Large Deviations in Discrete-time Systems with Control Signal Delay

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Abstract: The paper considers a problem of deviations (peak effect) in the free motion of linear discrete stable systems with a control signal delay. The problem consists in structure of eigenvectors of the state matrix. The control signal delay adds additional order to a discrete-time model and leads to the variation of eigenvectors structure and deviation increasing in the free motion of the system. A tracking discrete-time system is a subject of the research. An approach to the modal control law design taking into account the value of delay and the deviation is suggested in the paper. It is proposed to assess the upper bound of peaking processes in the system with the condition number of an eigenvectors matrix. The results are supported by an example.

1 INTRODUCTION

Large deviation problem in the free motion of a linear system is investigated for a long time (Feldbaum, 1948), (Izmailov, 1978). Firstly, the relationship between the large deviations of the motion of a system and its poles was observed. Then, the problem of large deviations for systems with observers were investigated (Polotskij, 1981). The large deviations problem in cascade control systems was considered also (Sussman and Kokotovic, 1991), where the result of R.N. Izmailov was generalized to obtain estimations of the deviations for the outputs.

Recently, the estimation of the deviations for the case of large and small values of the poles were obtained using the linear matrix inequalities technique (Polyak and Smirnov, 2016) and the state-space approach (Vunder et al., 2015, 2016). Thus, it is found that not only the multiplicity of eigenvalues (Vunder et al., 2015, 2016), but also the output method of control signal can cause significant deviations of norm of a free-motion state vector of the discrete-time system ((Whidborne and McKernan, 2007), (Vunder and Ushakov, 2016), (Halikias et al., 2010), (Francis and Glover, 1978), (Kimura, 1981)).

However, little number of publications is published on the deviations assessments for discrete-time systems. Therefore, the aim of this paper is to propose deviations estimations for the discrete-time system with the control system delay and design a modal control law taking into account the value of the delay and the deviation. The results of the paper can be useful for the stabilization problems solution like a stabilization problem for planes (Polyak at al., 2015) or switching systems (Liberzon, 2003).

The paper is laid out as follows. Firstly, the deviation assessments in discrete-time systems without control signal delay are presented. Then, the case of discrete-time systems with control signal delay is described and a modal control law taking into account the value of delay and the deviation is proposed. Thereafter, the example of a discrete-time plant is presented. The paper is finished with some concluding remarks.

2 DEVIATIONS IN DISCRETE-TIME SYSTEMS WITHOUT CONTROL SIGNAL DELAY

Any discrete-time control system is a composition of following parts: digital controller, a digital-to-analog

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converter, and a continuous-time plant. In order to obtain a single mathematical description of this composition, processes are studied at the time instances $t = k \cdot \Delta t$, where k is positive integer, it is called the discrete time; Δt is the sample time. This means that the discrete-time plant is said to be discrete time sampling from continuous-time state and output variables under a piecewise-constant control signal with the duration Δt. Note that a control signal from the digital controller can output both without and with delay τ. This fact gives rise to two discrete-time representations of the continuous-time plant.

Consider a linear continuous-time plant

$$\dot{x}(t) = Ax(t) + Bu(t); x(0) = x(t)\big|_{t=0}, \quad y(t) = Cx(t), \tag{1}$$

where $x \in R^n, u \in R^r, y \in R^m$ are state vector, input vector, output vector respectively; $A \in R^{n\times n}$, $B \in R^{n\times r}$, $C \in R^{m\times n}$ are state matrix, input matrix, output matrix. If the control of plant (1) for $t = k \cdot \Delta t$ is realized without delay, then it can be represented as follows

$$u(t) = u(k),\ k\Delta t \le t < (k+1)\Delta t \tag{2}$$

Combining (1) and (2), from (Zadeh and Desoer, 2008), we get following discrete-time model

$$x(k+1) = \bar{A}x(k) + \bar{B}u(k); x(0) = x(k)\big|_{k=0}, \quad y(k) = \bar{C}x(k), \tag{3}$$

where $k = \arg(t = k\Delta t)$ is discrete time; Δt is sample time; $\dim(\bar{A}) = \dim(A)$, $\dim(\bar{B}) = \dim(B)$, $\dim(\bar{C}) = \dim(C)$; $\bar{A} = \exp(A\Delta t)$, $\bar{B} = (\bar{A} - I)A^{-1}B$, $\bar{C} = C$.

