Asynchronous Control Design of Continuous-time Markovian Jump Systems with Bounded Time-varying Transition Rates

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Abstract: The asynchronous control design of continuous-time Markovian jump systems with bounded time-varying transition rates is addressed in this paper. According to the framework of parameterized linear matrix inequalities (PLMIs), essential stabilization conditions are established with consideration on dissipativity performance and then transform to solvable sets of linear matrix inequalities (LMIs) under our proposed method. Especially, our technique is derived from not only time-varying system modes but also asynchronous control modes transition rates. The effectiveness of our method is then illustrated through our numerical example.

1 INTRODUCTION

Markovian jump systems (MJSs) is a prosperous research area which have drawn energetic attraction in both academic and industries communities due to their advantages. Firstly, as a special class of hybrid systems, MJSs involve both time and event-driven mechanism to represent plants under abrupt changes in structure and parameters, for examples, sudden environment changes, component failures, package dropout, subsystems interconnection adjustment. The second advantageous aspect is that MJSs have been devoted to the study of practically diverse application such as communication networks, power systems, solar receiver control, networked control aircraft flight systems, robotics (see (Zhou et al., 2017; Liu et al., 2017; Mao et al., 2007; Zhai et al., 2016; Joo and Kim, 2015; Nguyen and Kim, 2019; Shi and Yu, 2009), etc). To dominate the variations among those systems, MJSs have been extended from Markov process which follows a certain transition rates matrix. Recently, to cover a wider range of practical applications systems, the term transition rates are defined as time-varying rather than ideally constant and time-invariant. Rasing from this research trend, the analyze technique emerged from bounded interval of time-varying transition rates has been took advantage to achieve many significant results in control community (refer to (Kim, 2014b; Kim, 2014a; Nguyen et al., 2016; Yin et al., 2018)), etc).

As for control synthesis, in past few decades, a lot of research pursues the mode-independent control method. Up to now, research trend has adopted to the mode-dependent Lyapunov method in designing controllers, with strict synchronization between plants and controllers. Nonetheless, with the growing in large scale and complexity of modern industrial process, the system dynamic and controller are quite different to each other. Therefore, a strict synchronization is likely to be hard in real time since sometimes controller can not access exact mode information from system, especially when the systems are insensitive to some amount of uncertainties, device failure or external perturbation. Under this circumstance, the necessity to discuss asynchronous phenomenon for control synthesis/filter of MJSs is desired to further explore.

Among those research on asynchronization matter on MJSs, the asynchronous $\mathcal{L}_2$-$\mathcal{L}_\infty$ filtering for discrete-time MJSs with occurred sensor nonlinearities satisfying the Bernoulli distribution are concerned in (Wu et al., 2014). On another research result, (see, for details, (Kim, 2019a) and (Kim, 2019b)) has discussed the issue of passivity-based asynchronous controller, synchronous controller as well as mode-independent controller for discrete-time MJSs. Recently, asynchronous stabilization problem for discrete-time MJSs was discussed at which both synchronous case and asynchronous ones have been surveyed to figure out sufficient and necessary stabilization conditions (Guan et al., 2019). However,

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there is still much room left on the asynchronous stabilization for continuous-time MJSs, which motivates us to conduct this paper.

The purpose of our paper is to address asynchronous control design for continuous-time Markovian jump systems with bounded time-varying transition rates since the difficulty emerged from both asynchronous phenomenon and unavailable transition rates need to be discussed and enhanced. The contribution of our paper can be itemized as follows:

- Our control method can deal with asynchronous controller, partially asynchronous controller, as well as perfectly synchronous controller based on the assumption of conditional probability matrix between system modes and controller modes.
- The robustness of controller has been taken to deal with external factors such as system uncertainties, disturbance so that the closed-loop system is stochastically stable with strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})$-$\gamma$-dissipative performance.
- The result is meaningful in practically real time application since this paper pursues to adapt time-varying transition rates, where system randomly switches from one mode to another at time-dependent pace.
- To be promise in our future research with the extension of application in nonhomogenous MJSs at which transition rates are not only fixed or bounded but also completely unknown.

