

Deriving Programs by Reliability Enhancement

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Keywords: Specification, Formal Approach, Program Derivation Process, Refinement, Relative Correctness, Reliable Software.

Abstract: This paper concerns the exploration of an approach that deals with formal program derivation in contrast to the traditional approach that begins with a formal specification, derive different refinements of that specification until generating the final correct program code. Hence, we use a rigorous theoretical framework which is based on the concept of relative correctness; the property of a program to be more correct than another program with respect to a specification. Program derivation process by relative correctness presents several advantages as for example deriving reliable software. In fact, for most software products, as for products in general, perfect correctness is not necessary; very often, adequate reliability threshold is sufficient. Our aim is to continue experimenting with the discipline of reliable program derivation by correctness enhancement by conducting an analytical and empirical study of this approach as a proof of concept. Then, to analyze the results and compare them (give feedback) to what is predicted and proposed by the analytical approach and decide on the usability of the approach and/or adjust/complete it. Finally, we propose a mechanism that helps and guides developer in the program derivation process using relative correctness.

1 INTRODUCTION

As we know, deriving programs from specifications by using a formal approach assures program correctness and better quality. Reuse based software engineering is essential nowadays where lot of software artifacts are available. Deriving programs from other existing programs can also be envisaged, more specifically, we can construct programs by successive program transformations. We are working on the program derivation process by correctness enhancement that was introduced by (Diallo et al., 2015) and refined by (Benabdelali et al., 2018). This process is orthogonal to traditional refinement-based program derivation processes (Back, 1978; Morgan, 1990) in the sense that instead of preserving correctness and enhancing executability (as traditional processes do), it preserves executability (i.e. proceeds from one executable program to the next) while enhancing correctness until it achieves absolute correctness. In this paper, we observe that the sequence of programs derived in the correctness enhancement process are increasingly reliable, and we argue that this process can be used to derive programs that are sufficiently reliable (for a given reliability threshold) even if they are not necessarily (absolutely) correct. Therefore, we

are conducting an analytical and empirical study as a proof of concept about deriving programs by enhancing correctness. We are going to present a set of relatively simple programs as a first sample. This latter will permit to illustrate the approach and to find mechanisms or guidelines that permit the transformation of a program to another. The paper is structured as follows: Section 2 discusses both approaches of program construction process according to a given specification. Section 3 presents the analytical and empirical analysis of the program derivation by correctness-enhancing transformations and describes the program derivation mechanism that our empirical study leads us to highlight. This mechanism will permit to guide how transformations can be done in program derivation by relative correctness process. Section 4 summarizes our findings and presents the future directions of our research.

2 PROGRAM DERIVATION PROCESS FROM SPECIFICATIONS

In computer science, program derivation aims to produce an executable and correct program from its

specification by applying mathematical correct rules. So, we are concerned with transformation process that takes a formal specification of what the computer is to do and produces a program which will cause the computer to do it. The program thus obtained is then correct by stepwise derivation. In the present paper, we are interested to explore a formal approach of program derivation that is based on relative correctness and consists in successive correctness-enhancing transformations in contrast to the traditional refinement-based process of successive correctness-preserving transformations. Before embarking on the study of the programs derivation process, we briefly introduce some elements of relational mathematics (Chris Brink and Schmidt, 1997) that we use throughout the paper to represent the specifications and programs functions. To represent specifications and programs, we need sets and relations. A set is the space S of the program and its elements is the *states* of the program. Indeed, given a program p that operates on *space* S , we let P be the function that defines the program, this function is represented as a set of pairs (s, s') where s is the start execution state of P and s' is the end execution state. A relation R on a set S is a subset of the cartesian product $S \times S$. Relations on a set S include the *identity* relation denoted by $I = \{(s, s') | s' = s\}$, *empty* relation denoted by $\emptyset = \{\}$ and the *universal* relation denoted by $L = S \times S$.

