 introduces the theoretical basis for modelling buyers' behaviour. A mathematical model for describing buyers' burstiness via gap processes is presented in Section 3. The estimation of burstiness is introduced in Section 4. The associated results of an empirical study of a medium size grocery shop in Lithuania are discussed in Section 5. Finally, some concluding remarks are provided in Section 6.

2 THEORETICAL BASIS

In this section the theoretical basis for describing business processes with independent events of buyers (i. e. the buyers appear independently from each other) is given. In general, any process including the process of buying in which binary decisions are made can be described by gaps as illustrated in Fig. 5 (Ahrens et al., 2019a). Once the process is modelled

```
-x--x--x---x x----x-
   2  2  3  0  4
```

Figure 5: Modelling of the buying process by gaps (a buyer (represented by "x") within a sequence of non-buying visitors (represented by "-")).

by gaps, a gap distribution function $u(k)$ defining the probability that a gap Y between two buyers is greater than or at least equal to a given number k, i. e.

$$u(k) = P(Y \geq k) \tag{1}$$

can be defined within the following boundaries

$$u(0) = 1 \quad \text{and} \quad \lim_{k\to\infty} u(k) = 0 \ . \tag{2}$$

Next to $u(k)$ a gap density function $v(k)$ defining the probability that a gap Y between two buyers is equal to a given number k, i. e.

$$v(k) = P(Y = k) \tag{3}$$

can be defined. Taking $v(k)$ into account, the function $u(k)$ results in

$$u(k) = v(k) + v(k+1) + v(k+2) + \cdots \ . \tag{4}$$

For situations with independent events, i. e. buyers, $u(k)$ can be defined, as a function of the buyers' probability p_e, as follows

$$u(k) = (1-p_\mathrm{e})^k = \overline{p}_\mathrm{e}^k \ . \tag{5}$$

Equation (5) is well-known in probability theory for independent events and is valid for any buyer probability p_e. Therein, the probability of non-buying visitors is defined as

$$\overline{p}_\mathrm{e} = \frac{\text{Number of Visitors - Number of Buyers}}{\text{Number of Visitors}} \ . \tag{6}$$

With (5) the probability can be derived that $\geq k$ consecutive visitors are non-buying visitors.

By calculating the average gap length $\mathbf{E}(Y)$, the interrelation between $u(k)$ and p_e becomes visible. Here, we get

$$\mathbf{E}(Y) + 1 = \frac{1}{p_\mathrm{e}} \ . \tag{7}$$

Calculating the sum of $u(k)$, we receive

$$\sum_{k=0}^{\infty} u(k) = u(0) + \sum_{k=1}^{\infty} u(k) = 1 + \sum_{k=1}^{\infty} u(k) \ . \tag{8}$$

With (4), the expression can be re-written as

$$\sum_{k=0}^{\infty} u(k) = 1 + \sum_{k=1}^{\infty} k \cdot v(k) \tag{9}$$

and the calculation of the average gap length $\mathbf{E}(Y)$ becomes

$$\mathbf{E}(Y) = \sum_{k=0}^{\infty} k \cdot v(k) = \sum_{k=0}^{\infty} u(k) - 1 \ . \tag{10}$$

The function $v(k)$ can be decomposed with (4) as

$$v(k) = u(k) - u(k+1) \ . \tag{11}$$

Combining (7) and (10), we get

$$\sum_{k=0}^{\infty} u(k) = \frac{1}{p_\mathrm{e}} \ . \tag{12}$$

By taking (5) into account, (12) can be verified as follows

$$\sum_{k=0}^{\infty} u(k) = \sum_{k=0}^{\infty} (1-p_\mathrm{e})^k = \frac{1}{1-(1-p_\mathrm{e})} = \frac{1}{p_\mathrm{e}} \ . \tag{13}$$

