Distributed Multi-objective Particle Swarm Optimization using Time-delayed Virtual Global Best Method

Yuji Sato, Shota Ueno and Toshio Hirotsu
Department of Computer and Information Sciences, Hosei University, Tokyo, Japan

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Abstract: To reduce the computational cost of particle swarm optimization (PSO) methods, research has begun on the use of Graphics Processing Units (GPUs) to achieve faster processing speeds. However, since PSO methods search based on a global best value, they are hampered by the frequent need for communication with global memory. Even using a standard PSO that uses a local best value does not solve this problem. In this paper, we propose a virtual global best method that speeds up computations by defining a time-delayed global best as a virtual global best in order to reduce the frequency of communication with low-speed global memory. We also propose a method that combines decomposition-based multi-objective PSO (MOPSO/D) with a virtual global best method to speed up multi-objective particle swarm optimization by running it in parallel while maintaining search accuracy, and we demonstrate the effectiveness of this approach by using a number of unimodal/multimodal single objective benchmark test functions and three classical benchmark test functions with two objectives.

1 INTRODUCTION

Particle swarm optimization (PSO) is an optimization algorithm that models the behavior of flocks of birds and schools of fish (Kennedy, 1995), and which is one of the population based non-deterministic optimization algorithms. There is a personal best and global best and both affect the direction in which particles are moving. PSO has attracted attention as a useful optimization tool due to its simple algorithm and other attributes. However, a problem with this algorithm is that when the number of particles is increased in order to deal with a more complex objective function, the computational cost also increases, resulting in longer search times. There have already been studies aimed at preventing this issue of increased search times by using GPUs to distribute the computational load (Zhou, 2009; Souza, 2011; Hussain, 2016). A study that uses parallel acceleration of PSO on a GPU based on a master-slave model to solve multi-objective problems has also been reported (Cao, 2017; Hussain, 2018).

However, since these previous studies use global memory to store particle swarm data, they are unable to achieve adequate speed increases even when running many cores in parallel. Alternatively, if attempts are made to speed up processing based on a master-slave model, the efficiency of processing speed improvements is liable to deteriorate as the number of cores increases. To address these issues, this paper proposes a method that can improve computation speeds by defining a time-delayed global best as a virtual global best for each streaming multiprocessor (SM) in order to reduce the frequency of communication with low-speed global memory. Next, compared with single-objective optimization where it is important to converge on just one optimal solution, in multi-objective optimization it is necessary to increase the speed of computation while maintaining diversity in the search process as well as convergence on the Pareto front. However, there is often a trade-off between convergence on the Pareto front and maintenance of diversity, and this problem cannot be solved simply by operating the proposed method in parallel. Therefore, we propose running a multi-objective particle swarm optimization method in parallel by combining a virtual global best method and decomposition-based multi-objective PSO (MOPSO/D) (Peng, 2008), which is capable of global searching, and we use benchmark problems to demonstrate the effectiveness of this approach.

2 OVERVIEW OF PARTICLE SWARM OPTIMIZATION

2.1 Original PSO

In original PSO, each particle has a position vector and a velocity vector that are used to calculate the evaluation values and the positions to which these particles move. After using the position vectors to calculate the evaluation value at the current position of each particle, the velocity vectors and position vectors are updated according to the following formula:

$$v(t+1) = v(t) + c_1 * rand_1 * \left(P_{bestx} - x(t)\right) + c_2 * rand_2 * \left(G_{bestx} - x(t)\right) \tag{1}$$

$$x(t+1) = x(t) + v(t+1) \tag{2}$$

where $v(t)$ and $v(t+1)$ are the velocity vectors in generations t and $t+1$ respectively, and $x(t)$ and $x(t+1)$ are the corresponding position vectors. P_{bestx} represents the personal best which is the best solution found by each particle up to generation t. G_{bestx} is the global best, which is the current best position found by the whole group. The coefficients c_1 and c_2 are positive constants, and rand1 and rand2 are random variables in the range from 0 to 1. After each particle has been moved, the calculation of evaluation values and the updating of vectors are repeated until the end condition is satisfied in order to find an approximation to the optimal solution.