2.1 Refinement-based Program Derivation

A program is a sequence of different operations and calculations. The difficulty in developing such a program is usually proportional to the size of the specification. It is often difficult to write the final version at the first shot. Traditionally, to solve this problem we proceed in stages using the refinement approach. The latter, was first introduced by (Back, 1978), (Morgan, 1990) and (Back and Wright, 1998) and is defined as a notation and a set of rules for deriving programs from their specifications. The basic idea behind this approach is to derive a correct program through a sequence of refinement steps, begins with a formal specification until generating the final correct program meeting the specification.

Definition 1: Given two programs P and P' , we write $P \sqsubseteq P'$ when P' is an effective program, meeting the specification more than P . The relation \sqsubseteq is called *refinement* and we say that P' refines P . To respond such definition, (Morgan, 1990) presents a catalogue of lemmas that determines how specification may be refined to an executable code. As an illustration of

this definition, we would like to construct a program that manipulates three variables of type float, say, u , v , w where v , w are positive. The program must give $\log(v)$ in u and any positive value in v .

Space, $S = \text{float } u, v, w;$
 $\{(s, s') | v > 0 \wedge w \geq 0 \wedge u' = \log(v) \wedge v' > 0 \wedge w' > 0\}$
 \sqsubseteq
 $\{(s, s') | v > 0 \wedge w \geq 0 \wedge u' = \log(v) \wedge v' > 0 \wedge w' = w\}$
 \sqsubseteq
 $\{(s, s') | v > 0 \wedge w \geq 0 \wedge u' = \log(v) \wedge v' = \text{sqrt}(w) \wedge w' = w\}$
 $= [u := \log(v); v := \text{sqrt}(w);] =$ a piece of the program that is conformed with the first specification.

Although refinement is a well-known approach for developing correct-by-derivation software that has proven its value in software development, nowadays the issues become no longer to construct applications from scratch and achieving the correctness, but rather to maintain and evolve them. Therefore, as for products in general, perfect correctness is not necessary; very often, adequate reliability (depending on the level of criticality of the application) is sufficient.

2.2 Relative Correctness-based Program Derivation

With the Relative Correctness-based program derivation approach, the objective becomes no longer to develop a program that is absolutely correct in relation to the specification but rather to achieve a satisfactory reliability threshold. The term *reliability* according to (O'Regan, 2017) is *the probability that the program works without failure for a specified length of time, and it is a statement of the future behaviour of the software*. So, in this context and while most approaches deal with informal program derivation techniques, the approach underlying this present work is based on a formal approach. Hence, we use a rigorous theoretical framework which is based on the concept of relative correctness. This latter, was introduced by (Diallo et al., 2015) as a viable alternative approach to the traditional refinement-based process of successive correctness-preserving transformations starting from the specification and culminating in a correct program (see Figure 1).

Among the many properties of relative correctness, the most intriguing is the property that program P' refines program P if and only if P' is more-correct than P with respect to a specification. This yield to reconsider program derivation by successive refinements: each step of this process mandates that we transform a program P into a program P' that refines P .

Definition 2: due to (Diallo et al., 2015) given two

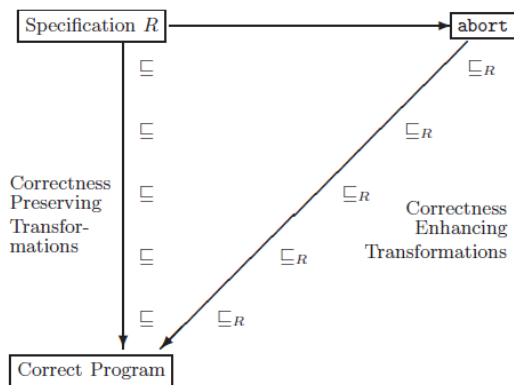


Figure 1: Program Derivation process (Diallo et al., 2015).

programs P , P' and a specification R ; we say that P' is *more – correct* than P if and only if P' obeys R for a larger set of inputs than P . This relation is denoted by $P \sqsubseteq_R P'$ which is equivalent to the relation: $(R \cap P) \circ L \subseteq (R \cap P') \circ L$. Also, we say that P' is *strictly more – correct* than P with respect to R if and only if $P \sqsubset_R P'$ which is equivalent to the relation $(R \cap P) \circ L \subset (R \cap P') \circ L$. \circ is the relative product of two relations.