Notations: The notations $X \geq Y$ and $X > Y$ mean that $X - Y$ is positive semi-definite and positive definite, respectively. In symmetric block matrices, the asterisk $(*)$ is used as an ellipsis for terms induced by symmetry. $\mathbf{E}\{\cdot\}$ denotes the mathematical expectation; $\mathbf{diag}(\cdot)$ stands for a block-diagonal matrix; $\mathbf{He}\{\mathcal{Q}\} = \mathcal{Q} + \mathcal{Q}^T$ for any square matrix $\mathcal{Q}$; and $\mathfrak{L}_2[0,\infty)$ stands for the space of square summable sequences over $[0,\infty)$. For $a_i \in \{1,2,\cdots\}$ such that $a_{i+1} > a_i$, $i \in \{1,2,\cdots,n\}$, the following notations are used: $\left[\mathcal{Q}_i\right]^T_{i\in\{a_1,\cdots,a_n\}} = \left[\begin{array}{ccc} \mathcal{Q}^T_{a_1} & \cdots & \mathcal{Q}^T_{a_n}\end{array}\right]$, $\left[\mathcal{Q}_i\right]^{\mathbf{D}}_{i\in\{a_1,\cdots,a_n\}} = \mathbf{diag}\left(\mathcal{Q}_{a_1},\cdots,\mathcal{Q}_{a_n}\right)$, where $\mathcal{Q}_i$ denotes real submatrix with appropriate dimensions or scalar values.

2 SYSTEM DESCRIPTION AND PRELIMINARIES

Let us consider the following continuous-time Markovian jump system defined on a complete probability space $(\Omega,\mathcal{F},\mathcal{P})$:

$$\begin{cases} \dot{x}(t) = \big(A(\phi(t)) + \Delta A(\phi(t)\big)x(t) \\ \qquad + \big(B(\phi(t)) + \Delta B(\phi(t))\big)u(t) + E(\phi(t))w(t), \\ z(t) = C(\phi(t))x(t) + D(\phi(t))u(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$, $u(t) \in \mathbb{R}^{n_u}$, $w(t) \in \mathbb{R}^{n_w}$, $z(t) \in \mathbb{R}^{n_z}$, and $\phi(t) \in \mathbf{N}_\phi = \{1,2,\ldots,n_\phi\}$ denote the state, the saturated control input, the disturbance input belonging to $\mathfrak{L}_2\,[0,\infty)$, the performance output, and the plant operation mode, respectively. Especially, the process $\{\phi(t), t \geq 0\}$ is characterized by a continuous-time nonhomogeneous Markov process governed by the following transition probabilities (TPs):

$$\mathbf{Pr}\big(\phi(t+\delta) = h \big| \phi(t) = g\big) = \begin{cases} \pi_{gh}(t)\delta + o(\delta) & \text{if } h \neq g, \\ 1 + \pi_{gg}(t)\delta + o(\delta) & \text{if } h = g, \end{cases}$$

where $\delta > 0$, $\lim_{\delta\to 0}(o(\delta)/\delta) = 0$, and $\pi_{gh}(t)$ denotes the transition rate (TR) from mode g to mode h at time $t+\delta$, satisfying that

$$\bullet \quad \sum_{h\in\mathbf{N}_\phi} \pi_{gh}(t) \equiv 0, \tag{2}$$

$$\bullet \quad \pi_{gh}(t) \geq 0,\ \forall h \in \mathbf{N}_\phi \setminus \{g\}. \tag{3}$$

Based on the property of TRs, this paper intends to address the following two finite sets such that $\mathbf{H}_g \bigcup \widetilde{\mathbf{H}}_g = \mathbf{N}_\phi$ holds: for $g \in \mathbf{N}_\phi$,

$$\mathbf{H}_g = \Big\{ h \;\big|\; \pi_{gh}(t) = \pi_{gh} \text{ is time-invariant and completely known} \Big\}, \tag{4}$$

$$\widetilde{\mathbf{H}}_g = \Big\{ h \;\big|\; \pi_{gh}(t) \text{ is unknown but bounded as } \underline{\pi}_{gh} \leq \pi_{gh}(t) \leq \overline{\pi}_{gh} \Big\}. \tag{5}$$

