Numerical Simulation of Internal Wave Attractors in Horizontally Elongated Domains with Sloping Boundaries

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Abstract: Global tides create continuous input of mechanical energy to the ocean, and produce inertia-gravity waves which change the processes of vertical mixing, affect background stratification, create mean and zonal currents. Internal and inertial wave attractors may arise in natural ocean and coastal topography, and due to energy accumulation they produce turbulence or increase turbulence intensity even for moderate values of the input energy. Previous studies of internal wave attractors at a laboratory scale were concerned mostly with basins of comparable vertical and horizontal dimensions, while in ocean we typically deal with horizontally elongated geometries and low ratios between the tidal and buoyancy frequencies. In this work we describe laminar and turbulent regimes of internal wave attractors in domains with large aspect ratio subject to forcing at low non-dimensional frequency.

1 INTRODUCTION

A large amount of energy is constantly injected into interior of the Ocean due to interaction of barotropic tides with the bottom topography. As a result the inertia-gravity waves radiate from generation sites, producing significant effects on the vertical mixing and stratification (Morozov, 1995; Dauxois et al., 2018). In addition, inertia-gravity wave motion resulting from tides can produce mean and zonal flows (Winters, 2015; Manders and Maas, 2004; Harlander and Maas, 2007). Inertia and gravity waves possess a very peculiar dispersion relation, which for certain frequency defines only the direction of wave propagation with respect to gravity or angular velocity vector. It was shown theoretically, experimentally and numerically, that in closed basins with sloping boundaries there may exist an attracting geometrical structure in physical space, a wave attractor. The theory of wave attractors in ideal fluids is well elaborated now, and in (Maas et al., 1997) one can see a diagram illustrating a rich variety of internal wave-ray patterns in a closed domain as function of two parameters, governing the geometry of the domain and the wave-ray slopes. While this diagram is very useful to identify the domains of parameters corresponding to strong energy focusing in ideal fluid, it is not fully relevant to real fluids, with effects due to viscosity and wave turbulence included. Previous works (Brouzet et al., 2016b; Brouzet et al., 2016a; Brouzet et al., 2017) considered trapezoidal geometry of the fluid domain, with comparable vertical and horizontal dimensions. Transition to nonlinear wave turbulence was studied in details, and scaling laws were investigated for such a geometric setup.

In the present paper we show some numerical results for internal wave attractors in a horizontally elongated domain at low ratio between forcing and buoyancy frequency, which is closer to typical situations in oceanology applications.

2 MATHEMATICAL MODEL

The subject of this study is the internal wave dynamics in a closed domain with one border inclined with respect to the vertical. Such a geometry can naturally occur near ridges, coastal areas, continental slopes and so on. We perform Direct Numerical Simulations

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within the framework of a mathematical model consisting of Navier – Stokes equations coupled to transport and diffusion of salt and continuity equations, and the boundary conditions given below:

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v},\nabla)\vec{v} = -\nabla \frac{\tilde{p}}{\rho_m} + \nu\Delta\vec{v} + \rho_s\vec{g}; \qquad (1)$$

$$\frac{\partial \rho_s}{\partial t} + (\vec{v},\nabla)\rho_s = \lambda_s\Delta\rho_s, \quad \mathrm{div}(\vec{v}) = 0, \qquad (2)$$

where $\vec{v}$ – velocity; $\rho = \rho_m + \rho_s$ – density; ρ_m – minimal density; ρ_s – salt density; $\tilde{p}$ - pressure ρ_m; ν – kinematic viscosity; λ_s – coefficient of salt diffusion.

We consider a continuously stratified initial state with constant buoyancy frequency

$$\mathrm{N}(z) = \sqrt{-\frac{\mathrm{g}}{\rho(z)}\frac{\mathrm{d}\rho(z)}{\mathrm{d}z}}$$

We prescribe the no-slip condition at all rigid boundaries except for one boundary where we apply a specific harmonic forcing. In the case of rotating fluid, different types of such forcing are introduced in (Sibgatullin et al., 2017), which describes for the first time the laminar and weakly turbulent inertial wave regimes in an annular frustum. In the case of stratified fluid, the boundary conditions and external forcing closely mimicking the experimental ones are described in (Brouzet et al., 2016b).

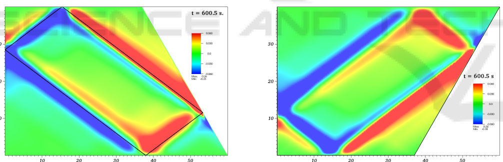

Figure 1: Internal wave attractors for two geometries, having opposite slopes of the right boundary with respect to the gravity direction. Vertical component of velocity is shown with color. The energy flows in opposite directions in left and right configurations, otherwise the dynamics is similar.