The relation $(R \cap P) \circ L$ refers to the set of initial states on which the behavior of P satisfies specification R . This set is denoted the *competence domain* of P with respect to R . Relative correctness of P' over P with respect to R simply means that P' has a larger competence domain than P . We illustrate this definition by a simple example; given a specification R defined on space $S = \{a, b, c, d, e, 1, 2, 3, 4, 5\}$

$$R = \{(a,2), (b,4), (c,5), (d,1), (a,3)\}$$

Where for the input a it gives the output 2 and 3, for the input b it gives the output 4, for the input c it gives the output 5 and for the input d it gives the output 1.

And let P and P' be the following candidates programs: $P = \{(a,2), (c,5)\}$. $P' = \{(a,2), (b,4), (c,5)\}$
 $(R \cap P) \circ L = \{a, c\} \times S$. $(R \cap P') \circ L = \{a, b, c\} \times S$
Hence P' is more-correct than P with respect to R .

(Khairredine et al., 2017) defined program reliability as the probability that a randomly element of the specification domain ($dom(R)$ =initial states on which candidate program must behaves according to R) selected according to the probability distribution (θ) falls within the competence domain of the program with respect to the specification. It can be written as: $\rho_R^\theta = \sum_{s \in dom(R \cap P)} \theta(s)$.

Definition 3: P' is more correct than P implies that P' has higher reliability than P . Hence, $P \sqsubseteq_R P' \Leftrightarrow (\forall \theta : \rho_R^\theta(P) \leq \rho_R^\theta(P'))$. So, the more the program is correct (P'), the more it is reliable compared to P .

With the absence of strong analytical and empirical evidence, (Diallo et al., 2015) have presented

some advantages of relative correctness based program derivation that may complement those of refinement. Indeed, the underlying approach is competent to model not only program development from scratch but also several software engineering activities including the development of sufficiently reliable programs, software upgrade, adaptive maintenance, program merger, corrective maintenance and software evolution. Table 1 present a comparison between the two programs derivation approaches. The table highlights the contributions of relative correctness concept in the program derivation process.

3 RELATIVE CORRECTNESS BASED PROGRAM DERIVATION: PROOF OF CONCEPT

This section presents our contributions. In fact, we discuss the feasibility of the relative correctness approach as an alternative to the refinement approach in the program derivation process. This, allows us to identify the strengths and weaknesses of such approach and may lead us to decide on the usability of the approach and/or adjust/complete it.

3.1 Analytical Study

The analytical study deals with the relational mathematics (Chris Brink and Schmidt, 1997) to proceed to the development process using relative correctness. Therefore, we present in the following a set of illustrative examples. In each example, we start with an abort program that fails to capture any functionality of the specification and we apply the relative correctness formula until reaching either a correct program or a program with a satisfying reliability threshold according to the specification.

3.1.1 MedianIndex

Let S be the space defined by; variable x of type `int`, variable $a[1..N]$ for some constant $N \geq 0$, and variable k of type `int`, used as an index variable into array a . And let R be the following specification on S :

$$R = \{(s, s') \mid a' = a \wedge 1 \leq k' \leq N \wedge a[k'] = x \wedge |(\#h : 1 \leq h < k' : a[h] = x) - (\#h : k' < h \leq N : a[h] = x)| \leq 1\} \cup \{(s, s') \mid a' = a \wedge (\forall h : 1 \leq h \leq N : a[h] \neq x) \wedge k' = -1\}.$$

The operator $\#$ means we count the number of times the condition is verified.

Table 1: Comparison between refinement and relative correctness.