2.2 Standard PSO

Standard particle swarm optimization (SPSO) (Bratton, 2007) is an algorithm that extends the original PSO algorithm described above. Based on the topology of the particle group defined by the designer, the best position from among each particle and its neighbors is used as a local best instead of using a global best.

In SPSO, changing the topology changes the search motion of the entire particle group. A simplified diagram of the ring topology that is generally used as the topology for SPSO is shown in Fig. 1. In a ring topology, a particle shares information with its neighbors on both sides. This is because convergence on the optimal solution is impaired if the particles are split into completely independent groups. In this case, with a reduced amount of shared information, the convergence is slightly worse compared with the original PSO, but the global search performance is maintained for a longer time. SPSO is therefore able to search for better solutions to problems that converge to a local solution in PSO.

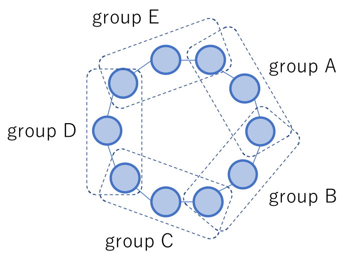

Figure 1: Ring topology example.

Furthermore, particle groups in PSO may fail to converge due to an excessive increase in velocity. Therefore, in order to guarantee that the particle group converges as the search proceeds, SPSO uses the following calculation formula taking the inertia weight χ into consideration. Here, L_{bestx} represents the local best.

$$v(t+1) = \chi * v(t) + \chi * c_1 * rand_1 * \left(P_{bestx} - x(t)\right) + \chi * c_2 * rand_2 * \left(L_{bestx} - x(t)\right) \tag{3}$$

$$\chi = \frac{2}{\left|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}\right|} \quad \varphi = c_1 + c_2 \tag{4}$$

When a search is performed using the above formula, the movement of the entire particle group changes according to the value of φ. When φ is less than 4, the particle groups repeatedly converge and diverge. When it is greater than 4, the particle groups are guaranteed to converge. Therefore, a combination of $c_1 = c_2 = 2.05$ is generally used in SPSO.

3 RUNNING PSO IN PARALLEL

3.1 Parallel PSO in Earlier Research

In a GPU-based parallel implementation of PSO, the computational load is distributed by allocating each particle to a CUDA core. The GPU can be used to parallelize parts that exist independently for each particle, such as calculating its evaluation value and updating its vectors. However, the global best calculation requires the collection of information on the whole particle group, which reduces the degree of parallelism. In previous studies by Zhou et al. (Zhou, 2009) and Hussain et al. (Hussain, 2016), SPSO was run in parallel by using a ring topology. Hussain et al.

also sought to achieve greater speed by using coreless access, which is an access technique for efficient communication with global memory.

In the methods proposed in these previous studies, there remains a problem in that the frequency of communication with global memory increases. In parallel SPSO, neighboring particles may exist in another SM. When using a topology where there is no sharing between particles in different SMs, it is not possible to perform searches using a global best, the overall convergence of the particle group is impaired, and it is not possible to search efficiently. Therefore, when SPSO is run in parallel, it must use global memory to share information between SMs. For these reasons, the previous studies basically used global memory to store data. This results in frequent communication with global memory while performing search calculations. When implementing parallel processing on a GPU, this communication with low-speed memory becomes a bottleneck, making it hard to improve the effective execution speed no matter how great the degree of parallelism.

3.2 Parallelization using Virtual Global Best

Figure 2 shows an outline of our proposed parallelization using virtual global best. The virtual global best method reduces the frequency of communication with global memory. First, the population (particle swarm) in PSO is separated into several new swarms. These new swarms are associated with each SM, with information such as the particle coordinates being stored in registers or shared memory. In order to minimize the number of data transfers between SMs, each particle searches using a virtual global best instead of the current global best. The initial value of the virtual global best is allocated to each SM as the global best of the particle swarm in the initial state before separation. The local best of each SM is compared with the virtual global best, and when the local best is better than the virtual global best, the virtual global best is updated and is simultaneously compared with the global best stored in global memory. When necessary, the global best and virtual global best are updated. In this way, it is no longer necessary to communicate with global memory when particle information is required. This can reduce the memory communication bottleneck, making parallel processing more effective. When the global best is updated in global memory, the timing of this event is delayed in each sub-swarm to prevent loss of data.

