Adaptive Controller for Uncertain Multi-agent System Under Disturbances

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Keywords: Adaptive Control, Robotics, Robust Control, Parametric Uncertain.

Abstract: This research is devoted to solving the problem of adaptive control algorithm synthesis for a mobile robots that is part of a multi-agent system. Proposed approach consists of trajectory planner and inner agent controller. The case of the passway intersection by the group of mobile robots is considered. Trajectory planner is based on intersection management approach. Adaptive consecutive compensator used for agent controller synthesis. Proposed approach provides control scheme which doesn’t depend on plant parameters. A group of mobile robots is built for experimental evaluation of proposed approach. Obtained results confirm effectiveness of the developed algorithms.

1 INTRODUCTION

Robotic systems are widespread in different spheres of human activity. They are widely used in industry, daily life, entertainment. Autonomous systems that simplify people’s lives are becoming increasingly popular. Among them there are multi-agent systems, which consist of many robots, connected in a common network. Today, mobile multi-agent systems are widely used by various major corporations, such as Aliexpress, Amazon, etc. Robots perform various functions, such as transportation of goods, cleaning of premises, delivery of correspondence. Different algorithms of automatic control are used for solving complex tasks for the movement of robots and goods. Besides, robots with different inner controllers are controlled by one system to solve different tasks. Agents have different parameters, for example, various engines, which will give a various moment of force on the motor shaft, can also have special wheels or alternative wheel bases. Moreover some parameters are nonstationary during functioning. All these parameters affect the synthesis of automatic control algorithms, thereby complicate the development of the whole system. If on the manufacture or warehouse moving a lot of robots the crossroad become bottle neck.

Scientific community conducts research in this field. The article (Li et al., 2011) is about the finite-time consensus problem for leaderless and leader–follower multi-agent systems with external disturbances. The paper (Olfati-Saber, 2006) describes theoretical framework for design and analysis of distributed flocking algorithms, two cases of flocking in free-space and presence of multiple obstacles are considered. The article (Lauer and Riedmiller, 2000) focuses on distributed reinforcement learning in cooperative multi-agent-decision-processes, where an ensemble of simultaneously and independently acting agents tries to maximize a discounted sum of rewards. Our team also have achievement in this area. The article (Bazylev et al., 2014) proposes a new control design of quadrotor with attached 2-DOF robotic arm. In this research we make new control system for new robotic agents.

The paper propose use of adaptive controller which doesn’t depend on agents parameters for control of mobile robots group. Formal problem statement is in the Section 2. Planing controller design for system is in Section 3. Inner controller synthesis and its stability analysis is in Section 4. Robots setup described in Section 5. The results of experiments of obtained control laws are shown in Section 6. Finally, the research is summarized in Conclusions.
2 PROBLEM STATEMENT

We have multi-agent system consists of different agents described by linear differential equations. All agents are under disturbances. Parameters of agents and disturbances are unknown but their upper and lower bounds are known. Let us consider case, when agents are moving through crossroad.

Main task is to develop control system which allow robots ride through crossroad without incident and under disturbances, minimize difference between acceleration of all agents in system. In this case, we allocate two separate tasks

- Develop main control system, which will keep all information about all agent and say to each who were will ride;
- Synthesis of controller which will have the same structure for all different robots neglecting agents parameters deviations. All necessary controller parameters should be tune automatically.

Goal of main control is to transfer agents from certain initial state to specified final state in a way that some functional \( Y \) raise to extreme value with excepted restriction. Specified \( Y \) is a bandwidth of a simple road element size like little agent in the system. Planning controller give to agent speed of moving \( y_m(t) \).

Every agent is described by equations

\[
Q_i(p)y_i(t) = R_i(p)u_i(t) + f_i(t) + \sum_{i=1,i\neq j}^{N} c_{ij}(p)y_j(t) + \sum_{j=1,j\neq i}^{M} \gamma_{ij}(p)u_j(t),
\]

where \( Q_i(p) \) and \( R_i(p) \) are linear differential operators with unknown parameters and degrees \( n_i \) and \( m_i \) respectively, \( y_i(t) \in \mathbb{R} \) are output signals, \( u_i \in \mathbb{R} \) are inputs, \( f_i(t) \) are external bounded disturbances, \( c_{ij}(p) \) and \( \gamma_{ij}(p) \) are linear differential operators with unknown coefficients which describe input and output cross couplings respectively, \( N \) and \( M \) are numbers of input and output signals, \( p = d/dt \) is a differential operator, \( p_i = n_i - m_i \geq 1 \) is a relative degree of \( i \)-th subplant.

