Numerical Modelling of High-frequency Internal Waves Generated by River Discharge in Coastal Ocean

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Abstract: A method for numerical simulation of internal waves generation by discharges of small and rapid rivers flowing into a coastal sea is proposed. The method is based on PFEM-2 (Particle Finite Element Method, 2nd version) and utilizes particles to simulate convection as well as transfer salinity. Main simplifying assumptions and the mathematical model are presented for this problem. The numerical scheme is split into predictor (particles motion) and corrector (finite element method solution) steps. The resulting method is expected to be efficient in terms of mesh fineness and length of simulation time.

1 INTRODUCTION

Satellite imagery detects internal waves with short wavelength (<100 m) that are generated in the areas adjacent to estuaries of small rivers and propagate offshore within river plumes (Fig. 1). This process is regularly observed in mountainous coastal areas, in particular, the Ring of Fire (the Pacific coasts of Mexico, Peru, and Chile; Taiwan, New Guinea, New Zealand), in Western Balkans, Western Caucasus.

A mechanism of generation of these internal waves by discharges of small and rapid rivers inflowing to coastal sea was recently described by Osadchiev (2018). Friction between river runoff at high velocity and the subjacent sea of one order of magnitude lower velocity causes abrupt deceleration of a freshened flow and increase of its depth, i.e., a hydraulic jump is formed. Transition from supercritical to subcritical flow conditions effectively transforms kinetic energy of river flow to potential energy and induces generation of high-frequency (65 – 90 s) internal waves. These internal waves propagate off a river mouth at a stratified layer between a buoyant river plume and subjacent ambient sea with phase speed equal to 0.45 – 0.65 m/s and dissipate within the plume or at its lateral border. These internal waves increase turbulence and mixing at this layer and, therefore, influence structure and dynamics of the river plume.

The process of generation of internal waves by river discharge described above was reported and analyzed for small river plumes located off the northeastern coast of the Black Sea (Osadchiev, 2018). It was shown that river runoff forms a hydraulic jump and generates internal waves under certain conditions defined by properties of a river flow, ambient sea water, and a local topography. In particular, a river current has to be fast enough to form a supercritical freshened flow in vicinity of a river mouth. On the other hand, kinetic energy of a freshened flow has to be low enough to be inhibited by friction with ambient sea along a strongly stratified bottom boundary of a river plume. This condition is satisfied if river discharge rate is low, i.e., river is rapid but small.

Despite a certain progress in study of high-frequency internal waves referred above, many aspects of their generation, propagation, and dissipation remain unaddressed. In particular, emerging of internal waves with a certain period is

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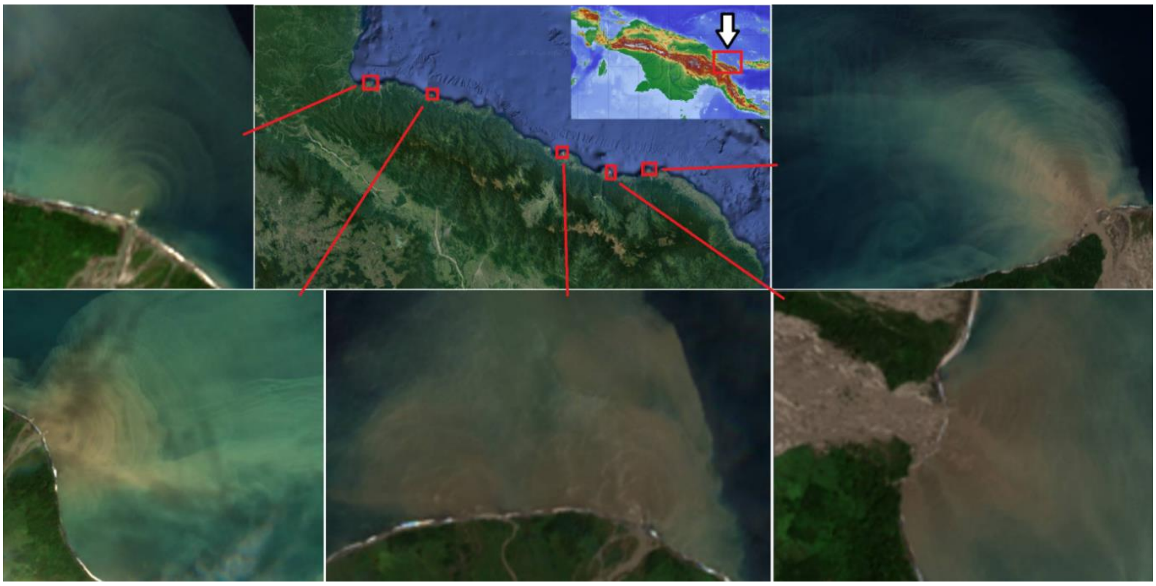