Then, from (2), (4), and (5), it is

$$\bullet \quad \sum_{h\in\widetilde{\mathbf{H}}_g} \pi_{gh}(t) + \Pi^+_g \equiv 0, \tag{6}$$

$$\bullet \quad \pi_{gh}(t) \in \left[\underline{\pi}_{gh}, \overline{\pi}_{gh}\right],\ \forall h \in \widetilde{\mathbf{H}}_g, \tag{7}$$

where $\Pi^+_g = \sum_{h\in\mathbf{H}_g} \pi_{gh}$. Further, in (1), the parameter uncertainties are represented as follows: $\Delta A(\phi(t)) = G(\phi(t))\Delta(\phi(t),t)H_1(\phi(t))$ and $\Delta B(\phi(t)) = G(\phi(t))\Delta(\phi(t),t)H_2(\phi(t))$, where $G(\phi(t) = g) = G_g$, $H_1(\phi(t) = g) = H_{1g}$, and $H_2(\phi(t) = g) = H_{2g}$ are known constant matrices with appropriate dimensions; and $\Delta(\phi(t) = g,t) = \Delta_g(t) \in \mathbb{R}^{n_p\times n_q}$ is an unknown matrix with Lebesgue measurable elements such that

$\|\Delta_g(t)\| \leq 1$ holds. In what follows, let us consider an asynchronous mode-dependent control law of the following form:

$$u(t) = F(\rho(t))x(t), \quad (8)$$

where $\rho(t) \in \mathbf{N}_\rho = \{1, 2, \cdots, n_\rho\}$ denotes the control mode, and $F(\rho(t) = \ell) = F_\ell$ denotes the control gain to be designed later. Further, since the control mode $\rho(t)$ is definitely associated with the plant mode $\phi(t)$, it is supposed to be characterized by the following conditional probability:

$$\mathbf{Pr}\big(\rho(t) = \ell \big| \phi(t) = g\big) = \varpi_\ell^g, \ \forall g \in \mathbf{N}_\phi, \ \ell \in \mathbf{N}_\rho, \quad (9)$$

which satisfies $\sum_{\ell \in \mathbf{N}_\rho} \varpi_\ell^g = 1$ and $0 \leq \varpi_\ell^g \leq 1$ for all g and ℓ. As a result, the closed-loop system with (1) and (8) is described as follows:

$$\begin{cases} \dot{x}(t) = (\bar{A}_{g\ell} + \Delta\bar{A}_{g\ell})x(t) + E_g w(t), \\ z(t) = \bar{C}_{g\ell} x(t), \end{cases} \quad (10)$$

where $\bar{A}_{g\ell} = A_g + B_g F_\ell$, $\bar{C}_{g\ell} = C_g + D_g F_\ell$, $\Delta\bar{A}_{g\ell} = G_g \Delta_g(t) \bar{H}_{g\ell}$, and $\bar{H}_{g\ell} = H_{1g} + H_{2g} F_\ell$.

Remark 2.1. As reported in (Choi et al., 2017), the strict $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-$\gamma$-dissipativity performance can be reduced into two special performances: 1) $\mathcal{H}_\infty$ performance by setting $\mathcal{Q} = -I$, $\mathcal{S} = 0$, and $\mathcal{R} = (\gamma^2 + \gamma)I$; and 2) passivity performance by setting $\mathcal{Q} = 0$, $\mathcal{S} = I$, and $\mathcal{R} = 2\gamma I$.

3 CONTROL DESIGN

Let us consider the following mode-dependent Lyapunov function:

$$V(t, \rho(t)) = x^T(t) P(\rho(t)) x(t), \quad (11)$$

where $0 < P(\rho(t) = \ell) = P_\ell \in \mathbb{R}^{n_x \times n_x}$. Then the weak infinitesimal operator of the process $\{x(t), \phi(t), \rho(t)\}$ acting on $V(t) := V(t, \rho(t))$ is given by