Figure 1 presents the sketch of the generic geomerical setup used in analytical, numerical and experimental studies of the wave attractors. The fluid is confined inside a trapezoidal prism, with two bases of the trapeze perpendicular to gravity, the left wall being vertical and the right wall tilted by a certain angle α. The first attempts to model wave attractors assumed two-dimensionality of the flow (Grisouard et al., 2008). This assumption allowed a fully relevant qualitative simulation of wave attractors, with relatively poor quantitative agreement between numerical and experimental results. Later, the first fully 3D DNS greatly improved the quantitative agreement between experiments and simulations both in linear and non-linear regimes, and showed the significance of the dissipation at the lateral walls of the fluid domain (Brouzet et al., 2016b; Brouzet et al., 2016a; Beckebanze et al., 2018). Interestingly, numerical simulations of wave attractors generated by a spanwise localized wave maker in the otherwise two-dimensional setup demonstarted a strong qualitative trend toward two-dimensionality of wave motion, but with specific non-uniform energy distribution in spanwise direction and significant effects due to the lateral walls (Pillet et al., 2018).

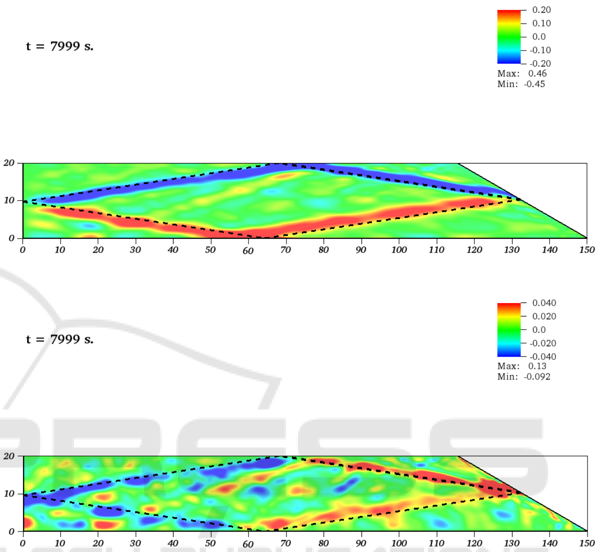

Figure 2: $\alpha = 60^\circ$, a=0.02 cm, horizontal and vertical components of velocity.

Substituting the solution in form of monochromatic travelling waves into linearised Euler equations describing the motion of a uniformly stratified ideal fluid, one can obtain the following dispersion relation:

$$\omega_0^2 = N^2\sin^2(\theta), \qquad (3)$$

where ω_0 – wave frequency. It can be seen that the dispersion relation admits the propagation of waves in form of oblique wave beams, but does not contain any length scale. In many practical problems a qualitative description of wave patterns can be obtained by neglecting the width of wave beams and considering the propagation of wave rays. The problem of wave-ray billiard in the geometric setup presented in Figure 1 can be re-scaled by introducing two quantities d and τ, d being the parameter responsible for the slope (with the limiting values -1 and $+1$ corresponding to triangular and rectangular geometry), and $\tau = \frac{2H}{L}\sqrt{\left(\frac{N}{\omega_0}\right)^2 - 1}$ being the parameter responsible for the forcing frequency. In ideal fluid, a diagram of Lyapunov exponents of internal wave rays (Maas et al., 1997) characterises the wave regimes, and show the domains of strong convergence toward attractors. However, in real world viscosity and wave turbulence come into play, adding a rich variety of linear and non-linear regimes.

In numerical simulations of linear regimes one needs to resolve the boundary layers at rigid boundaries, and the structure of the attractor beams, which appear in form of oblique viscous shear layers. In simulations of strongly nonlinear regimes, one needs to resolve the small-scale patterns resulting from a cascade of wave-wave interactions. In a density-stratified fluid the numerical simulations are complicated by necessity to resolve the diffusion of the stratifying agent, and at high values of the Prandtl-Schmidt number the small-scale structures appear, which greatly complicate the interaction of the waves beams of the attractors with the walls (Sibgatullin and Kalugin, 2016). In direct numerical simulations described in the present paper we use the spectral element method, which proves to be highly efficient for this type of problems (Brouzet et al., 2016b; Brouzet et al., 2016a; Beckebanze et al., 2018).

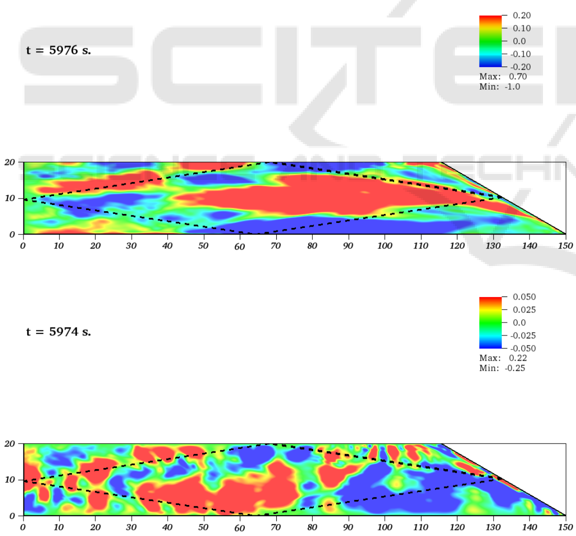

Figure 3: $\alpha = 60^{\circ}$, a=0.05 cm, horizontal and vertical components of velocity.