Analytically, control (2) can be written as

$$u(k) = \bar{K}_g g(k) - \bar{K}x(k), \tag{4}$$

where $g \in R^m$ is an external input; $\bar{K}_g \in R^{r\times m}$, $\bar{K} \in R^{r\times n}$ are the feed forward matrix, the feedback matrix respectively. Combining (4) and (3), we get discrete-time closed-loop system

$$\left.\begin{aligned} & x(k+1) = \bar{F}x(k) + \bar{G}g(k); x(0) = x(k)\big|_{k=0} \\ & y(k) = \bar{C}x(k) \\ & \varepsilon(k) = g(k) - y(k) \end{aligned}\right\}, \tag{5}$$

where

$$\bar{F} = \bar{A} - \bar{B}\bar{K},\ \bar{G} = \bar{B}\bar{K}_g, \tag{6}$$

$\varepsilon(k)$ is a tracking error. Eigenvalues and eigenvectors of the state matrix $\bar{F}$ is given by

$$\sigma\{\bar{F}\} = \left\{\begin{aligned} & \bar{\lambda}_i = \arg\left(\det\left(\bar{\lambda}I - \bar{F}\right) = 0\right): \\ & \operatorname{Im}\left(\bar{\lambda}_i\right) = 0, \bar{\lambda}_i \ne \bar{\lambda}_j; i, j = \overline{1,n}; i \ne j \end{aligned}\right\}, \tag{7}$$

$$\bar{F}\xi_i = \bar{\lambda}_i \xi_i; i = \overline{1,n}. \tag{8}$$

If the external input is not available to direct measurement, then control (4) is presented as

$$u(k) = \bar{K}_g g(k) - \bar{K}x(k) = \bar{K}_\varepsilon \varepsilon(k) - \bar{K}_r x(k), \tag{9}$$

where

$$\bar{K}_\varepsilon = \bar{K}_g = \arg\left(\bar{C}\left(I - \bar{F}\right)^{-1}\bar{B}\bar{K}_g = I\right) = \left(\bar{C}\left(I - \bar{F}\right)^{-1}\bar{B}\right)^{-1},$$

$$\bar{K}_r = \bar{K} - \bar{K}_y \bar{C}\big|_{\bar{K}_y = \bar{K}_g},$$

$\bar{K}_r$ is feedback matrix for a part of the state vector components.

Block diagram representation of system (5) with control (9) is shown in figure 1.

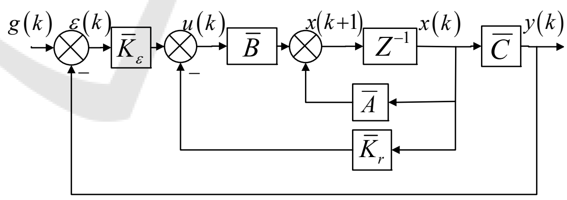


Figure 1: Block diagram of system without control delay.

Note also that the modification of control in form (9) does not change the mathematical representation of system (5).

Let us single out the autonomous component in the discrete-time system (5)

$$x(k+1) = \bar{F}x(k); x(0). \tag{10}$$

The solution of equation (10) takes the form

$$x(k) = \bar{F}^k x(0). \tag{11}$$

For the state matrix $\bar{F}$ the following condition satisfies

$$M\bar{\Lambda} = \bar{F}M, \tag{12}$$

where $\bar{\Lambda} = \exp(\Lambda\Delta t) = diag\left\{\bar{\lambda}_i = \exp(\lambda_i \Delta t); i = \overline{1,n}\right\}$ $M = row\left[M_i = \xi_i : \bar{F}\xi_i = \lambda_i \xi_i; i = \overline{1,n}\right]$ is a square matrix whose columns are the n linearly independent eigenvectors of $\bar{F}$. Using (11) and (12), we get

$$M\bar{\Lambda}^k = \bar{F}^k M, \tag{13}$$

Now, combining (11) and (13), we obtain

$$x(k) = \bar{F}^k x(0) = M\bar{\Lambda}^k M^{-1} x(0). \tag{14}$$

From (14) it follows

$$\|x(k)\| = \left\|\bar{F}^k x(0)\right\| \le \left\|\bar{F}^k\right\| \|x(0)\|_{\|x(0)\|=1} = \left\|\bar{F}^k\right\|, \tag{15}$$

where $\|*\|$ is any consistent norm here and elsewhere.