$$\begin{aligned} \nabla V(t) &= \lim_{\delta \to 0} \frac{1}{\delta} \mathbf{E}\Big\{ V\big(t+\delta, \rho(t+\delta) = m \big| \phi(t) = g\big) \\ &\qquad - V(t, \rho(t) = \ell | \phi(t) = g) \Big\} \\ &= \lim_{\delta \to 0} \frac{1}{\delta} \left\{ \sum_{m \in \mathbf{N}_\rho} \left(\sum_{h \in \mathbf{N}_\phi \setminus \{g\}} \varpi_m^h \cdot (\pi_{gh}(t)\delta + o(\delta)) \right. \right. \\ &\qquad \times V(t+\delta, m) \\ &\qquad \left. + \varpi_m^g \cdot \big(\pi_{gg}(t)\delta + o(\delta) + 1\big) \times V(t+\delta, m) \right) \\ &\qquad \left. - \sum_{\ell \in \mathbf{N}_\rho} \varpi_\ell^g V(t, \ell) \right\} \\ &= \sum_{m \in \mathbf{N}_\rho} \sum_{h \in \mathbf{N}_\phi} \varpi_m^h \pi_{gh}(t) V(t, m) + \sum_{\ell \in \mathbf{N}_\rho} \varpi_\ell^g \dot{V}(t, \ell) \\ &= \sum_{\ell \in \mathbf{N}_\rho} \varpi_\ell^g \left(\sum_{h \in \mathbf{N}_\phi} \sum_{m \in \mathbf{N}_\rho} \pi_{gh}(t) \varpi_m^h V(t, m) + \dot{V}(t, \ell) \right). \end{aligned}$$

According to (10),

$$\nabla V(t) = \sum_{\ell \in \mathbf{N}_\rho} \varpi_\ell^g \eta^T(t) \begin{bmatrix} \Psi_{g\ell} & P_\ell E_g \\ E_g^T P_\ell & 0 \end{bmatrix} \eta(t), \quad (12)$$

where

$$\begin{aligned} &\eta(t) = \begin{bmatrix} x^T(t) & w^T(t) \end{bmatrix}^T \in \mathbb{R}^{n_\eta}, (\text{i.e.}, n_\eta = n_x + n_w), \\ &\Psi_{g\ell} = \mathcal{P}_g + \mathbf{He}\Big\{ P_\ell (\bar{A}_{g\ell} + \Delta\bar{A}_{g\ell}) \Big\}, \\ &\mathcal{P}_g = \sum_{h \in \mathbf{N}_\phi} \sum_{m \in \mathbf{N}_\rho} \pi_{gh}(t) \varpi_m^h P_m. \end{aligned}$$

The following lemma provides the robust stochastic stability condition for (10) with $w(t) \equiv 0$.

Lemma 3.1. *Suppose that there exist $P_\ell > 0$ and F_ℓ such that the following condition holds:*

$$0 > \Psi_{g\ell}, \ \forall g \in \mathbf{N}_\phi, \ \ell \in \mathbf{N}_\rho. \quad (13)$$

Then closed-loop system (10) with $w(t) \equiv 0$ can be said to be robustly stochastically stable.

The following lemma provides the robust asynchronous dissipative control synthesis conditions for (10), formulated in terms of parameterized linear matrix inequalities (PLMIs).

Lemma 3.2. *For a prescribed scalar μ, suppose that there exist matrices $0 < \bar{P}_\ell = \bar{P}_\ell^T \in \mathbb{R}^{n_x \times n_x}$, $W_{\ell m} = W_{\ell m}^T \in \mathbb{R}^{n_x \times n_x}$, $\bar{F}_\ell \in \mathbb{R}^{n_u \times n_x}$; and scalar variables γ, $\alpha_g > 0$ such that the following conditions hold: for $g \in \mathbf{N}_\phi$, $\ell \in \mathbf{N}_\rho$,*

$$\begin{aligned} 0 > &\begin{bmatrix} \mathbf{He}\{A_g \bar{P}_\ell + B_g \bar{F}_\ell\} + \alpha_g G_g G_g^T & (*) & (*) & (*) \\ E_g^T - \mathcal{S} C_g \bar{P}_\ell - \mathcal{S} D_g \bar{F}_\ell & -\mathcal{R} + \gamma I & 0 & 0 \\ \mathcal{Q}_1 C_g \bar{P}_\ell + \mathcal{Q}_1 D_g \bar{F}_\ell & 0 & -I & 0 \\ H_{1g} \bar{P}_\ell + H_{2g} \bar{F}_\ell & 0 & 0 & -\alpha_g I \end{bmatrix} \\ &+ \mathbf{e}^T \left(\sum_{h \in \mathbf{N}_\phi \setminus \{g\}} \pi_{gh}(t) \bar{W}_h(\mu) \right) \mathbf{e}, \end{aligned} \quad (14)$$