Criterion	Refinement-based program derivation	Relative correctness-based program derivation
Starting condition	Client specification.	Program that does not meet the specification.
Ending condition	Correct program.	Reliable program.
Transformation process	In each transformation step, We have a partially defined program.	In each transformation step, we have an executable program that satisfies R in some initial states.
Designer decision	Determined/ restricted	Undetermined/ expanded
Goal	Derives executable and correct program from its specification.	Models various software engineering activities.
Development; cost and complexity	High cost and complexity because it aims to produce an absolutely correct program.	Low cost and complexity because we are not trying to produce all the program functionalities but rather to achieve a sufficiently reliability level.
Fault removal	When an error occurs, it can easily propagate in the development process which generates a defective program that does not satisfying the client specification.	Since each transformation step represents a program that is more correct to the previous one, it is easy to detect its defaults and correct them immediately.
Correctness and executability	Correctness preservation and executability enhancement.	Correctness enhancement and executability preservation.

The specification mandates to return in k' a median index where x occurs (i.e. if x occurs three times in a , return in k' the second position; and if x occurs for example six times then return in k' the 3rd or fourth index where x occurs) if x is in $a[1..N]$. else (if x does not occur in $a[1..N]$) to return -1 in k' . In addition, the specification mandates to preserve a . We start from the initial program P_0 whose competence domain is the empty set and we let the next program P_1 be the program that assigns the value -1 to variable k' . In fact, remember that $k'=-1$ is the value mentioned in the specification R when x is not in the array a .

```
P1: int main ()
    {int K; k=-1; return k;}
```

We compute the function of this program and we find:

$$P_1 = \{(s, s') | k' = -1 \wedge a' = a \wedge x' = x\}.$$

Whence, we compute the competence domain of P_1 with respect to R :

$$\begin{aligned} & (R \cap P_1) \circ L \\ &= \{ \text{substitution, simplification} \} \\ & \{(s, s') | (\forall h : 1 \leq h \leq N : a[h] \neq x) \wedge k' = -1 \wedge a' = a \wedge x' = x\} \circ L. \\ &= \{ \text{taking the domain} \} \\ & \{(s, s') | (\forall h : 1 \leq h \leq N : a[h] \neq x)\}. \end{aligned}$$

program P_1 satisfies specification R for all initial states that have no instance of x in a . We now consider the case where the number of instances of x in $a[1..N]$ does not exceed 2. This yields the program P_2 which is derived from P_1 .

```
P2: int main ()
    {loaddata(); // loads a and x
```

```
int a[N+1]; int x; int k; int N; k = N;
while ((a[k] != x) && (k >= 0)) {k = k - 1;}
return k;}
```

The program preserves a , and returns in k' the largest index where x occurs if x is in $a[1..N]$. else (if x does not occur in $a[1..N]$) it return -1 in k' . We write its function as follows:

$$P_2 = \{(s, s') | a' = a \wedge x' = x \wedge a[k'] = x \wedge 1 \leq k' \leq N \wedge (\forall h : k' < h \leq N : a[h] \neq x)\} \cup \{(s, s') | a' = a \wedge x' = x \wedge (\forall h : 1 \leq h \leq N : a[h] \neq x) \wedge k' = -1\}.$$

We compute the competence domain of P_2 with respect to R :

$$\begin{aligned} & (R \cap P_2) \{(s, s') | a' = a \wedge x' = x \wedge a[k'] = x \wedge 1 \leq k' \leq N \wedge (\#h : 1 \leq h < k' : a[h] = x) \leq 1 \wedge (\forall h : k' < h \leq N : a[h] \neq x)\} \cup \{(s, s') | (\forall h : 1 \leq h \leq N : a[h] \neq x) \wedge a' = a \wedge x' = x \wedge k' = -1\}. \end{aligned}$$

Whence

$$\begin{aligned} & (R \cap P_2) \circ L \\ &= \{ \text{Domain of a union} \} \\ & \{(s, s') | (\#h : 1 \leq h \leq N : a[h] = x) \leq 2\} \cup \{(s, s') | (\forall h : 1 \leq h \leq N : a[h] \neq x)\}. \\ &= \{ \text{The first term is a superset of the second} \} \\ & \{(s, s') | (\#h : 1 \leq h \leq N : a[h] = x) \leq 2\}. \end{aligned}$$