Let us form an upper bound of (15)

$$\begin{aligned}\|x(k)\| &= \left\|M\bar{\Lambda}^k M^{-1} x(0)\right\| \le \\ &\le \|M\| \left\|\bar{\Lambda}^k\right\| \left\|M^{-1}\right\| \|x(0)\| = \\ &= C\{M\} \left\|diag\left(\lambda_i^k\right); i = \overline{1,n}\right\| \|x(0)\| \le \\ &\le C\{M\} \bar{\lambda}_{\max}^k \|x(0)\|,\end{aligned} \tag{16}$$

where $C\{M\} = \|M\| \left\|M^{-1}\right\|$ is condition number (Gantmacher, 2000), (Golub and Van Loan, 2012) of the matrix M ; $\bar{\lambda}_{\max}$ is a maximum eigenvalue of the matrix $\bar{F}$ that satisfies conditions $\operatorname{Im}\left(\bar{\lambda}_{\max}\right) = 0, \bar{\lambda}_{\max} > 0$. Thus, by $\|x(0)\| = 1$, we have the upper bound

$$\sup\left(\|x(k)\|\right)\Big|_{\|x(0)\|=1} = C\{M\} \bar{\lambda}_{\max}^k. \tag{17}$$

From (17) the following properties have to be considered:

1. The condition number $C\{M\}$ of the eigenvectors matrix M determines quality of processes in the system on free motion norm.

2. If eigenvalues are orthogonal to each other, then the condition number $C\{M\}$ is equal to one. As a result, processes in the system (5) start from the point $\|x(0)\|$ and then decrease monotonic.

3. If even one pair of close to collinear eigenvectors exists, then the condition number $C\{M\}$ can be sufficiently large. Processes in the system (5) start from the point $\|x(0)\|$ too but then there can appear significant deviation of free motion trajectory. In this case the upper bound $\sup\left(\|x(k)\|\right)$ start from the point $C\{M\}\|x(0)\| >> \|x(0)\|$.

3 DEVIATIONS IN DISCRETE-TIME SYSTEMS WITH CONTROL SIGNAL DELAY

The case of a discrete-time system with the control signal delay is characterized by the increased dimension of the matrices and modification of eigenvector structure of the state matrix.

If the control $u(t)$ of plant (1) for $t = k \cdot \Delta t$ realizes with delay $\tau \le \Delta t$, then it can be represented as follows (Grigoriev et al., 1983)

$$u(t) = \begin{cases} u(k-1), & k\Delta t \le t < k\Delta t + \tau; \\ u(k), & k\Delta t + \tau \le t < (k+1)\Delta t. \end{cases} \tag{18}$$

Combining (18) and (1), we get following discrete-time model (Grigoriev et al., 1983) of plant

$$\begin{aligned} x(k+1) &= \bar{A}x(k) + \bar{B}_1(\tau)u(k-1) + \bar{B}(\tau)u(k) \\ y(k) &= \bar{C}x(k), \end{aligned} \tag{19}$$

where

$$\begin{aligned} \bar{B}_1(\tau) &= \bar{A}\left(I - e^{-A\tau}\right)A^{-1}B, \\ \bar{B}(\tau) &= \left(\bar{A}e^{-A\tau} - I\right)A^{-1}B. \end{aligned} \tag{20}$$

Block diagram representation of model (19) is shown in figure 2.

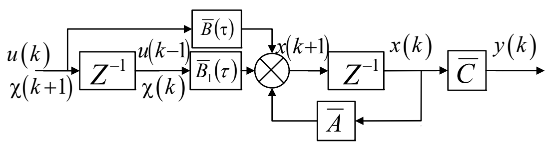


Figure 2: Block diagram of plant with control delay.