$$0 \leq \begin{bmatrix} W_{\ell m} & (*) \\ \bar{P}_\ell & \bar{P}_m \end{bmatrix}, \ \forall m \in \mathbf{N}_\rho. \quad (15)$$

where $\mathbf{e} = \begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix} \in \mathbb{R}^{n_x \times (n_x + n_w + n_z + n_q)}$, and $\bar{W}_h(\mu) = \sum_{m \in \mathbf{N}_\rho} \left(\varpi_m^h W_{\ell m} - 2\mu \varpi_m^g \bar{P}_\ell + \mu^2 \varpi_m^g \bar{P}_m \right)$. Then closed-loop system (10) is robustly stochastically stable and strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-$\gamma$-dissipative, and the control gains are reconstructed as follows: $F_\ell = \bar{F}_\ell \bar{P}_\ell^{-1}$, $\forall \ell \in \mathbf{N}_\rho$.

The following theorem provides the relaxed robust asynchronous control synthesis conditions for (10) with strict $(\mathcal{Q},\mathcal{S},\mathcal{R})$-γ-dissipativity performance, formulated in terms of linear matrix inequalities (LMIs).

Theorem 3.1. *For a prescribed scalar μ, suppose that there exist symmetric matrix $0 < \bar{P}_\ell \in \mathbb{R}^{n_x\times n_x}$, $W_{\ell m} \in \mathbb{R}^{n_x\times n_x}$; matrices $\bar{F}_\ell \in \mathbb{R}^{m\times n_x}$, $X_g \in \mathbb{R}^{n_x\times n_x}$, $Y_{gh} \in \mathbb{R}^{n_x\times n_x}$; and γ, $\alpha_g > 0$ such that the following conditions hold: for $g \in \mathbf{N}_\phi$, $\ell \in \mathbf{N}_\rho$,*

$$0 > \bar{\Gamma}_{g\ell} = \begin{bmatrix} \Gamma_{g\ell} & (*) \\ \left[\frac{1}{2}\bar{W}_h(\mu)\mathbf{e}\right]_{h\in\widetilde{\mathbf{H}}_g\setminus\{g\}} & 0 \end{bmatrix} + \begin{bmatrix} \mathbf{X}_g^{(1,1)}+\mathbf{Y}_g^{(1,1)} & (*) \\ \mathbf{X}_g^{(2,1)}+\mathbf{Y}_g^{(2,1)} & \mathbf{Y}_g^{(2,2)} \end{bmatrix}, \quad (16)$$

$$0 \le \begin{bmatrix} W_{\ell m} & (*) \\ \bar{P}_\ell & \bar{P}_m \end{bmatrix}, \ \forall m \in \mathbf{N}_\rho, \quad (17)$$

where

$$\Gamma_{g\ell} = \begin{bmatrix} \Gamma_{g\ell}^{(1,1)} & (*) & (*) & (*) \\ E_g^T - \mathcal{S}C_g\bar{P}_\ell - \mathcal{S}D_g\bar{F}_\ell & -\mathcal{R}+\gamma I & 0 & 0 \\ \mathcal{Q}_1 C_g\bar{P}_\ell + \mathcal{Q}_1 D_g\bar{F}_\ell & 0 & -I & 0 \\ H_{1g}\bar{P}_\ell + H_{2g}\bar{F}_\ell & 0 & 0 & -\alpha_g I \end{bmatrix},$$

$$\Gamma_{g\ell}^{(1,1)} = \mathbf{He}\{A_g\bar{P}_\ell + B_g\bar{F}_\ell\} + \alpha_g G_g G_g^T + \sum_{h\in\mathbf{H}_g\setminus\{g\}} \pi_{gh}\bar{W}_h(\mu),$$

$$\mathbf{X}_g^{(1,1)} = \begin{cases} \mathbf{e}^T\mathbf{He}\{\Pi_g^+ X_g\}\mathbf{e}, & \text{if } g \in \mathbf{H}_g \\ 0, & \text{otherwise} \end{cases},$$