R mandates to return the median index when x is found in $a[1..N]$ whereas P_2 returns the largest index; hence P_2 satisfies R for all initial states in which the number of instances of x in $a[1..N]$ does not exceed 2. For the third program, we want to satisfy the specification R for any initial state s in S . We consider the following program:

```
P3: int main ()
    {loaddata(); // loads a and x
```

```

int a[N+1]; int x; int k; int N; int k2;
k = N + 1; k2 = N + 1;
while (k2 > 0) { k = k - 1; k2 = k2 - 1;
while ((a[k] != x) && (k >= 0))
{k = k - 1;}; while ((a[k2] != x)
&& (k2 >= 0)) {k2 = k2 - 1;}; if ((k2 >= 0))
{k2 = k2 - 1;}; while ((a[k2] != x)
&& (k2 >= 0)) {k2 = k2 - 1;};} return k;
}end{verbatim}

```

We write the function of P_3 as follows:

$$P_3 = \{(s, s') \mid a' = a \wedge x' = x \wedge 1 \leq k' \leq N \wedge a[k'] = x \wedge (\#h : 1 \leq h < k' : a[h] = x) - (\#h : k' < h \leq N : a[h] = x) \mid \leq 1\} \cup \{(s, s') \mid a' = a \wedge x' = x \wedge (\forall h : 1 \leq h \leq N : a[h] \neq x) \wedge k' = -1\}.$$

We compute the competence domain of P_3 with respect to R and we find that $(R \cap P_3) \circ L = RL = S$

So, it's easy to see that the competence domain of P_3 is S , since it may satisfies R for any initial state s in S . Therefore we conclude that P_3 is the more correct program with respect to R . Hence P_3 is correct with respect to R hence it is more-correct than P_2 with respect to R . So we do have: $P_0 \sqsubseteq_R P_1 \sqsubseteq_R P_2 \sqsubseteq_R P_3$. Furthermore, we find that P_3 is correct with respect to R ; this concludes the derivation.

3.1.2 BinaryCode

We let space S be defined by a natural variable n and we let specification R be the following relation on S . $R = \{(s, s') \mid n' = \text{binary}(n) \wedge n \leq 1024 \wedge n' = n\}$, with $\text{binary}(n)$; a function that represents natural numbers as sequences of digits 0 or 1 in the usual way (binary representation). Thereby, our specification R mandates to return in n' the binary code of a given natural number n between 0 and 1024. Starting from the initial program P_0 that is always false according to R , and we let the next program P_1 be the program that returns the binary code of given n that must be ≤ 100 .

```

P1: int main ()
{ int n; long long binaryNumber = 0;
if (n <= 100) { binaryNumber = binary(n);}
return binaryNumber;}

```

The function of this program and its competence domain are given as:

$$P_1 = \{(s, s') \mid n' = \text{binary}(n) \wedge n \leq 100 \wedge n' = n\} \cup \{(s, s') \mid n' = 0 \wedge n > 100 \wedge n' = n\}.$$

$$(R \cap P_1) \circ L = \{(s, s') \mid n \leq 100 \wedge n' = \text{binary}(n) \wedge n' = n\} \circ L = \{\text{taking the domain}\} = \{(s, s') \mid n \leq 100\}.$$

P_1 works only when $n \leq 100$, and when it does, it returns the binary code of the given n . Therefore, the competence domain of P_1 , is the set of n whose value does not exceed 100. For the next programs; P_2, P_3, P_4 and P_5 we do the same thing as P_1 only,

at each transition from one program to another we keep the same functionalities of the program then we increase the domain of n until it reaches 1024. Therefore, we obtain in the end of derivation a program P_5 ($\{(s, s') \mid n' = \text{binary}(n) \wedge n \leq 1024 \wedge n' = n\} \cup \{(s, s') \mid n' = 0 \wedge n > 1024 \wedge n' = n\}$.) that works on the whole interval of n defined by the specification R . Hence we do have: $P_0 \sqsubseteq_R P_1 \sqsubseteq_R P_2 \sqsubseteq_R P_3 \sqsubseteq_R P_4 \sqsubseteq_R P_5$. Furthermore, we find that P_5 is correct with respect to R ; this concludes the derivation.