Let us introduce an additional state vector χ , then, by figure 2, we get a following discrete-time model

$$\begin{aligned}\tilde{x}(k+1)&=\begin{bmatrix}x(k+1)\\ \chi(k+1)\end{bmatrix}=\\ &=\begin{bmatrix}\bar{A} & \bar{B}_1(\tau)\\ 0 & 0\end{bmatrix}\begin{bmatrix}x(k)\\ \chi(k)\end{bmatrix}+\begin{bmatrix}\bar{B}(\tau)\\ I\end{bmatrix}u(k)=\\ &=\tilde{A}\tilde{x}(k)+\tilde{B}u(k);\, y(k)=\tilde{C}\tilde{x}(k),\end{aligned} \tag{21}$$

where

$$\tilde{A}=\begin{bmatrix}\bar{A} & \bar{B}_1(\tau)\\ 0 & 0\end{bmatrix};\tilde{B}=\begin{bmatrix}\bar{B}(\tau)\\ I\end{bmatrix};\tilde{C}=\begin{bmatrix}\bar{C} & 0\end{bmatrix}, \tag{22}$$

$\dim(\chi)=\dim(u)=r$.

It should be noted, it is recommended to take the discrete interval $\Delta t \geq \tau$. At the same time, the discrete interval is limited as $\Delta t \leq \pi/\Delta\omega$ by the Nyquist–Shannon–Kotelnikov theorem, where $\Delta\omega$ is a system bandwidth. If it is impossible to take the discrete interval $\Delta t \geq \tau$ due to the theorem, then the order of the discrete model (19) increases by more than 1. This case is not considered in the paper.

Consider two cases. *The first case* is called "unpredictable delay" (or unaccounted delay). The control is given by (4), but on account of the modification of plant model (21), (22) the discrete-time system takes the form

$$\begin{aligned}&\tilde{x}(\kappa+1)=\tilde{F}(\tau)\tilde{x}(k)+\tilde{G}(\tau)g(k);\tilde{x}(0)=\tilde{x}(k)\big|_{k=0};\\ &y(k)=\tilde{C}\tilde{x}(k),\end{aligned} \tag{23}$$

where

$$\begin{aligned}\tilde{F}(\tau)&=\begin{bmatrix}\bar{A}-\bar{B}(\tau)\bar{K} & \bar{B}_1(\tau)\\ -\bar{K} & 0\end{bmatrix},\\ \tilde{G}(\tau)&=\begin{bmatrix}\bar{B}(\tau)\bar{K}_g\\ \bar{K}_g\end{bmatrix},\tilde{C}=\begin{bmatrix}\bar{C} & 0\end{bmatrix}.\end{aligned} \tag{24}$$

Block diagram representation of system (23) with control (9) is shown in figure 3.

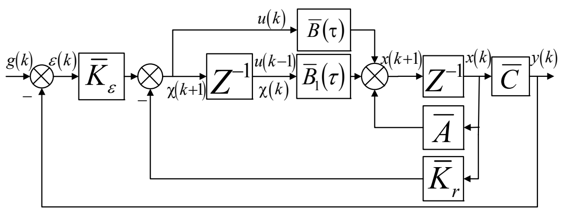


Figure 3: Block diagram of system with unpredictable delay in control.

Free motion of system (23) can be represented by

$$\tilde{x}(\kappa)=\left(\tilde{F}(\tau)\right)^k\tilde{x}(0)=\tilde{M}\tilde{\Lambda}^k\tilde{M}^{-1}\tilde{x}(0). \tag{25}$$

From (25) it follows

$$\begin{aligned}\left\|\tilde{x}(k)\right\|&=\left\|\left(\tilde{F}(\tau)\right)^k\tilde{x}(0)\right\|=\\ &=\left\|\tilde{M}\tilde{\Lambda}^k\tilde{M}^{-1}x(0)\right\|\leq C\left\{\tilde{M}\right\}\tilde{\lambda}_{\max}^k\left\|\tilde{x}(0)\right\|,\end{aligned} \tag{26}$$

where $C\left\{\tilde{M}\right\}$ is condition number of a $(n+1)\times(n+1)$ -matrix $\tilde{M}$; $\tilde{M}=row\left\{\tilde{M}_i=\arg\left(\tilde{F}(\tau)\tilde{M}_i=\tilde{\lambda}_i\tilde{M}_i\right);i=\overline{1,n+1}\right\}$ is eigenvectors matrix of state matrix $\tilde{F}$; $\tilde{\lambda}_{\max}=\max_i\left\{\tilde{\lambda}_i=\arg\left(\tilde{\lambda}I-\tilde{F}(\tau)=0\right);i=\overline{1,n+1}\right\}$. As a result, the upper bound of free motion of system (23) takes the following form

$$\sup\left(\left\|\tilde{x}(k)\right\|\right)\Big|_{\|\tilde{x}(0)\|=1}=C\left\{\tilde{M}\right\}\tilde{\lambda}_{\max}^k. \tag{27}$$