The virtual global best search algorithm (Algorithm 1) is as follows:

```
1  let N = the number of particles in the sub swarm
2  let s = the index of sub swarm
3  let B = the number of sub swarms
4  let T = the number of migration interval
5  store global best in global memory
6  store virtual global best in shared memory
7  if the index of thread is N+1 then
8    for j = 1 to iterations do
9      if (j%(B*T)) is (s*T) then
10       update global best and virtual global best
11     end if
12   end for
13 end if
14 else
15   for j = 1 to iterations do
16     update velocity and position
17     calculate fitness, personal best, and virtual global
   best
18   end for
19 end else
```

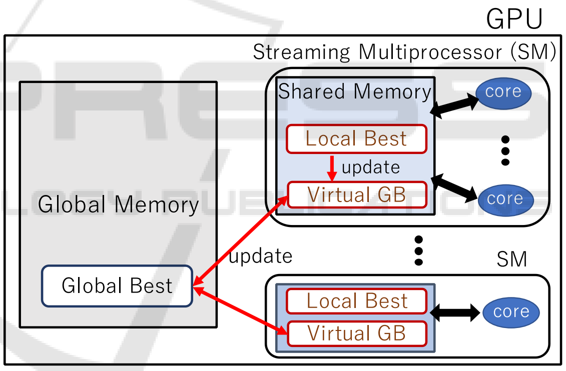

Figure 2: Outline of virtual global best method.

4 PARALLEL MOPSO USING VIRTUAL GLOBAL BEST

4.1 Multi-objective Optimization

Multi-objective optimization is a method that simultaneously optimizes multiple objective functions in a trade-off relationship. Figure 3 shows a conceptual illustration of the optimization of two objectives. In multi-objective optimization, one of the currently known solutions that has a good evaluation value for any particular objective function is called a non-inferior solution and is regarded as an optimal solution. As shown in Fig. 3, there are usually multiple non-inferior solutions. The set of non-inferior solutions is called the Pareto optimal solution. The surface formed by the Pareto optimal solution is called the Pareto front, and the purpose of a search algorithm such as evolutionary computation is to conduct searches so that the Pareto front takes a form that better satisfies the criteria of the objective function. Basically, PSO is a population based non-deterministic optimization algorithms as a single objective optimizer. On the other hand, research on PSO strategy for solving multi-objective problems (Multi-Objective PSO, MOPSO) has already began (Moore, 1999; Hussain, 2018).

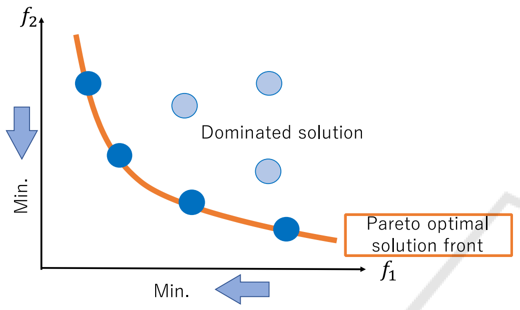

Figure 3: Conceptual illustration of the optimization of two objectives.

4.2 Proposal of Parallel MOPSO/D using Virtual Global Best

In MOPSO, the global best cannot be updated in the same way as in single-objective PSO because the optimal solution of the whole particle swarm is not uniquely determined. Thus, when the virtual global best algorithm is simply implemented as parallel MOPSO, we can consider a method that uses an archive. The archive stores the coordinates of multiple candidate best solutions out of the personal best solutions of each particle. A particle updates its velocity by using the coordinates of one candidate selected from the archive as the virtual global best.