$$\mathbf{Y}_g^{(1,1)} = \sum_{h\in\widetilde{\mathbf{H}}_g\setminus\{g\}} \mathbf{e}^T\mathbf{He}\{\underline{\pi}_{gh}\overline{\pi}_{gh}Y_{gh}\}\mathbf{e},$$

$$\mathbf{X}_g^{(2,1)} = \begin{cases} [X_g\mathbf{e}]_{h\in\widetilde{\mathbf{H}}_g\setminus\{g\}}, & \text{if } g \in \mathbf{H}_g \\ 0, & \text{otherwise} \end{cases},$$

$$\mathbf{Y}_g^{(2,1)} = \left[(-\underline{\pi}_{gh}-\overline{\pi}_{gh})Y_{gh}\mathbf{e}\right]_{h\in\widetilde{\mathbf{H}}_g\setminus\{g\}},$$

$$\mathbf{Y}_g^{(2,2)} = \left[\mathbf{He}\{Y_{gh}\}\right]^{\mathbf{D}}_{h\in\widetilde{\mathbf{H}}_g\setminus\{g\}},$$

$$\mathbf{e} = \begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix} \in \mathbb{R}^{n_x\times(n_x+n_w+n_z+n_q)},$$

$$\bar{W}_h(\mu) = \sum_{m\in\mathbf{N}_\rho} \left(\varpi_m^h W_{\ell m} - 2\mu\varpi_m^g\bar{P}_\ell + \mu^2\varpi_m^g\bar{P}_m\right).$$

Then closed-loop system (10) is robustly stochastically stable and strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})$-γ-dissipative, and the control gains are reconstructed as follows:

$$F_\ell = \bar{F}_\ell\bar{P}_\ell^{-1}, \ \forall \ell \in \mathbf{N}_\rho. \quad (18)$$

4 ILLUSTRATIVE EXAMPLE

Example 1: Let us consider the following continuous-time nonhomogenous MJS with four modes, used in (Zong et al., 2013):

$$A_1 = \begin{bmatrix} 2 & 2 \\ 1 & -3 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}, A_4 = \begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, B_3 = \begin{bmatrix} 3 \\ 1 \end{bmatrix},$$
$$B_4 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}, E_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, E_2 = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix},$$
$$E_3 = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix}, E_4 = \begin{bmatrix} 0.4 \\ 0 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 2 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}, C_3 = \begin{bmatrix} 1 & 3 \end{bmatrix}, C_4 = \begin{bmatrix} 0 & 1 \end{bmatrix},$$
$$G_1 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, G_2 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, G_3 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},$$
$$G_4 = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, H_{11} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix},$$
$$H_{12} = \begin{bmatrix} 0.2 & 0.3 \end{bmatrix}, H_{13} = \begin{bmatrix} 0.2 & 0.3 \end{bmatrix},$$
$$H_{14} = \begin{bmatrix} 0.2 & 0.4 \end{bmatrix}, H_{21} = 0.1,$$
$$H_{22} = 0.2, H_{23} = 0.3, H_{24} = 0.4,$$
$$D_1 = 0.1, D_2 = 0.2, D_3 = 0.3, D_4 = 0.4. \quad (19)$$

Further, the TR matrix is taken as follows:

$$\left[\pi_{gh}(t)\right]_{g,h\in\mathbf{N}_\phi} = \begin{bmatrix} \times & 0.3 & \times & 0.4 \\ \times & -1 & 0.3 & \times \\ 0.8 & \times & -1.3 & \times \\ 1.0 & \times & \times & -1.5 \end{bmatrix}, \quad (20)$$

where all × denote the bounded TRs with upper and lower bound values listed in Table 1.

Table 1: Interval of bounded transition rates.