It should be noted that a change of condition number $C\left\{\tilde{M}\right\}$ happens even by $\tau=0$ although eigenvalues set of the matrix $\tilde{F}(\tau)=\tilde{F}(0)$ is increased a zero eigenvalue $\tilde{\lambda}_{n+1}=0$. Let us show that. Consider the matrix $\tilde{F}(0)$

$$\tilde{F}(0)=\begin{bmatrix}\bar{A}-\bar{B}\bar{K} & 0\\ -\bar{K} & 0\end{bmatrix}. \tag{28}$$

Obviously, by property (Gantmacher, 2000) of a block-triangular matrix, the eigenvalues set of matrix (28) consists of the eigenvalues set of the matrix $\sigma\left\{\bar{F}=\bar{A}-\bar{B}\bar{K}\right\}=\left\{\tilde{\lambda}_i=\bar{\lambda}_i;i=\overline{1,n}\right\}$ and $\tilde{\lambda}_{n+1}=0$. For eigenvectors $\tilde{\xi}_i$ of the matrix $\tilde{F}(0)$ the following condition satisfies

$$\tilde{F}(0)\tilde{\xi}_i=\tilde{\lambda}_i\tilde{\xi}_i\Rightarrow\left(\tilde{F}(0)-\tilde{\lambda}_iI\right)\tilde{\xi}_i=0. \tag{29}$$

The eigenvector $\tilde{\xi}_i$ belongs to null space N $\left\{\tilde{F}(0)-\tilde{\lambda}_iI\right\}$ of a characteristic matrix $\left(\tilde{F}(0)-\tilde{\lambda}_iI\right)$. Null space implementation for each eigenvalue $\tilde{\lambda}_i$, including $\tilde{\lambda}_{n+1}=0$, which corresponds a nonzero eigenvector. Eigenvectors $\tilde{\xi}_i$ correspond to $\left\{\tilde{\lambda}_i=\bar{\lambda}_i;i=\overline{1,n}\right\}$ but they don't preserve components of eigenvectors ξ_i. As a result, the condition number $C\{\tilde{M}\}$ significant changes even by $\tau = 0$.

The second case is called "predictable delay" (or accounted delay). The control of plant (21) takes the form

$$u(k) = \tilde{\tilde{K}}_g g(k) - \tilde{\tilde{K}}_x x(k) - \tilde{\tilde{K}}_\chi \chi(k) = \\ = \tilde{\tilde{K}}_\varepsilon \varepsilon(k) - \tilde{\tilde{K}}_{xr} x(k) - \tilde{\tilde{K}}_\chi \chi(k). \tag{30}$$

Combining (21) and (30), we get following discrete-time closed-loop system

$$\tilde{x}(k+1) = \tilde{\tilde{F}}\tilde{x}(k) + \tilde{\tilde{G}}g(k); \tilde{x}(0) = \tilde{x}(k)\big|_{k=0}; \\ y(k) = \tilde{C}\tilde{x}(k). \tag{31}$$

where

$$\tilde{\tilde{F}}(\tau) = \begin{bmatrix} \bar{A} - \bar{B}(\tau)\tilde{\tilde{K}}_x & \bar{B}_1(\tau) - \bar{B}(\tau)\tilde{\tilde{K}}_\chi \\ -\tilde{\tilde{K}}_x & -\tilde{\tilde{K}}_\chi \end{bmatrix}, \\ \tilde{\tilde{G}}(\tau) = \begin{bmatrix} \bar{B}(\tau)\tilde{\tilde{K}}_g \\ \tilde{\tilde{K}}_g \end{bmatrix}, \tilde{C} = \begin{bmatrix} \bar{C} & 0 \end{bmatrix}. \tag{32}$$

Control law (30) is formed such that an eigenvalues set of matrix $\tilde{\tilde{F}}(\tau)$ consists of eigenvalues set of matrix $\bar{F}$ (6) and an eigenvalue $\tilde{\tilde{\lambda}}_{n+1}$. The eigenvalue $\tilde{\tilde{\lambda}}_{n+1}$ is taken to much less than $\bar{\lambda}_i, i = \overline{1,n}$.