3.1.3 StringCombinations

Let space S be defined by; str : string of char, a and b : char. Also, we let R be the following specification: $R = \{(s, s') \mid str' = (a \vee b)^* \vee \text{epsilon}\}$.

With str' ; a string formed of epsilon or any combination of "a" and/or "b". The specification mandates to return all string formed from $\{a, b\}$ or epsilon. We start with P_0 whose competence domain is the empty set. For the next program, we choose:

```

string P1(string str)
{string r = ""; if (str == "a") {r = str;}
return r;}

```

The function of this program and its competence domain are given as:

$$P_1 = \{(s, s') \mid str' = a \vee \text{epsilon}\}.$$

$$(R \cap P_1) \circ L = \{(s, s') \mid str' = a \vee \text{epsilon}\}.$$

The competence domain of P_1 is the set of string that are constructed only from one a and also from epsilon . For the next programs; P_2, P_3 and P_4 , at each transition from one program to another we improve the output of the string str' by adding some functionalities to the next program compared to its predecessor. For P_2 , it satisfies the specification R only for the string that are formed from either one a or one b or epsilon ($\{(s, s') \mid str' = a \vee b \vee \text{epsilon}\}$.) For P_3 it satisfies R for the string that are constructed only from multiple a or multiple b or epsilon ($\{(s, s') \mid str' = a^* \vee b^* \vee \text{epsilon}\}$.) And for P_4 it satisfies our R for all the string that are formed from either a or b or epsilon ($\{(s, s') \mid str' = (a \vee b)^* \vee \text{epsilon}\}$.) Hence we do have: $P_0 \sqsubseteq_R P_1 \sqsubseteq_R P_2 \sqsubseteq_R P_3 \sqsubseteq_R P_4$. Therefore we conclude that P_4 is the more correct program with respect to R .

Due to the lack of space, we find in (Benabdelali, 2019) the detailed development of the example BinaryCode and StringCombinations. We find also various other examples that illustrate the program derivation by correctness enhancement.

3.2 Empirical Study

In this section, we conduct an empirical experimentation using c++ language of the examples presented in the previous section and some others examples that have been presented in (Benabdelali et al., 2018). To implement the specifications, we use the concept of *test oracle* that checks the programs reliability threshold according to specification R. Therefore, in all examples implementations, our test oracle always contains the c ++ code source of our specifications. We define a general format, which we instantiate it for each example to calculate the reliability of programs with respect to given specification:

```
typedef struct {int n, x, y;}state;
state sinitial;state sfinal;
//-----
bool domR (state sinitial)
{ }// domain of specification
//-----
bool R (state sinitial, state sfinal)
{ }// specification
//-----
bool oracle (state sinitial, state sfinal)
{ }//oracle of correctness with respect to R
//-----
state p1(int n)
{ }// The candidates programs p1, p2, p3, etc...
//-----
int randomgeneration(int n)
{ }//random generator
//-----
void tesdriver(int testdatasize)
{int counters1=0;int counters2=0;...//initialize
for(int testindex=1; testindex <= testdatasize;
++testindex){sinitial.n=randomgeneration(10000);
...}// run the test oracle on our programs
std::cout << "reliability of p1 : " <<
(counters1/static_cast<double>(testdatasize))
<< std::endl;
std::cout << "reliability of p2 : " <<
(counters2/static_cast<double>(testdatasize))
<< std::endl;
...}// check the reliability of each program
int main(){tesdriver();}
// run the tesdriver function n number times
```