In MOPSO using an archive, when selecting a solution stored in the archive or a global best, it is possible to improve diversity by considering parameters such as the particle congestion factor. However, if this solution requires complex computations, it will increase the computational load and adversely affect the execution time. Furthermore, to prevent the loss of information stored in memory, the addition of information to the archive must be performed sequentially for each particle, making it difficult to conceal the added computational load in parallel processing. Accordingly, the above trade-off relationship cannot be resolved in the virtual global best method using an archive. Therefore, in the proposed method, we use a combination of MOPSO/D, which is capable of performing global searches, and a virtual global best approach that effectively operates at high speed in parallel by avoiding communication with low-speed memory. This not only ensures the diversity of solutions, but can also reduce the execution time.

4.2.1 MOPSO/D

In MOPSO/D, a partition function is uniformly distributed in evaluation value space based on the objective function, and the particles optimize this partition function. In this way, by having each particle optimize a single objective function, the overall particle swarm find uniform and diverse solutions in evaluation value space by optimizing a single objective function. The Tchebycheff function (Zhang, 2007) is used as a partition function. This function is calculated as follows,

$$g(\boldsymbol{x}|\boldsymbol{\lambda},\boldsymbol{z}^*) = max_{1\leq j\leq m}\{\lambda_j|f_j(\boldsymbol{x}) - z_j^*|\} \quad (5)$$

where j represents the number of objective functions, λ is a weighting vector corresponding to each partition function, and the search direction changes according to its value. z* is a reference point, and the coordinates of this point are determined by the values of the whole particle swarm. An overview of MOPSO/D is shown in Fig. 4. When the reference point is set as the minimum value of the particle swarm in a minimization problem as shown in Fig. 4, the optimal solution can be obtained by determining the minimum value of the partition function.

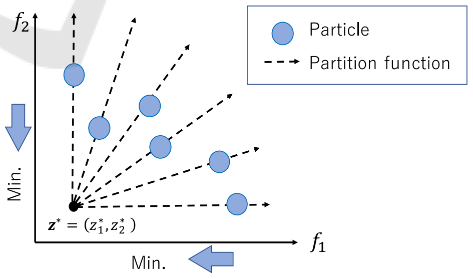

Figure 4: Outline of MOPSO/D.

The difference between this method and ordinary PSO lies in the way in which the global best solution is updated. Since MOPSO/D uses a different function for the evaluation of each particle, it is not possible to compare each of these calculated evaluation values directly. Therefore, each particle calculates an evaluation value by inputting the coordinates of its neighboring particles into its own partition function.

When the evaluation value is better than the current global best, the coordinates of the referenced particle are set as the new global best coordinates. In MOPSO/D, adjustment of parameters is required in order to influence the solution accuracy and execution speed of neighboring particles that are referenced when updating the global best and the number of partition functions.

4.2.2 Proposed Parallelization of MOPSO/D using Virtual Global Best

Figure 5 shows an outline of the distributed MOPSO/D with virtual global best method. In the proposed method, the particle swarm is partitioned between each SM as in the virtual global best method. This makes it possible to perform the calculations to update coordinates and personal best solutions without using global memory. Furthermore, the number of partition functions is assumed to be equal to the number of particles in an SM, and the distribution of partition functions is assumed to be the same for any sub-swarm. Therefore, in SM there is a one-to-one correspondence between particles and partition functions, and in the overall particle swarm, a single partition function is searched by the number of particles allocated to an SM. By sharing the global best with particles searching for the same partition function, it is thought that this will improve the convergence so that an optimal solution can be found in fewer generations. Furthermore, by storing each particle's virtual global best solution in a register, the frequency of communication with global memory can be reduced. By delaying the global best update time for each sub-swarm, we can expect to maintain the global search performance in each partition function.

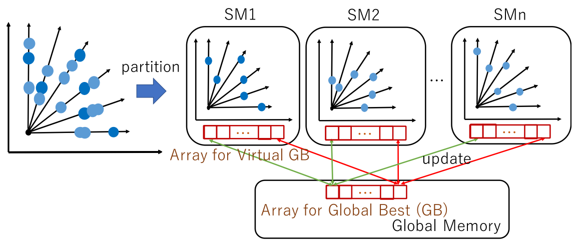

Figure 5: Outline of the distributed MOPSO/D with virtual global best method.