Block diagram representation of system (31) with control (30) is shown in figure 4.

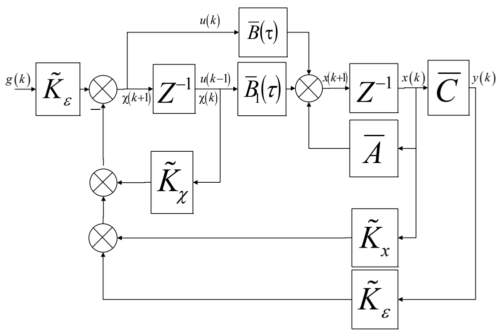


Figure 4: Block diagram of system with predictable delay in control.

Free motion of system (31) can be represented

$$\tilde{x}(\kappa) = \left(\tilde{\tilde{F}}(\tau)\right)^k \tilde{x}(0) = \tilde{\tilde{M}}\tilde{\tilde{\Lambda}}^k \tilde{\tilde{M}}^{-1}\tilde{x}(0). \tag{33}$$

From (33) it follows

$$\|\tilde{x}(k)\| = \left\|\left(\tilde{\tilde{F}}(\tau)\right)^k \tilde{x}(0)\right\| = \\ = \left\|\tilde{\tilde{M}}\tilde{\tilde{\Lambda}}^k \tilde{\tilde{M}}^{-1} x(0)\right\| \le C\left\{\tilde{\tilde{M}}\right\}\tilde{\tilde{\lambda}}^k_{\max}\|\tilde{x}(0)\| \tag{34}$$

where $C\left\{\tilde{\tilde{M}}\right\}$ is condition number of a $(n+1)\times(n+1)$-matrix $\tilde{\tilde{M}}$;

$\tilde{\tilde{M}} = row\left\{\tilde{\tilde{M}}_i = \arg\left(\tilde{\tilde{F}}(\tau)\tilde{\tilde{M}}_i = \tilde{\tilde{\lambda}}_i\tilde{\tilde{M}}_i\right); i = \overline{1, n+1}\right\}$ is eigenvectors matrix of state matrix $\tilde{\tilde{F}}$; $\tilde{\tilde{\lambda}}_{\max} = \max_i\left\{\tilde{\tilde{\lambda}}_i = \arg\left(\tilde{\tilde{\lambda}}I - \tilde{\tilde{F}}(\tau) = 0\right); i = \overline{1, n+1}\right\}$. As a result, the upper bound of free motion of system (30) takes the following form

$$\sup\left(\|\tilde{x}(k)\|\right)\Big|_{\|\tilde{x}(0)\|=1} = C\left\{\tilde{\tilde{M}}\right\}\tilde{\tilde{\lambda}}^k_{\max}. \tag{35}$$

4 EXAMPLE

Consider discrete-time plant (3) for $\Delta t = 0.01\text{s}$ with matrices $\bar{A} = \begin{bmatrix} 1 & 0.01 \\ 0 & 1 \end{bmatrix}$; $\bar{B} = \begin{bmatrix} 0.0001 \\ 0.01 \end{bmatrix}$; $\bar{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

Design modal control law for discrete-time systems with and without delay gives the following.

1. Assume the required quality indicators of the closed loop system are provided by assigning the following eigenvalues $\sigma\{\bar{F}\} = \{\bar{\lambda}_1 = 0.9802; \bar{\lambda}_2 = 0.9048\}$, that corresponds to the eigenvalues $\sigma\{F\} = \{\lambda_1 = -2; \lambda_2 = -10\}$ for the continuous-time analog of the plant. The satisfactory eigenvalues are achieved by the modal control with the feedback matrix $\bar{K} = \begin{bmatrix} 18.84 & 11.4 \end{bmatrix}$. Then, the state matrix of the close loop system (10) is obtained as $\bar{F} = \begin{bmatrix} 0.999 & 0.0094 \\ -0.1884 & 0.886 \end{bmatrix}$ with the eigenvectors $\xi_1 = \begin{bmatrix} 0.4472 \\ -0.8944 \end{bmatrix}; \xi_2 = \begin{bmatrix} 0.0995 \\ -0.995 \end{bmatrix}$, and the condition number of eigenvectors matrix $C\{M\} = C\{[\xi_1, \xi_2]\} = 5.434$. Norm (15) of free motion of the system (10) is shown in figure 5.