So, for each experimentation, we just instantiate *domR*, *R* and *oracle*, redefine our variables, and of course write the programs. we define a test driver and a random test data generator. Then we apply each test driver to the programs generated in the corresponding derivation. We take the example of Fermat Decomposition, we generate 4 000 random numbers (as a test data size) between 1 and 10 000. For each random number, we execute the programs P_0 , P_1 , P_2 , P_3 and we see which satisfies the specification R (with the oracle test that translates the specification R). For each random number, if the program is correct with respect

to R, the program counter is incremented and at the end (until we finished the generation of 4000 random number) the counter sum of each program is divided on test data size. As a result, for each program we obtain a probability that will be the reliability threshold according to R. Table 2 summarizes the reliability results of the examples that we have used for the program derivation based relative correctness.

The results obtained in each example, shows that the reliability evolves from one program to another. We start from a program abort that never runs successfully since, it always gives 0 for reliability with respect to R(it's useless to mention it in the table), and we enhance the transformation until we reach a correct program, or a sufficiently reliable program. Note when we obtain reliability percentage strictly greater than 0 and strictly less than 1 the program is partially correct with respect to R. Furthermore, in the program derivation process by correctness enhancement, deriving a reliable program follows the same process as deriving a correct program, except that the derivation terminates as soon as the required reliability threshold matches or exceeds the selected threshold. According to the programs reliability results obtained in each example, we argue that for most software products, as for products in general, absolute correctness is not necessary; often, a high reliability threshold is sufficient. This could greatly reduce the development costs and complexity.

3.3 Towards a Mechanism for Program Derivation Process

In the previous sections and with samples examples, we have realized a proof of concept regarding the program derivation process using relative correctness. We have showed that when going from P_i to P_{i+1} we improved the reliability of the latter according to the specification R and we end the derivation where the desired level of reliability has been achieved and/or when we attain program correctness. However, until there, we have reasoned as if the programs that we have derived are at our disposal in some way or other. Therefore, the objective of this section is to find a mechanism that helps and guides the developer in his program derivation process. So, we propose three possible attempts.

3.3.1 Different Possible Scenarios

The various examples presented above and in (Benabdelali et al., 2018), lead us to conclude that we can use the program derivation process using relative correctness in at least four scenarios that will show how

Table 2: Programs reliability results.

Programs	Test Data Size	Input Data	Output of each program according to R				
			P_1	P_2	P_3	P_4	P_5
Median Index	3000	Random array with a size N between 5 and 10 and with cells between 0 and 50. Random variable x between 60 and 80. An occurrence number of x between 0 and 4.	0.2620	0.5803	1.0000		
Even number (Benabdelali, 2019)	1000	Random array of size between 1 and 5 and with value between 1 and 20.	0.2140	0.6050	0.7480		
BinaryCode (Benabdelali, 2019)	4000	Random natural number n between 0 and 1024.	0.0985	0.2917	0.4860	0.8790	1.0000
StringCombinations (Benabdelali, 2019)	5000	File of strings formed from {a, b}.	0.2486	0.3	0.5078	1.0000	
MinMax (Benabdelali, 2019)	3000	Random array of size between 5 and 25 and with value between 0 and 20. Random variable z between 1 and 30.	0.0593	0.1103	1.0000		
Decimal number (Benabdelali, 2019)	5000	Random binary numbers (n) range between 0 and 10.	0.1782	0.7248	0.9068		
Palindrome stringAB (Benabdelali, 2019)	5000	Random string formed from {a, b}.	0.2022	0.3898	0.5034		
Fermat Decomposition (Benabdelali et al., 2018)	4000	Random natural variable range between 1 and 10 000.	0.2535	0.3445	1.0000		
The Ceiling of the Square Root (Benabdelali et al., 2018)	4000	Random natural variable range between 1 and 10 000.	≈0.0000	0.0102	1.0000		
Analyzing a String (Benabdelali et al., 2018)	100	ASCII file (for example: the latex source of an article, with symbols and numbers).	0.0057	0.2790	0.2917	1.0000	
Word Wrap (Benabdelali et al., 2018)	3000	ASCII file file (for example: the latex source of an article, with symbols and numbers).	0.0363	0.0873	0.1023	0.8990	1.0000

to derive a program that is more correct than another: **Domain Enlargement:** using this scenario, we keep the same program functionalities but at each transition from one program to another, we increase the domain of $P (dom(P))$ with respect to the domain of $R (dom(R))$ (as our binary code example).