Furthermore, defining a given interval n with at least one buyer, the block buyer probability $p_\mathrm{B}(n)$ is formulated as

$$p_\mathrm{B}(n) = 1 - (1-p_\mathrm{e})^n \tag{14}$$

and can be approximated for small p_e as follows

$$p_\mathrm{B}(n) \approx 1 - (1 - n\,p_\mathrm{e}) = n\,p_\mathrm{e} \tag{15}$$

with

$$\lim_{n\to\infty} p_\mathrm{B}(n) = 1 \ . \tag{16}$$

The block buyer probability $p_\mathrm{B}(n)$ results from the probability of having no buyers in a block of the length n, defined as $(1-p_\mathrm{e})^n$.

In a double-logarithmic representation the linear relationship between $log_{10}(p_\mathrm{B}(n))$ and $log_{10}(n)$ becomes for $p_\mathrm{B}(n) \leq 1$ evident. Here we get

$$\log_{10}(p_\mathrm{B}(n)) = \log_{10}(n) + \log_{10}(p_\mathrm{e}) \ . \tag{17}$$

as it was confirmed by practical measurements (Wilhelm, 1976).

Furthermore, the block buyer probability $p_\mathrm{B}(n)$ can be calculated by taking the distribution function $u(k)$ into account as shown in Fig. 6 and results in

$$p_\mathrm{B}(n) = p_\mathrm{e} \cdot \sum_{k=0}^{n-1} u(k) \tag{18}$$

Together with $p_\mathrm{B}(n) = p_\mathrm{e}\,n$, derived in (15), we get

$$\sum_{k=0}^{n-1} u(k) = n \ . \tag{19}$$

The searched distribution $u(k)$ can now be obtained iteratively

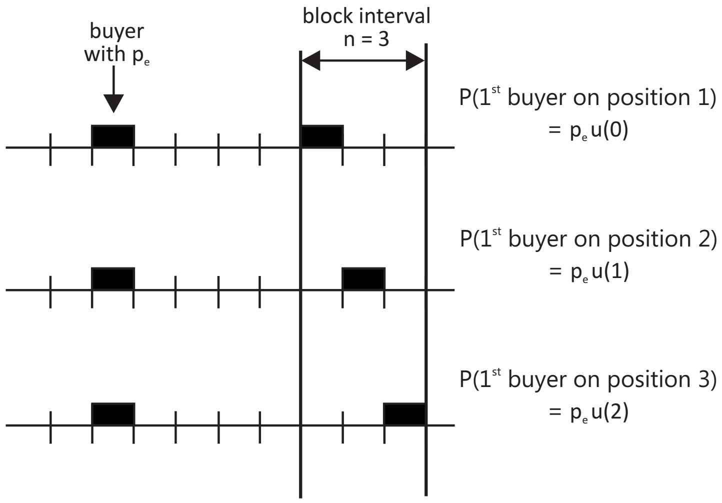

Figure 6: Calculating the block-buyer probability $p_{\mathrm{B}}(n)$ using the gap distribution function $u(k)$.

$$
\begin{array}{llll}
n=1 & : & u(0) & = 1 \\
n=2 & : & u(0)+u(1) & = 2 \\
n=3 & : & u(0)+u(1)+u(2) & = 3 \\
\cdots & : & \cdots & = \cdots \\
n & : & u(0)+u(1)+\cdots+u(n-1) & = n
\end{array}
$$

and formulated as

$$u(k) = (k+1) - k = 1 \ . \tag{20}$$

In order to fulfil (2) and (12), the function $u(k)$, defined in (20), can be multiplied by

$$(1-p_{\mathrm{e}})^k \approx \mathrm{e}^{-p_{\mathrm{e}} \cdot k} \tag{21}$$

resulting in (5).

The resulting block buyers' probability $p_{\mathrm{B}}(n)$ is highlighted in Fig. 7 when analysing the differences in (15) and (18). The linear dependency between $log_{10}(p_{\mathrm{B}}(n))$ and $log_{10}(n)$ for $p_{\mathrm{B}}(n) \leq 1$ forms a basis for buyers' simulation model.