It is necessary to build inner controller which satisfies following condition

\[
|y_i(t) - y_m(t)| \leq \delta, \forall t \geq T,
\]

where \( \delta \) is a required accuracy, \( T \) is a time of transients.

Introduce following assumptions:

**Assumption 1.** All subplants of (1) are minimum phase, i.e. \( R_i(\lambda) \) are Hurwitz polynomials, where \( \lambda \) is a complex variable.

**Assumption 2.** Unknown coefficients of operators \( Q_i(p) \) and \( R_i(p) \) belong to the known compact set \( \Xi \).

**Assumption 3.** Only the output variable is available for measurements. Its derivatives are unmeasurable.

**Assumption 4.** The relative degree of the plant model is assumed to be known.

**Assumption 5.** Maximum amplitude of disturbance are known all disturbances are piecewise smooth.

3 PLANNING CONTROLLER DESIGN

For a complete understanding of the intersection management, we should describe crossroad. It shows on Fig1. It contains an intersection of four roads, each may move in one direction only.

We have some agents stay on different parts of crossroad. During the distributed intersection management robots collaborate at some time point. Let’s the road consist of the set of elementary areas \( p_0, \ldots, p_n \). The crossroad is located on the area \( p_i \), \( 0 < i < n \). In this context \( c \) represents a maximum speed on the one road. It is necessary to provide the intersection with other robots from the moment when the agent has enough distance for safe braking.

Therefore, we need \( c + 1 \) steps to stop. Hence, the full path of the braking is \( (c(c+1))/2 + 1 \) elementary areas. In an example that present on the Fig1, the robot must start the process of interaction with other vehicles at the moment when they located on position \( p_i((c(c+1))/2 + 1) \).

Robot routing is based on the following basic principles:

**Figure 1:** Schematic view of crossroad.
1. Turning is performed according to scheme on Fig.2;
2. Speed on the turning areas equals 1;
3. Speed of moving robot on the previous or next time point may differ from the actual speed by more than 1;
4. The robot tries moving with the maximum speed. At low speeds, it seeks the opportunity to restore it to maximum value;
5. Speed more than maximum is unacceptable.

In addition, when choosing variant of priority passage need to ensure maximum bandwidth of crossroad for robots. In practice increasing bandwidth lead to decrease interval between agents. And the endowment as, in the same time interval $T$ crossroad must pass as possible robots, provided that in any random moment of time $(t_n + k) \in T$ would not be applying for one simple area more then two agents. Then bandwidth of road area will be higher the greater next treatment

$$Y = \frac{\sum_{i=1}^{L} \sum_{j=1}^{N_i} \sum_{l=1}^{M} n_{lji}}{M},$$

where $N$ is count of robots on crossroad, $L$ is count of simple areas on crossroad, $M$ is count of time interval, for which $N$ robots pass crossroad,

$$n_{lji} = \begin{cases} 1, & \text{if } j \text{ is robot in time } i \text{ on the } l \text{ element}, \\
0, & \text{if } n_{lji-1} \neq n_{lji} \text{ and } \text{in other case.} 
\end{cases}$$

Conflict is the point in time when two or more robots locate in the same elementary area. Rerouting proceed according to order of conflict situations occurrence. Each car involved in the conflict must change route using speed reduction. It allows to solve all other conflicts. In a general case, each conflict has two possible crossroad intersection plans. We use a variety of criteria to determine the best possible options.

According to Fig.2, when the agents driving up to the crossroad elementary area after which it will have to change the motion direction, the agent must reduce speed to the minimum possible value. Speed reduction is carried out according to the general principles of smooth braking. After the robot passed the intersection, it accelerates smoothly to the maximum desired speed.

This approach computing by table methods(Viksnin et al., 2016).

4 INNER CONTROLER DESIGN

4.1 Controller Design

For controller synthesis we use consecutive compensator approach (Margun and Furtat, 2015), (Margun et al., 2017a). Compensator applied in decentralized manner (independent controller for all sub-plants). Choose control law as follows

$$u_i(t) = -(\alpha_i + \beta_i)K_i(p)\hat{e}_i(t),$$

where $\alpha_i, \beta_i > 0, K_i(\lambda)$ are such Hurwitz polynomials of degrees $\rho_i - 1$ that $(Q_i(\lambda) + \alpha R_i(\lambda)K_i(\lambda))$ are Hurwitz polynomials, $\hat{e}_i(t)$ are estimates of errors $e_i(t) = y_i(t) - y_m(t)$.