$\pi_{11}(t)$	[-1.4, -0.7]	$\pi_{32}(t)$	[0, 0.5]
$\pi_{13}(t)$	[0, 0.7]	$\pi_{34}(t)$	[0, 0.5]
$\pi_{21}(t)$	[0, 0.7]	$\pi_{42}(t)$	[0, 0.5]
$\pi_{24}(t)$	[0, 0.7]	$\pi_{43}(t)$	[0, 0.5]

That is, it is given that $\mathbf{H}_1 = \{2,4\}$, $\widetilde{\mathbf{H}}_1 = \{1,3\}$, $\mathbf{H}_2 = \{2,3\}$, $\widetilde{\mathbf{H}}_2 = \{1,4\}$, $\mathbf{H}_3 = \{1,3\}$, $\widetilde{\mathbf{H}}_3 = \{2,4\}$, $\mathbf{H}_4 = \{1,4\}$, $\widetilde{\mathbf{H}}_4 = \{2,3\}$. Meanwhile, the conditional probability matrix is taken as follows:

$$\left[\varpi_{g\ell}\right]_{g\in\mathbf{N}_\phi,\ell\in\mathbf{N}_\rho} = \begin{bmatrix} 0.4 & 0.2 & 0.3 & 0.1 \\ 0.1 & 0.5 & 0.3 & 0.1 \\ 0.2 & 0.3 & 0.4 & 0.1 \\ 0.1 & 0.3 & 0.4 & 0.2 \end{bmatrix}. \quad (21)$$

The goal of this example is to design an asynchronous mode-dependent state-feedback control (8) such that the closed-loop system is stochastically stable with strict strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})$-γ-dissipativity performance.

To this end, by Theorem 1, the following feasible solution is obtained: $\gamma = 0.4470$, and

$$F_1 = \begin{bmatrix} -7.9921 & -11.8143 \end{bmatrix},$$
$$F_2 = \begin{bmatrix} -7.8669 & -11.6618 \end{bmatrix},$$
$$F_3 = \begin{bmatrix} -7.9062 & -11.7685 \end{bmatrix},$$
$$F_4 = \begin{bmatrix} -7.8386 & -11.4010 \end{bmatrix}.$$

Furthermore, to show the validity of the obtained control gains, Figure 1 demonstrates the state response of the closed-loop system, where $x_0 = [0.5 \quad -0.4]^T$, $w(t) = 0.1e^{-0.3t}\sin(0.2\pi t)$, and the applied control input is plotted in Figure 2. Clearly, from Figure 1, it can be found that the state gradually converges to zero as time increases despite the presence of the asynchronous controller and incomplete knowledge of transition rates.

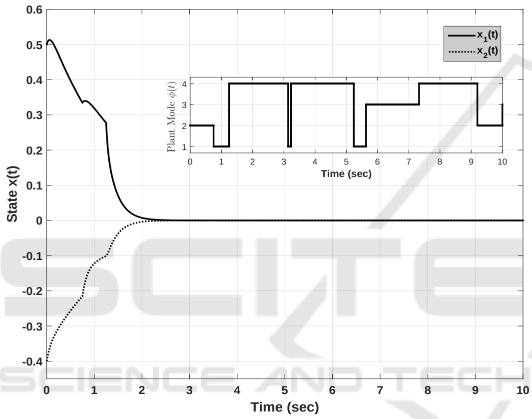

Figure 1: State response $x(t)$ and mode evolution $\rho(t)$ used in Example 1.

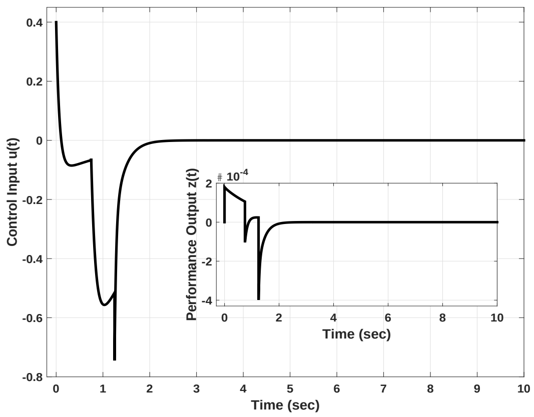

Figure 2: Control input $u(t)$ and performance output $z(t)$.

5 CONCLUSIONS

This paper has investigated the problem of asynchronous control for continuous-time MJSs with bounded time-varying transition rates and system uncertainties. In order to obtain a finite set of solvable LMIs from mode-dependent PLMIs, our method has been proposed to conduct the impact of both system modes and controller modes on stabilization conditions. Eventually, this paper has presented the LMIs-based stabilization conditions to design an asynchronous controller with dissipativity performance.

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