3 DESCRIPTION OF BURSTY BUSINESS PROCESSES

The bursty business processes have to consider that the block buyer probability decreases for a given *n* as the buyers become more and more concentrated. By introducing a buyers' concentration $(1-\alpha)$ as shown in (Ahrens, 2000) and (Ahrens and Zaščerinska, 2016) the block buyer probability can be approximated as

$$p_{\mathrm{B}}(n) = p_{\mathrm{e}}\, n^{\alpha} \tag{22}$$

for any interval with $p_{\mathrm{B}}(n) \leq 1$. In the double-logarithmic representation we get

$$\log_{10}(p_{\mathrm{B}}(n)) = \alpha \log_{10}(n) + \log_{10}(p_{\mathrm{e}}) \ . \tag{23}$$

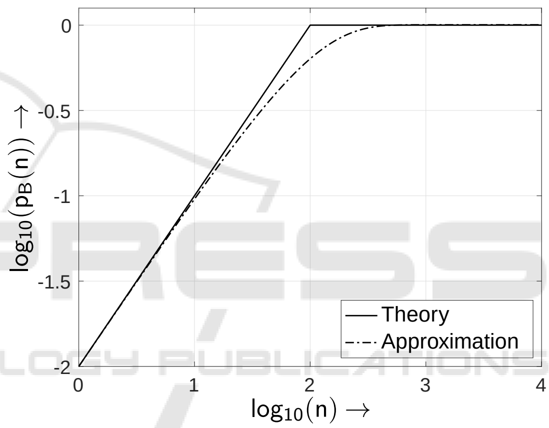

Figure 7: Approximated block buyer's probability $p_{\mathrm{B}}(n)$ as a function of the interval length *n* for $(1-\alpha) = 0$ at a buyer's probability of $p_{\mathrm{e}} = 10^{-2}$.

with the parameter α defining the gradient of the line. Practically relevant buyers' concentrations are in the range of $0 < (1-\alpha) \leq 0.5$, whereas a buyers' concentration of $(1-\alpha) = 0$ describes the beforehand studied situation with independent buyers. Using (18), the distribution function $u(k)$ results in

$$
\begin{array}{llll}
n=1 & : & u(0) & = 1 \\
n=2 & : & u(0)+u(1) & = 2^{\alpha} \\
n=3 & : & u(0)+u(1)+u(2) & = 3^{\alpha} \\
\cdots & : & \cdots & = \cdots \\
n & : & u(0)+u(1)+\cdots+u(n-1) & = n^{\alpha}
\end{array}
$$

and can be calculated as

$$u(k) = (k+1)^{\alpha} - k^{\alpha} \ . \tag{24}$$

In order to satisfy the condition

$$\lim_{k\to\infty} \sum_{\kappa=0}^{k} u(\kappa) = \frac{1}{p_{\mathrm{e}}} \tag{25}$$

(24) can be multiplied with the asymptote $e^{-\beta \cdot k}$ with

$$\lim_{k\to\infty} e^{-\beta\cdot k} = 0 \ . \tag{26}$$

By multiplying $u(k)$ with the factor $e^{-\beta\cdot k}$, the parameter β has to be calculated in order to fulfil the condition

$$\sum_{k=0}^{\infty}\left[(k+1)^{\alpha}-k^{\alpha}\right]\cdot e^{-\beta\cdot k} = \frac{1}{p_e} \ . \tag{27}$$