Figure 1: Sentinel-2 ocean color composite from 28 December 2017 indicating internal waves generated by small rivers along the northeastern coast of New Guinea.

presumed to be caused by oscillation of a quasi-stationary hydraulic jump. However, the exact background physical mechanism is largely unknown and requires a detailed study. Thorough description of this mechanism will reveal dependence of parameters of internal waves (frequency, phase speed, wavelength) on external conditions (river inflow velocity, river plume height, salinity and vertical stratification, salinity of ambient sea, etc.). These dependences are crucial for evaluation of wave-induced turbulence and mixing intensity in bottom and lateral frontal zones of a river plume that govern its structure and mixing dynamics (Osadchiev and Korshenko, 2017; Osadchiev and Sedakov, 2019). In this study we apply a novel Eulerian-Lagrangian approach to reveal these issues. Based on results of numerical experiments we present new insights into the processes describe above.

2 GOVERNING EQUATIONS

For numerical simulation of such phenomenon the following simplifying assumptions can be taken into account (Milne-Thomson, 2011):

1) the water is incompressible, so the velocity field $\vec{u}$ is divergence-free:

$$\nabla \cdot \vec{u} = 0 \; ; \qquad (1)$$

2) the temperature of the water is considered to be constant, its density ρ depends only on the salinity S (in per mille), we confine the approximate dependency to linear term

$$\rho \approx \rho_0 + \alpha S \, , \qquad (2)$$

3) where ρ_0 is the density of fresh water (at $S = 0$), $\alpha \approx 0.65$ kg/m^3 for salt water;
4) the Boussinesq-type approximation is considered in order to take into account the buoyancy-driven flow which arises due to the density difference caused by non-uniform salinity distribution:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho_0} \nabla p + \nu \Delta \vec{u} - \vec{g}\,\alpha S \quad , \qquad (3)$$

where p is the pressure field, ν is the kinematic viscosity coefficient, which we assume to be constant ($\nu \approx 10^{-6}$ m^2/s for the water); $\vec{g}$ is the gravity acceleration;

5) the salinity distribution in general case is described by the convection-diffusion equation

$$\frac{\partial S}{\partial t} + \vec{u} \cdot \nabla S = D \Delta S \, , \qquad (4)$$

where D is the diffusivity coefficient; for saline solution it weakly depends on concentration and can be considered equal to $D \approx 1.1 \cdot 10^{-9}$ m^2/s. The latter means that the Schmidt number

$$\mathrm{Sc} = \frac{\nu}{D}, \qquad (5)$$

which defines the ratio of momentum diffusivity (kinematic viscosity) and mass diffusivity, has the order of 10^3, so the salinity diffusion process can be neglected. As the result, we have the equation for salinity

$$\frac{dS}{dt} = \frac{\partial S}{\partial t} + \vec{u} \cdot \nabla S = 0, \qquad (6)$$

which means that salinity is being transferred along the velocity streamlines as a passive impurity.

3 COMPUTATIONAL DOMAIN AND BOUNDARY CONDITIONS

Preliminary numerical simulation can be provided for 2D and 3D axisymmetric (a semicircular sector is considered) cases. It seems reasonable to consider a rectangular computational domain shown in the Fig. 2.

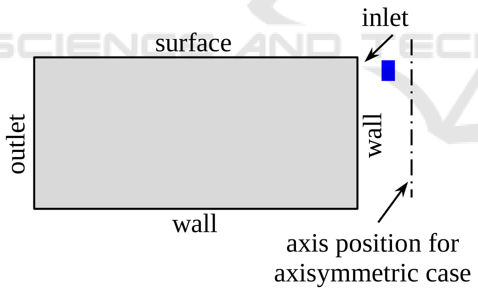

Figure 2: Computational domain.