3 NUMERICAL RESULTS

Many previous studies on dynamics of internal wave attractors used the trapezoidal geometry depicted in Figure 1 (Mass et al., 1997; Scolan et al., 2013; Brouzet et al., 2016b; Brouzet et al., 2016a; Beckebanze et al., 2018; Brouzet et al., 2017; Dauxois et al., 2017), where the largest base of the trapezium varied between 30 to 150 cm and the fluid depth was about 2/3 of the largest base. The angle of the slope with the gravity was typically about 30°. The external forcing was realized by moving vertical boundary of the fluid volume or by oscillating the tank filled with the density stratified water.

Significant change of aspect (depth-to-length) ratio of the geometrical setup may introduce new dynamical effects in the behaviour of the system at laminar and turbulent regimes. Of particular interest is the horizontally elongated geometry, which is relevant to typical applications in oceanography.

In the following, the tidal forcing is simulated in a highly simplified way by imposing small-amplitude oscillations of the largest base of the trapezium L in the following form:

$$\xi = a \sin(2\pi x/L) \sin(\omega_0 t)$$

We take the length of the domain $L = 150$ cm and the depth is $H = 20$ cm. The value of the buoyancy frequency is $N = 1$ rad/s. The kinematic viscosity of the fluid is taken to be $\nu = 0.002$ cm^2/s, five times smaller than the viscosity of water. The global Stokes number, which can be introduced as $St = H^2 N/\nu$ is $2 \cdot 10^5$. In the present paper we are mostly concerned with the qualitative effects observed in horizontally elongated domains. Therefore we perform two-dimensional simulations.

Figure 2 shows the horizontal and vertical components of velocity field at small forcing amplitude $a = 0.02$ cm. The narrow wave beams are localized in the vicinity of the theoretical sceleton of the attractor given by solution of the wave-ray billiard and marked with a black dashed line. One can identify the presence of nonlinear effects, in particular, wave beams at double frequency $2\omega_0$ emitted at the points of reflection of the primary wave beam. Note that the possibility of super-harmonic effects is an important feature of the horizontally elongated setup. In previous studies of wave turbulence in internal wave attractors (Scolan et al., 2013; Brouzet et al., 2016b; Brouzet et al., 2016a) the typical non-dimensional forcing frequency was $\omega_0/N > 0.5$. Therefore the super-harmonics could exist only in form of evanescent waves. The present study considers a more realistic case where the super-harmonic waves are propagative.

Figure 3 shows the horizontal and vertical components of the velocity field for the forcing amplitude $a = 0.05$ cm (i.e. only 2.5 times larger than in the case shown in Figure 2). Note that the forcing amplitude remains very small compared to the size of the fluid domain: a/L is about $3 \cdot 10^{-4}$. The wave pattern now is completely different: we can see at the same time the small-scale structures located in the upper right corner, and the large-scale structures in the bulk of the domain. This observation suggests the existence of the inverse cascade resulting from the complex turbulent wave motion. This qualitative consideration is to be confirmed by a careful Fourier-Hilbert analysis being performed at present time. Importantly, the observed snapshot bears no resemblance to a classic pattern of internal wave attractor. Therefore, at high energy input into the system the fact that a particular geometry admits the existence of wave attractors may be completely hidden by wave turbulence although the concentration of energy at the attractor is at the origin of the observed energy cascade. This is particularly important for interpretation of oceanographic measurements which typically deal rather with the large-scale patterns seen Figure 3 than with the narrow-beam patterns presented in Figure 2. Establishing the link between the two patterns may significantly enrich our understanding of the underlying nonlinear dynamics. Broad numerical calculations are now under way, focused at the analysis of the spectra of the wave motions in space and time and other characteristics of wave turbulence in different areas of horizontally elongated fluid domains.

4 CONCLUSIONS

This communication describes the peculiarities of internal wave turbulence in horizontally elongated confined fluid domains with one sloping wall. The wave attractors in such a geometry typically have a lower amplitude threshold corresponding to onset of wave turbulence as compared to conventional setups where length and depth of the fluid domain are comparable.

The most important result of this study consists in revealing the inverse cascade. We consider the time-evolution of the internal wave regime from the state of rest toward formation of the attractor and transition to a fully developed wave turbulence via a cascade of wave-wave interactions. In the regime of wave turbulence we observe the small scale structures to merge and form larger structures. This result may explain in particular why internal wave attractors are hard to detect in natural conditions: we see here that they initially act as a source of wave turbulence, but afterwards they are hidden behind the large scale turbulence arising due to development of the inverse cascade.

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