Taking the series expansion of the expression

$$(k+\Delta k)^{\alpha} = k^{\alpha}\,(1+\frac{\alpha}{k}\,\Delta k+\ldots) \tag{28}$$

into account, the expression (24) simplifies with $\Delta k = 1$ to

$$(k+1)^{\alpha}-k^{\alpha} \approx \alpha\cdot k^{\alpha-1} \ . \tag{29}$$

Using the integral instead of the sum, the following equation has to be solved in order to determine the parameter β. Here we get

$$U = \alpha\int_{0}^{\infty} k^{\alpha-1}\,e^{-\beta\cdot k}\,\mathrm{d}k = \frac{\alpha\,\Gamma(\alpha)}{\beta^{\alpha}} \quad , \tag{30}$$

with the parameter $\Gamma(\cdot)$ describing the Gamma function. With the approximation

$$\alpha\,\Gamma(\alpha) \approx 1 \tag{31}$$

we get

$$\sum_{k=0}^{\infty}\left[(k+1)^{\alpha}-k^{\alpha}\right]\cdot e^{-\beta\cdot k} = \frac{1}{\beta^{\alpha}} \ . \tag{32}$$

Together with (27) the following approximation for parameter β has been found

$$p_e \approx \beta^{\alpha} \ . \tag{33}$$

For bursty buying processes, the buyers' gap distribution function results in

$$u(k) = \sum_{k=0}^{\infty}\left[(k+1)^{\alpha}-k^{\alpha}\right]\cdot e^{-\beta\cdot k} \ . \tag{34}$$

For independent buyers, i. e. $\alpha = 1$, the parameter β equals the buyer probability p_e as derived in section 2.

Fig. 8 demonstrates the buyers' gap distribution functions. With increasing buyers' concentration $(1-\alpha)$, the appearance of gaps of shorter lengths increases whereas at the same time the probability of longer gaps decreases.

Fig. 9 shows the calculated block buyers' probabilities $p_B(n)$ as a function of the interval length n for different parameters of the $(1-\alpha)$ at buyers' probability of $p_e = 10^{-2}$. With increasing buyers' concentration, the buyers appear more and more concentrated and block buyer's probability $p_B(n)$ decreases for a given n as stated before. Furthermore, the linear dependence between $log_{10}(n)$ and $log_{10}(p_B(n))$ becomes obvious for small n.

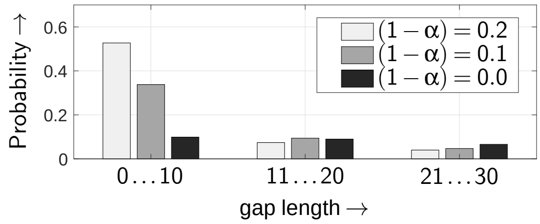

Figure 8: Probability of different gap length for different parameters of the $(1-\alpha)$ at a buyer's probability of $p_e = 10^{-2}$.

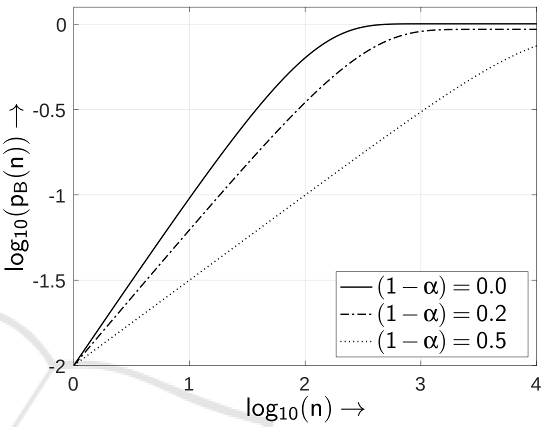

Figure 9: Approximated block buyer's probability $p_B(n)$ as a function of the interval length n for different parameters of the $(1-\alpha)$ at a buyer's probability of $p_e = 10^{-2}$.