The boundary conditions on the computation domain boundaries are the following:
Inlet: given values of the velocity and salinity which refer to the fresh water from the river.
Wall: zero velocity and zero gradient of salinity and pressure.
Outlet: mixed boundary condition: a) zero gradient of velocity and salinity if normal component of velocity is directed outside the flow domain, b) given value of the total pressure and given value of salinity, which refers to the salt water in the sea, when normal component of velocity is directed inside the flow domain.
Surface: it is assumed that the surface is flat which means that the surface waves are not simulated; normal (vertical) velocity is zero, pressure is equal to a fixed value which refers to atmospheric pressure, shear stress is equal to zero. Surface waves can still be simulated in a more precise simulation, that would require description of the water surface position and the Boussinesq boundary conditions consideration on the free surface. However, even in this case the surface tension effect should be neglected because the Weber number $\mathrm{We} = \frac{\rho u^2 l}{\sigma}$ has the order of $10^3 \ldots 10^4$

4 NUMERICAL METHOD

The described problem can be solved numerically by using, for example, finite volume method, implemented in a number of computational codes. However, the flow in the considered case is convection-dominated, it means that the convective term in the Navier–Stokes equation is prevalent over all other terms: pressure gradient and viscous diffusion. Moreover, the salinity distribution evolution is considered as pure convective transport, as it was mentioned above. It means that the precision of approximation of the convective terms is much more crucial than of the pressure and diffusive ones. The computational domain can be rather large (in comparison to the size of the river outlet), and physical time in simulation also should be long enough to obtain a quasi-steady regime of flow.

Such requirements make it reasonable to use hybrid Lagrangian–Eulerian methods for numerical simulation. PFEM-2 (particle finite element method, the 2-nd version; Idelsohn, Onate and Del Pin, 2004, as well as Idelsohn et al., 2013) seems to be the most suitable among them. Its main features are the following:

1) Particles, which move along the streamlines of the velocity field, are introduced in order to approximate the convective term in the Navier–Stokes equation. These particles are ‘immaterial’, i.e. they do not transfer the mass and other intrinsic properties except of marker of the phase (for multiphase flows). In the considered problem, it seems to be convenient to consider salinity as the other property transferred by the particles.

2) Fixed mesh, which can be rather coarse, is introduced in the flow domain for finite-element approximation of the other terms – pressure gradient, viscous term and buoyancy term.

3) Interpolation technique is used in order to transmit velocities and salinity from the particles to mesh nodes and vice versa.

So each time step is split into two stages. The first one is predictor, which consists of the particles motion phase. Note that the step size can be rather high (CFL number is allowed to be more than 1), while for particles motion simulation it is normally split into sub-steps in order to provide CFL not more than 0.10…0.15. At the same time, the velocity field is considered to be known at the mesh nodes from the previous time step, so its finite element reconstruction and Euler (or Runge–Kutta) explicit method for particles motion integration make this operation not time-consuming and easy to parallelize.

After the particles motion sub-steps have been performed, the velocity and salinity values, having been transferred with the particles, should be projected onto mesh nodes. Now the convection (predictor) stage is finished.

The correction stage consists of solution of the equation

$$\frac{\partial \vec{u}}{\partial t} = -\frac{1}{\rho_0}\nabla p + \nu\Delta\vec{u} - \vec{g}\,\alpha S\,, \qquad (7)$$

where the initial value is known from the prediction step. This equation no longer contains the convection term, which is highly sensitive to numerical approximation. Implicit finite element scheme is used, both monolithic and coupled strategies (including fractional step approach) can be used for calculation of new values of $\vec{u}$ and p. The velocity field $\vec{u}$ should satisfy the incompressibility equation (1) while this property is broken for the velocity field after ‘predictor’ stage.

5 CONCLUSIONS

The proposed method is expected to be an efficient means of simulation of a mechanism of generation of internal waves by discharges of small and rapid rivers inflowing to coastal sea. The heavy use of particles makes computation possible on rather coarse meshes as well as long periods of simulation time, which is required to reach a quasi-steady state. Current implementation has been provided for 2D and axisymmetric problems but can be further developed for solution of fully 3D simulation cases.

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