Particular Case: the program does something else but in particular cases, it does what the specification R requires (e.g in the MedianIndex example, the specification mandates to return the median index where x is found whereas P2 does something else indeed, it returns the largest index where x is found hence P2 satisfies R only in particular cases which are the initial states in which the number of instances of x does not exceed 2. beyond this, P2 no longer satisfies R.

Changing Behavior: the behavior of the program changes depending on the type of input data(e.g; Fermat decomposition).

Improve Program Functionality: improve the program functionalities from one program to another

with respect to the specification(from one program to another, we add a little bit of code to the program until we reach a program that is absolutely correct with respect to R or we reach a sufficiency reliability threshold. (as string analysis, StringAB, PalindromeStringAB).

Hence, the transformation from one program to another which is more correct according to a given specification can be done as mentioned by the different scenarios. The only calculus to be done is about the competence domains between the programs and the specification R. These scenarios help the developer to decide from the beginning about the strategy adopted in the program derivation process using relative correctness.

3.3.2 Reusable Programs Stored in a Repository

The program derivation process starts with an abort program and then we search in the repository for pro-

grams that are more and more correct according to the specification R by competence domain calculations. The process finishes when a reliability threshold is reached. It seems to be very interesting because it encourages reuse-based development in one hand and it capitalizes the multitude of existing programs that can be reused.

3.3.3 Test Driven Development as an Instance of Relative Correctness

The idea of Test Driven Development (commonly shortened to TDD) was popularized by (Beck and Andres, 2004) in the Extreme Programming (XP) method. This agile software development methodology is considered as a short iterative software development process. It starts by developing the test before writing the code source which specifies and validates what the code will do (Beck, 2002). The TDD process revolves around five simple steps: 1) write the first test, 2) confirm the test fails because the code he is testing does not exist, 3) write write enough code to pass the test, 4) confirm the test passes, and 5) refactor: that is to say improve it while keeping the same functionalities.

Despite the fact that the program derivation process using TDD is different to that of relative correctness, the results obtained by both processes are the same, where we obtain a sequence of programs that are respectively more-correct with respect to a specification R . Our aim is to draw inspiration from TDD process to derive programs using relative correctness. So with a series of tests $\sum_{i=1}^n T_i$, we create a list of programs $\sum_{i=1}^n P_i$ such as each P_i is an upgrade of P_{i-1} . As a result, the competence domain increases from one program to another. Note that to create P_n programs, we need T_{n-1} tests. So, the notion of testing before coding can be a way that helps the developer to derive programs by using the approach of relative correctness. In a short term research perspective, we will better focus on developing this idea with illustrative examples.

4 CONCLUSION

Program derivation or construction by relative correctness enhancement seems to be another way to produce executable programs sufficiently reliable but not necessary correct for a given specification. As mentioned previously in this paper, this can be very attractive in a world where people don't develop from scratch but reuse existing reliable programs. This paper tried first to present concrete programs and speci-

fications used to illustrate the analytical aspect of relative correctness. Second, an empirical approach was conducted in order to establish a proof of concept and demonstrates the viability of the approach. Now, we think that development based on relative correctness is a promising tentative for constructing reliable programs. As a short term perspective, we are going to continue the experimentation by cases from the real world. Also, we try to find other mechanisms and scenarios for the program transformation process based on relative correctness. In fact, we are conscious that this latter process is not efficient as program derivation by refinement calculus which is older and more mature. Hence, we still need to investigate if we can find a kind of calculus or a set of more sophisticated guidelines for program transformations in the Relative Correctness-based reliable program construction approach. Furthermore, we would like to use this latter concept in the context of software maintenance and test driven development.

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