4 ESTIMATION OF BURSTINESS

The practical evaluation of bursty buying processes requires the calculation of the level of burstiness. A possible approach can be defined by the estimation of the buyers' concentration $(1-\alpha)$ as shown in (Ahrens and Zaščerinska, 2017). For this, the gap density function $v(k)$ has to be analysed. Analysing the probability that after a buyer, in the distance of zero another buyer appears, i. e. $v(0) = u(0) - u(1)$, the buyers' concentration $(1-\alpha)$ can be analysed by taking (34) into account. Here we ge with $u(0) = 1$

$$v(0) = 1 - u(1) = 1 - \left[(2^{\alpha}-1)\,e^{-\beta}\right] \ . \tag{35}$$

With the assumption of small *beta* the expression can be simplified as follows

$$e^{-\beta} \approx 1 \quad \text{for} \quad \beta \ll 1 \ , \tag{36}$$

and the parameter $v(0)$ simplifies to

$$v(0) \approx 2 - 2^{\alpha} \ . \tag{37}$$

From this equation, the buyers' concentration $(1-\alpha)$ is estimated as

$$(1-\alpha) \approx 1 - \log_2\left[2 - v(0)\right] \ . \tag{38}$$

However, it requires the calculation of the gap density function $v(k)$. In comparison to (38), Goh & Barabasi (Goh and Barabási, 2008) provided an alternative solution for estimating burstiness in business processes. In (Goh and Barabási, 2008) burstiness is defined by taking the mean value m (average gap length or average length of a time interval between two buyers) as well as the standard deviation σ of the length of time intervals or gaps into account. The definition by Goh & Barabasi (Goh and Barabási, 2008) results in

$$B = \frac{\sigma - m}{\sigma + m} \quad . \tag{39}$$

with $-1 \leq B \leq 1$.

Goh & Barabasi pointed out that $B = 1$ corresponds to a bursty environment whereas $B = 0$ is referred to a neutral environment. Regular (periodic) signals are described by negative parameters of B. Analysing a buying process with independent buyers, i. e. $(1-\alpha) = 0$, the gap distribution function $u(k)$ results in

$$u(k) = \mathrm{e}^{-p_\mathrm{e} \cdot k} \quad . \tag{40}$$

Taking the gap density function $v(k) = u(k) - u(k+1)$ into account, the mean value $m = \mathbf{E}(Y)$ can be calculated as follows

$$m = \sum_{k=0}^{\infty} k\, v(k) = \sum_{k=0}^{\infty} k \left(\mathrm{e}^{-p_\mathrm{e} \cdot k} - \mathrm{e}^{-p_\mathrm{e} \cdot (k+1)}\right) \tag{41}$$

and results in

$$m = \frac{\mathrm{e}^{-p_\mathrm{e}}}{1 - \mathrm{e}^{-p_\mathrm{e}}} \quad . \tag{42}$$

Together with the standard deviation

$$\sigma = \sqrt{\mathbf{E}(Y^2) - m^2} = \frac{\mathrm{e}^{-p_\mathrm{e}/2}}{1 - \mathrm{e}^{-p_\mathrm{e}}} \tag{43}$$

the parameter B results in

$$B = \frac{\sigma - m}{\sigma + m} = \frac{\mathrm{e}^{p_\mathrm{e}/2} - 1}{\mathrm{e}^{p_\mathrm{e}/2} + 1} \quad . \tag{44}$$

Fig. 10 shows the dependence of the parameter B on the buyers' probability p_e. As shown by Goh & Barabasi the parameter B is close to zero indicating the independence of the buyers. However, the dependence of the parameter B on the buyers' probability shows the weakness of the burstines definition as independent buyers' scenarios are solely described by the buyer probability. On the other hand the parameter B can be easily calculated for an empirically obtained gap sequence.