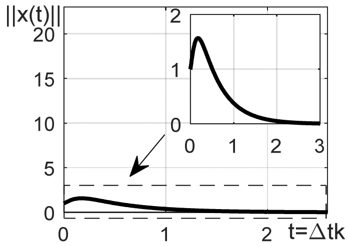


Figure 5: Norm $\left\|x(t)\right\|$ of free motion of the system(10).

The condition number $C\{M\}$ is much greater than one. Thus, there is a deviation of free motion trajectories from a monotone decreasing curve.

2. The case of "unpredictable delay". If the control $u(t)$ of plant (1) for $t = k \cdot \Delta t = k \cdot 0.01s$ is realized with delay $\tau = 0.5\Delta t \leq \Delta t$, then we get the discrete-time system (23) with the same feedback matrix $\bar{K}$ and the following state matrix:

$$\tilde{F}(\tau) = \begin{bmatrix} \bar{A} - \bar{B}(\tau)\bar{K} & \bar{B}_1(\tau) \\ -\bar{K} & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 - 18.84(0.5\Delta t - 1.5\tau)(\Delta t - \tau) & 0.01 - 11.4(0.5\Delta t - 1.5\tau)(\Delta t - \tau) & (\Delta t - 0.5\tau)\tau \\ -18.84(\Delta t - \tau) & 1 - 11.4(\Delta t - \tau) & \tau \\ -18.84 & -11.4 & 0 \end{bmatrix}.$$

Here, the eigenvalues and eigenvectors are derived as $\sigma\left\{\tilde{F}(\tau)\right\} = \left\{\tilde{\lambda}_1 = 0.9801; \tilde{\lambda}_2 = 0.8987; \tilde{\lambda}_3 = 0.0644\right\}$,

$$\tilde{\xi}_1 = \begin{bmatrix} -0.2179 \\ 0.4352 \\ -0.8735 \end{bmatrix}; \tilde{\xi}_2 = \begin{bmatrix} 0.0089 \\ -0.0933 \\ 0.9956 \end{bmatrix}; \tilde{\xi}_3 = \begin{bmatrix} 0 \\ 0.0057 \\ -1 \end{bmatrix}.$$

The condition number of eigenvectors matrix is $C\left\{\tilde{M}\right\} = C\left\{\left[\tilde{\xi}_1, \tilde{\xi}_2, \tilde{\xi}_3\right]\right\} = 70.674$. Norm of free motion of the system (23) is shown in figure 6.

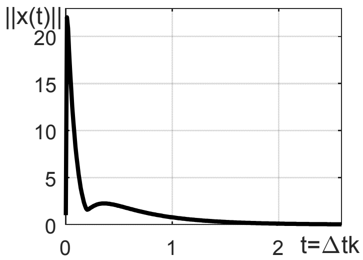


Figure 6: Norm $\left\|x(t)\right\|$ of free motion of the system (23).

The condition number $C\left\{\tilde{M}\right\}$ is sufficiently large. Thus, there is significant deviation of free motion trajectories from a monotone decreasing curve.

3. The case of "predictable delay". Assume system (31) is realized with the control law (18). The third eigenvalue is assigned as $\tilde{\tilde{\lambda}}_3 = 0.8187$. Then the feedback matrix $\tilde{\tilde{K}} = \begin{bmatrix} 3.4157 & 2.247 & -0.715 \end{bmatrix}$ provides the state matrix

$$\tilde{\tilde{F}}(\tau)\Big|_{\tau = 0.5\Delta t = 0.005} = \begin{bmatrix} 1 & 0.01 & 0 \\ -0.0171 & 0.9888 & 0.0086 \\ -3.4157 & -2.2468 & 0.715 \end{bmatrix},$$

with eigenvalues

$$\sigma\left\{\tilde{\tilde{F}}(\tau)\right\} = \left\{\tilde{\tilde{\lambda}}_1 = 0.9802; \tilde{\tilde{\lambda}}_2 = 0.9048; \tilde{\tilde{\lambda}}_3 = 0.8187\right\},$$