The basic algorithm is the same as in the virtual global best method, but with two main changes. The first is that a different method is used to update the virtual global best. In MOPSO/D, the coordinates of neighboring particles are required when updating the global best. Essentially, storing the coordinates in shared memory allows the operations related to these particles to be completed inside the SM. However, due to shared memory capacity limits, it is not possible to store the coordinates of sub-swarms. Therefore, in the proposed method, the particle coordinates are stored in global memory for the purpose of sharing information, and are retrieved when updating the virtual global best. In this way, communication with the global memory takes place at each generation, which is liable to adversely affect the execution speed.

The second change is that there is no thread responsible for updating the global best solution (thread *N*+1 in Algorithm 1). In MOPSO/D, the global best is stored individually by each particle, so when one thread communicates with global memory, the amount of communication is large, and the execution time becomes longer. Therefore, the global best is updated by the thread that updates each particle.

5 EVALUATIONS

5.1 Evaluation Method

Table 1 shows the environment used in the experiment. In this study, we performed three types of comparative experiments. First, we solved the five benchmark problems of previous studies shown in the appendix in the same environment (Zhou, 2009; Hussain, 2016), and we compared the execution times and solution accuracy by solving five benchmark problems with three global best methods. The results shown here are the average values from 20 runs of this experiment with 7,936 particles, a 100-dimensional sphere function, and other functions with 50 dimensions, and 2,000 generations.

Table 1: Experimental environment.

CPU	Intel core i7-6700 3.40 GHz
RAM	16.00 GB
GPU	NVIDIA GeForce GTX 960
OS	Windows 10 Home

Second, by comparing the execution speeds and solution distributions of MOPSO/D using a virtual global best method when implemented on a CPU and when implemented in parallel on a GPU, we verified that it maintains the same level of accuracy while reducing the execution time. The benchmark problems with two objectives used in this experiment were ZDT1, ZDT2 and ZDT3 (Zitzler, 2000) shown below. The experiments related to execution speed were conducted with 4,096 particles, 30 dimensions and 250 generations, and the experiments related to distribution were conducted with the number of dimensions changed to 200.

[ZDT1 function]

min $f_1(x) = x_1$

min $f_2(x) = g \cdot h$

where, $g = 1 + 9.0\sum_{i=2}^{n}(x_i/n - 1)$,

$h = 1 - \sqrt{f_1/g}$,

$x_i \in [0, 1]$

[ZDT2 function]

min $f_1(x) = x_1$

min $f_2(x) = g \cdot h$

where, $g = 1 + 9.0\sum_{i=2}^{n}(x_i/n - 1)$,

$h = 1 - (f_1/g)^2$,

$x_i \in [0, 1]$

[ZDT3 function]

min $f_1(x) = x_1$

min $f_2(x) = g \cdot h$

where, $g = 1 + 9.0\sum_{i=2}^{n}(x_i/n - 1)$,

$h = 1 - \sqrt{f_1/g} - (f_1/g)sin(10\pi \cdot f_1)$,

$x_i \in [0, 1]$

Third, we imposed a time limit on the CPU execution, and compared the resulting distribution with the results obtained by running on a GPU. In the limited time experiment, the GPU execution was set to either the time taken to reach the set number of generations, or the time taken for all the particles to reach the Pareto front of the benchmark problem. The number of particles was 4,096, and the number of generations was 250. We used the same benchmark problem as in the second experiment, with the number of dimensions set to 30.

5.2 Experimental Comparison of Virtual Global Best Method with Previous Studies

As shown in Table 2, the virtual global best method yields much more accurate solutions for the Sphere function and Griewank function than the parallel PSO method used in previous studies. This is thought to be because the virtual global best method achieves the same high level of convergence as the original PSO method. The Sphere function is a unimodal function without any local solutions, while the Griewank function is similar to a unimodal function in that it features a large global gradient. Since a highly convergent search algorithm is effective for searching unimodal functions, the proposed method was able to find better solutions. It also found very similar solutions for other multimodal functions. This is thought to be because the division of the particle swarm into sub-swarms allows greater global search performance to be maintained than with ordinary PSO. Also, according to Table 3, the virtual global best method is about four times faster than the conventional parallel PSO, except for the Sphere function. This is due to the effect of using virtual global memory to reduce the frequency of communication with global memory.