Taking different parameters of the buyers' concentration $(1-\alpha)$ into account, Fig. 11 shows the obtained values for the parameter B. As obtained by computer simulations, the parameter B can be used

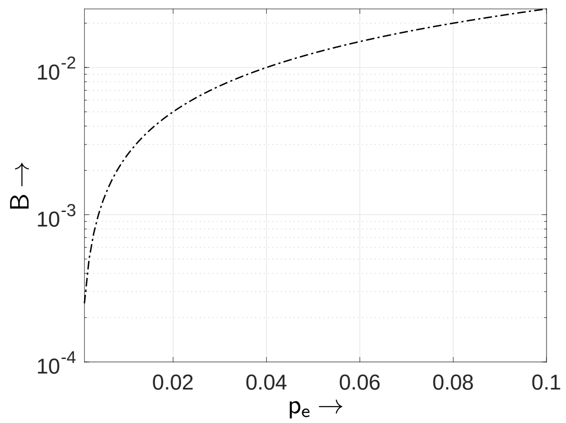

Figure 10: Dependence of the parameter B on the buyer's probability p_e for independent buyers.

as an indicator regarding the level of burstiness. According to Fig. 11 a rough estimation leads to the following condition

$$B \approx (1-\alpha) \quad . \tag{45}$$

Unfortunately, the plot depicted in Fig. 11 shows that the parameter B depends on the buyers' probability p_e and buyers' concentration $(1-\alpha)$, i. e.

$$B = B(p_\mathrm{e}, (1-\alpha))$$

is used as an indicator for the expected buyers' concentration.

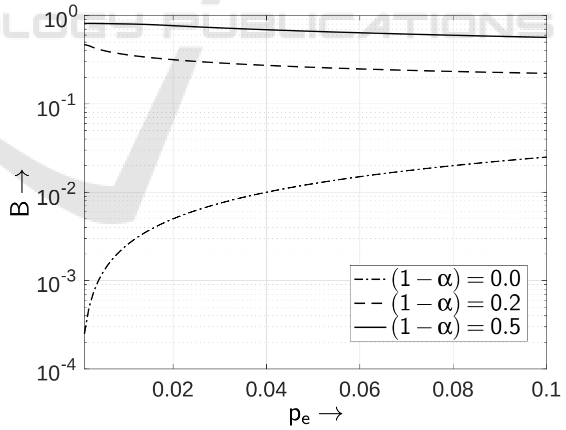

Figure 11: Dependence of the parameter B on the buyer's probability p_e for different parameters of the buyers' concentration $(1-\alpha)$.

5 PRACTICAL APPLICATION

In real world business processes, the probability p_e of visitor to buy a good as well as the correlation between buyers, described by the buyers' concentration $(1-\alpha)$, may be not available. The solution is to use a statistical approach to estimate the buyers' concentration $(1-\alpha)$.

In this section the service of the buyers at the cash register as a practical example of bursty processes is analysed. In this example, the duration of the service of the buyers at the cash register of a medium size grocery shop in Lithuania is studied. The cash register data collected contain the data about the operation time, the amount of goods purchased, their codes, and the prices paid by each buyer. The data collection was carried out in June 2018. At that time 2575 buyers were served.

Unfortunately, the cash registers do not record the start time of the operation. Therefore, the service duration time was not available from the database. To cope with this problem we observed buyers' service durations with different quantities of goods (see Tab. 2). It appeared, that the service duration t_s depends not only on the quantity of the goods, but also on the type of goods, individual characteristics of the buyer and other random factors, i. e. the dependence is statistical.

Table 2: Duration of the service at the cash register.

Amount of Goods g	Service Time t_s (in s)
3	44
1	18
10	30
1	11
18	61
1	37

The correlation coefficient between g and t_s equals $0,72$ and the regression equation is given by

$$t_s = 1{,}9g + 22{,}8 \quad . \tag{46}$$

The equation yields that for one good about 1,9 seconds and additionally about 22,8 seconds for each buyer are required.

Knowing the quantity of goods and (46), the start and end times of each buyer can be calculated. This allows us to analyse the free time between two buyers' service, if any. When it appeared that there was no free time interval between two or several buyers' service, then the sequential service times were merged into one continuous service time interval.