eigenvectors

$$\tilde{\tilde{\xi}}_1 = \begin{bmatrix} 0.2192 \\ -0.4363 \\ 0.8727 \end{bmatrix}; \tilde{\tilde{\xi}}_2 = \begin{bmatrix} -0.0102 \\ 0.0996 \\ -0.995 \end{bmatrix}; \tilde{\tilde{\xi}}_3 = \begin{bmatrix} -0.0026 \\ 0.05 \\ -0.9987 \end{bmatrix},$$

and the condition number of eigenvectors matrix $C\left\{\tilde{\tilde{M}}\right\} = C\left\{\left[\tilde{\tilde{\xi}}_1, \tilde{\tilde{\xi}}_2, \tilde{\tilde{\xi}}_3\right]\right\} = 131.9$. Norm of free motion of this system is shown in figure 7a.

If we take another third eigenvalue, that is close to 1, then the feedback matrix $\tilde{\tilde{K}} = \begin{bmatrix} 0.1875 & 0.3014 & -0.8766 \end{bmatrix}$ provides the state matrix

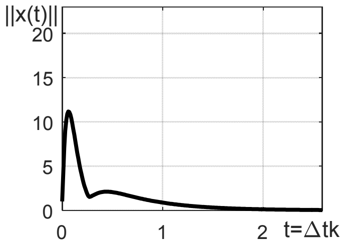
$$\tilde{\tilde{F}}(\tau)\Big|_{\tau = 0.5\Delta t = 0.005} = \begin{bmatrix} 1 & 0.01 & 0 \\ -0.0009 & 0.9985 & 0.0094 \\ -0.1875 & -0.3014 & 0.8766 \end{bmatrix}$$

with eigenvalues $\sigma\left\{\tilde{\tilde{F}}(\tau)\right\} = \left\{\tilde{\tilde{\lambda}}_1 = 0.9802; \tilde{\tilde{\lambda}}_2 = 0.9048; \tilde{\tilde{\lambda}}_3 = 0.99\right\}$, and eigenvectors

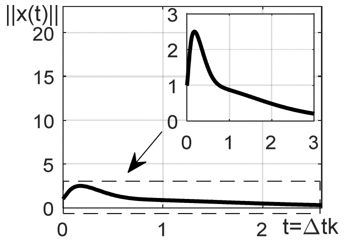
$$\tilde{\tilde{\xi}}_1 = \begin{bmatrix} 0.2192 \\ -0.4363 \\ 0.8727 \end{bmatrix}; \tilde{\tilde{\xi}}_2 = \begin{bmatrix} -0.0102 \\ 0.0996 \\ -0.995 \end{bmatrix}; \tilde{\tilde{\xi}}_3 = \begin{bmatrix} -0.5783 \\ 0.5769 \\ -0.5769 \end{bmatrix}$$

of the system (31). In this case the condition number of eigenvectors matrix is $C\left\{\tilde{\tilde{M}}\right\} = C\left\{\left[\tilde{\tilde{\xi}}_1, \tilde{\tilde{\xi}}_2, \tilde{\tilde{\xi}}_3\right]\right\} = 17.5787$. Norm of free motion of this system is shown in figure 7b.

From received curves it follows that the greater modulus of eigenvalue $\tilde{\tilde{\lambda}}_3$ is, the smaller condition number $C\left\{\tilde{\tilde{M}}\right\}$. This indicates a damping of the deviation.



a



b

Figure 7: Norm $\left\| x\left(t\right) \right\|$ of free motion of the system (31) with different values of the third eigenvalue.

5 CONCLUSIONS

Stable discrete-time systems with control signal delay were considered in the paper and the problem of large deviations in the free motion of the systems was investigated. The upper bound of peaking processes in the system was estimated with the condition number of its eigenvectors matrix. The modal control law was designed taking into account the delay and the deviation. It was shown relationship between the delay and the eigenvectors structure, and the value of the deviations in the free motion of the discrete-time system. Modification of the additional eigenvalue due to the control signal delay gives opportunity to affect the deviation and reduce it. The level of the deviation reducing depends on requirements to dynamic quality indicators of the researched system.

In future, it is supposed to expand the results of the paper to the case of stabilization and control discrete-time systems by observers with unknown initial conditions.

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