Table 2: Comparison of the accuracy of previous studies and the proposed method.

Function	Zhou, 2009	Hussain, 2016	Proposed method
Sphere	1.06e-01	1.32e-01	**4.51e-29**
Rosenbrock	**2.11e+01**	**2.11e+01**	2.22e+01
Rastrigin	1.45e+02	1.38e+02	**8.42e+01**
Griewank	8.32e-10	1.29e-09	**1.40e-45**
Ackley	9.44e-05	1.17e-04	**3.81e-06**

Table 3: Comparison of the execution time of previous studies and the proposed method.

Function	Zhou, 2009 [ms]	Hussain, 2016 [ms]	Proposed method [ms]
Sphere	9463.55	2767.95	**1148.17**
Rosenbrock	2758.78	1495.40	**314.77**
Rastrigin	2898.74	1579.15	**409.13**
Griewank	2927.17	1599.27	**445.46**
Ackley	2891.06	1596.29	**414.08**

5.3 Experimental Comparison with Parallel MOPSO/D

Tables 4 through 6 show the average execution times for 20 trials of each benchmark problem when the number of generations is 250 and dimension size is 30. According to these tables, we achieved a speed improvement factor of at least 14 for every function. It shows an average higher performance improvement rate than related work (Hussain, 2018). When implemented on a CPU, the execution speed varies with the function parameters, whereas on a GPU there is almost no variation. This is because the evaluation values have to be calculated sequentially on a CPU, which has a direct effect on the function's computation time. On the other hand, since the GPU performs the function calculations in parallel, the variation of computation time with the difficulty of the function can be concealed.

In addition, when implemented on a GPU, the acceleration effects of parallel processing do not vary greatly with the number of cores. This can be attributed to various factors, including the proposed algorithm's higher proportion of computations involving neighboring particles, the use of global memory when updating the virtual global best solutions, and the fact that the global best is not updated in parallel. Furthermore, the execution time increases in proportion to the number of particles. This is thought to be due to the large number of registers used by a single thread. The number of threads that are run in parallel by the GPU is scheduled according to how much each thread occupies memory resources such as shared memory and registers. Therefore, when a single thread requires more memory, fewer threads can be processed at a time, and the overall execution becomes sequential. Since parallel MOPSO/D stores virtual global best coordinates in addition to the particle coordinates and personal best coordinates, each thread requires a large number of registers. As the number of particles increases, the number of threads that are executed sequentially also increases by a corresponding amount, so it is thought that the execution time increases in proportion to the number of particles.

Table 4: Speed improvement factors for ZDT1.

Num. of particles	CPU	GPU	Speed improvement factor
1024	144.9	9.8	14.71
2048	288.1	16.7	17.27
4096	650.0	32.5	17.38
8192	1133.2	68.7	16.50

Table 5: Speed improvement factors for ZDT2.

Num. of particles	CPU	GPU	Speed improvement factor
1024	159.3	9.6	16.51
2048	263.0	16.8	15.67
4096	504.6	31.7	15.94
8192	1001.7	67.4	14.86

Table 6: Speed improvement factors for ZDT3.

Num. of particles	CPU	GPU	Speed improvement factor
1024	167.0	10.3	16.23
2048	331.9	17.4	19.09
4096	650.0	33.2	19.58
8192	1283.6	70.7	18.15

Figures 6 through 8 compare the Pareto fronts of the solution distributions obtained when optimizing each problem on a CPU and on a GPU. The accuracy is almost the same for any function, but in the solution distributions of problems ZDT1 and ZDT3, there are slight differences in uniformity and solution accuracy. This seems to be due to minute calculation discrepancies between the CPU and the GPU that are amplified when the search process is repeatedly iterated.