Therefore, it was possible to investigate service duration times and time gaps between successive buyers. It appeared that the average service duration is 37 seconds and the most frequent duration takes 32 seconds. The distribution of service duration time is given in Fig. 12.

Similarly, the duration of free time intervals (gaps) can be processed. The average free time interval (a gap) equals $m = 234$ s, i. e. 4 minutes and the standard deviation $\sigma = 620$ s. The histogram is given in Fig. 13.

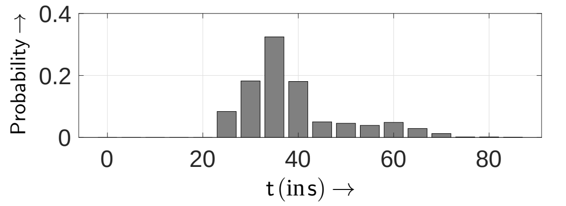

Figure 12: Distribution of service duration times.

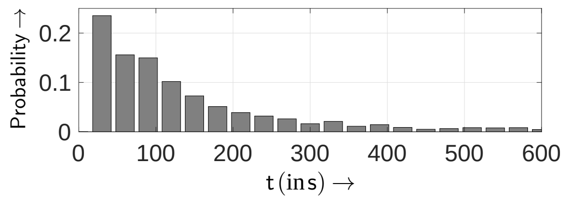

Figure 13: Distribution of free times of cash register (grouped).

This histogram is different from the histogram of service times. The frequencies of free times (gaps of various lengths) are constantly decreasing while it is not true for histogram of service times given in Fig. 12.

The small mode of free time (gap lengths) and the histogram of free times testifies that the time gaps between services usually are short. The histogram of free times' durations up to 30 seconds, i. e. half minute, reveals that these durations are distributed quite similarly (see Fig. 14.). One of the measures of burstiness is the parameter B defined in (39). Applying this formula to free times (gaps), it yields that $B = 0,45$. Therefore, the free times between services are quite bursty. On the other hand, the burstiness level of free times up to 30 seconds states that this process is close to neutral. The high burstiness of free times during the whole month was determined by the longer free time at the beginning and end of the shop open time.

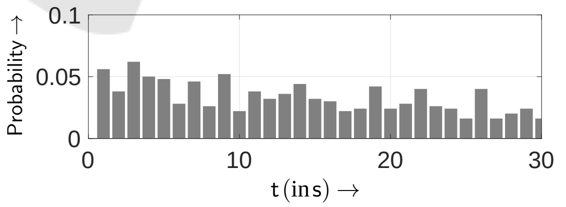

Figure 14: Distribution of free time duration up to 30 seconds.

6 CONCLUSIONS

In this work, on the example of the cash register of a medium size grocery shop in Lithuania, different approaches to estimation burstiness are presented and analysed. The proposed solution of burstiness estimation takes the mean value and the standard deviation into account and avoids the complex estimation of distribution or density functions.

The discussed probabilistic models and their approximations of business processes can be evaluated by the burstiness parameter B. It revealed, the burstiness is positive, i. e. between neutral and bursty process in the investigated case of a grocery shop in Lithuania.

In real world business processes, the probability p_{e} of visitor to buy a good as well as the buyers' concentration $(1-\alpha)$ may be not available. Nevertheless, it is possible to process statistically the cash register data. Usually the cash equipment just registers one time moment of the service of the buyer and number of goods and their codes in the basket, but not the service duration. Therefore, the shop's database does not contain lengths of busy intervals and lengths of free time intervals. The solution of the problem is an additional observation of the cashier's work, the registration of the number of goods in the buyer's basket and the service time of the basket. Then the regression equation between the number of goods in the basket and service duration was derived. Using this equation, it becomes possible to estimate the service time lengths, to compute the free times and to apply statistical analysis including calculation of burstiness parameter.

Our plan on the future research is to investigate the interrelationship between business process and visitor decisions influenced by the behaviour of other visitors and buyers.

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