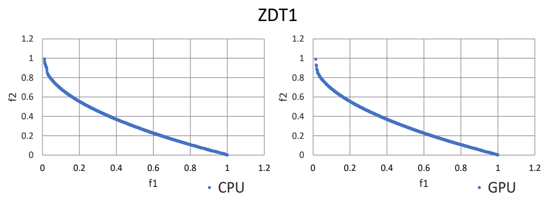

Figure 6: Comparison of ZDT1 solutions on a distribution chart.

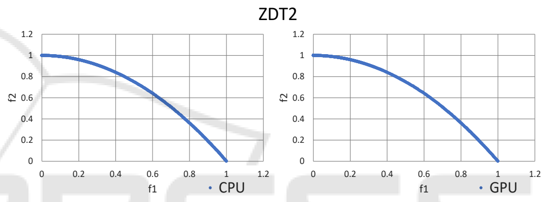

Figure 7: Comparison of ZDT2 solutions on a distribution chart.

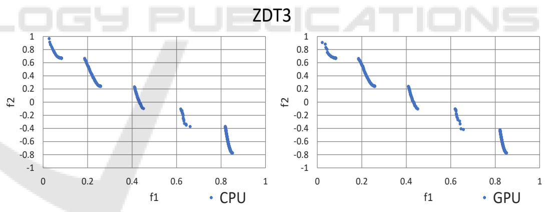

Figure 8: Comparison of ZDT3 solutions on a distribution chart.

5.4 Comparison with the Case Where Time Limits Are Imposed

Figures 9 through 11 compare the distributions of solutions obtained by the CPU and GPU implementations when subject to time constraints. Although the GPU implementation converges on the Pareto front of the benchmark problems, the CPU version found few Pareto solutions and yielded similar solution distributions for any function. This is because the CPU version can only process about 10 generations in the time it takes for the GPU version to process 250 generations, so it was not possible to perform sufficient searching to be able to ascertain the shape of the Pareto front. These results suggest that parallelization by the proposed method is effective even for real-time applications.

We experimented by assigning the same execution parameter settings to different algorithms this time. In the future, we think that the comparison in the case of performing parameter adjustment for each algorithm is also necessary. It is also necessary to compare the execution time and the search accuracy when the number of dimensions is changed.

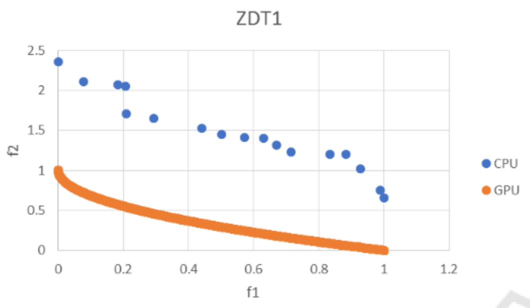

Figure 9: Comparison of ZDT1 solutions on a distribution chart.

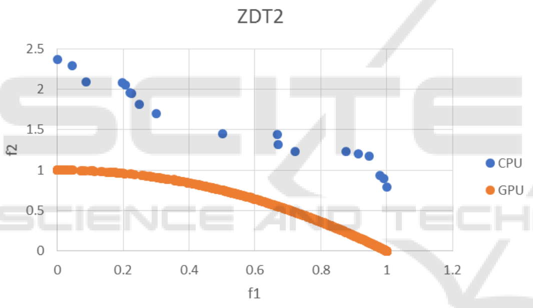

Figure 10: Comparison of ZDT2 solutions on a distribution chart.

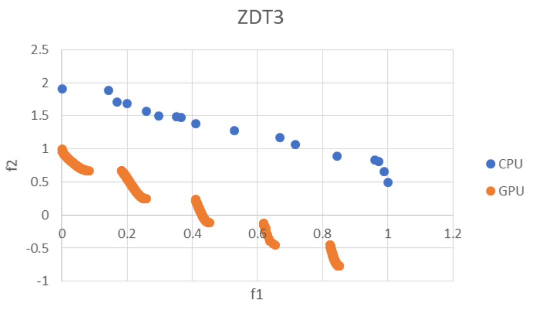

Figure 11: Comparison of ZDT3 solutions on a distribution chart.

6 CONCLUSION

In this study, we have shown that the proposed virtual global method can reduce processing time by up to 90% and perform searches at least with the same level of accuracy as the parallel processing methods of previous studies.

We have also proposed a parallel MOPSO algorithm that runs quickly while maintaining the diversity of solutions by performing parallel distributed processing based on a virtual global best method for MOPSO/D where multiple particles search a single partition function when performing multi-objective optimization using virtual global best solutions. We performed experiments to compare the performance of the proposed method when implemented on a CPU and on a GPU, and showed that the proposed method can adapt to changes in the Pareto front and can improve the processing speed by a factor of at least 14 without loss of precision when run in parallel.

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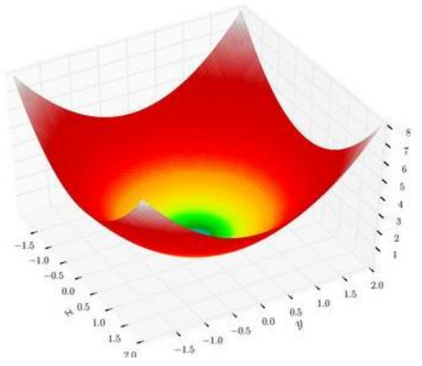
APPENDIX

The five benchmark functions for single-objective optimization (Wikipedia: Test functions for optimization, 2019) for compare with previous studies (Zhou, 2009; Hussain, 2016).

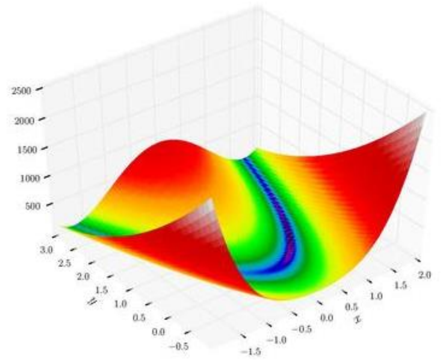
[Sphere function]

$$f(x) = \sum_{i=1}^{n} x_i^2$$



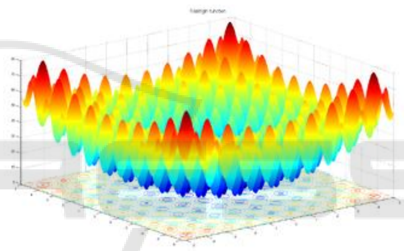
[Rosenbrock function]

$$f(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (1 - x_i)^2]$$



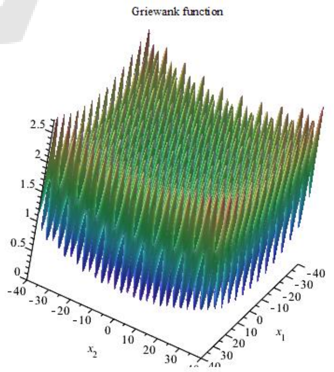
[Rastrigin function]

$$f(x) = An + \sum_{i=1}^{n} [x_i^2 - A\cos(2\pi x_i)],\ A = 10$$



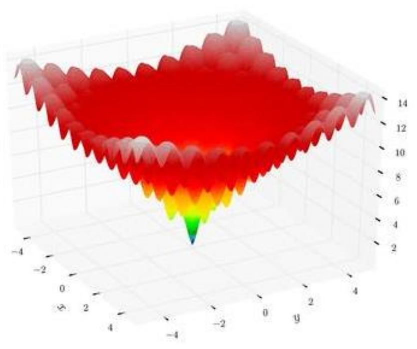
[Griewank function]

$$f(x_1, x_2) = 1 + \frac{1}{4000} x_1^2 + \frac{1}{4000} x_2^2 - \cos(x_1)\cos\left(\frac{1}{2} x_2 \sqrt{2}\right)$$



[Ackley function]

$$f(x, y) = -20\exp\left[-0.2\sqrt{0.5(x^2 + y^2)}\right] - \exp[0.5(\cos 2\pi x + \cos 2\pi y)] + e + 20$$


14
12
10
8
6
4
2
−4
−2
0
x
2
4
−4
−2
0
y
2
4