Taking into account (1) and (4) obtain errors dynamics in the form

$$(Q_i + \alpha R_i K_i)\hat{e}_i = R_i K_i(- (\alpha_i + \beta_i)(e_i - \hat{e}_i) - \beta_i e_i) +$$

$$+ \varphi_i(t) + \sum_{i=1}^{N} c_{ij} e_j +$$

$$+ \sum_{j=1, i \neq j}^{M} \gamma_{ij}(\alpha_i + \beta_i)K_i(e_j - \hat{e}_j) -$$

$$- \sum_{j=1, i \neq j}^{M} \gamma_{ij}(\alpha_i + \beta_i)K_i e_j,$$

$$\varphi_i(t) = -Q_i y_{m_i} + \sum_{i=1, i \neq j}^{N} c_{ij} y_{m_j} + f_i,$$

where $\varphi_i(t)$ is a bounded function.
Rewrite (5) in state space representation

\[
\begin{align*}
\dot{e}_i &= A_i e_i + B_i (\beta_i e_i + (\alpha_i + \beta_i)(e_i - \hat{e}_i)) + \\
&+ \sum_{j=1, j \neq i}^{N} W_{ij} e_i + \sum_{j=1, j \neq i}^{N} U_{ij} (e_i - \hat{e}_i) + \\
&+ \sum_{j=1, j \neq i}^{N} D_{ij} \hat{e}_j,
\end{align*}
\]

where \( \hat{e}_i \) is an error state vector of \( i \)-th subplant, \( A_i, B_i, W_{ij}, U_{ij}, D_{ij} \) are matrices obtained from (5) to (6) transition, \( L_a = [1 \ 0 \ ... \ 0] \).

Rewrite Closed loop system (6) in matrix form

\[
\begin{align*}
\dot{\xi}_i(t) &= \sigma_i \Gamma_i \xi_i(t) + \sigma_i G_i e_i(t), \\
\tilde{e}_i(t) &= L_i \xi_i(t),
\end{align*}
\]

where \( \xi_i(t) \in \mathbb{R}^{N_i} \) is an observer state vector, \( \Gamma_i = \begin{pmatrix} \Gamma_i & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{pmatrix} \) are Hurwitz matrices, \( G_i = \begin{pmatrix} 0 \\ L_p \end{pmatrix} \) is a identity matrix of order \( p_i - 2 \), \( L_a = [1 \ 0 \ ... \ 0] \), \( \sigma_i > \alpha_i + \beta_i \). Rewrite (8) in matrix form

\[
\begin{align*}
\dot{\xi}_i(t) &= \sigma_i \Gamma_i \xi_i(t) + \sigma_i G_i e_i(t), \\
\tilde{e}_i(t) &= L_i \xi_i(t),
\end{align*}
\]

where \( \xi_i(t) = \text{diag}\{\xi_i(t)\} \), \( \sigma = \text{diag}\{\sigma_i\} \), \( L = \text{diag}\{L_i\} \).

Introduce error of observer estimates

\[
\begin{align*}
\hat{\eta}_i(t) &= L_i^T \xi_i(t) - \xi_i(t), \\
\hat{\eta}_i(t) &= \sigma_i \Gamma_i \eta_i(t) + L_i^T \tilde{e}_i(t).
\end{align*}
\]

and rewrite it in matrix form

\[
\begin{align*}
\hat{\eta}_i(t) &= L_i^T \xi_i(t) - \xi_i(t), \\
\hat{\eta}_i(t) &= \sigma_i \Gamma_i \eta_i(t) + L_i^T \tilde{e}_i(t),
\end{align*}
\]

where \( \eta_i(t) = \text{diag}\{\eta_i(t)\} \).

Finally closed loop system with observer takes the form

\[
\begin{align*}
\dot{\xi}_i(t) &= \sigma_i \Gamma_i \xi_i(t) + L_i^T \tilde{e}_i(t), \\
\dot{\eta}_i(t) &= \sigma_i \Gamma_i \eta_i(t) + L_i^T \tilde{e}_i(t),
\end{align*}
\]

Let us analyse stability of (12).

Introduce Lyapunov function candidate

\[
V = \xi^T(t) P \xi(t) + \eta^T(t) H \eta(t),
\]

where \( P \) and \( H \) are solutions of Lyapunov equations

\[
\begin{align*}
\dot{P} &= -\Phi_1 \Gamma^T H + H \Gamma - \sigma_2, \\
\Phi_1 &\text{ and } \Phi_2 \text{ are positive defined symmetric matrices.}
\end{align*}
\]

Differentiating (13) along trajectories (12) gives us

\[
\begin{align*}
\dot{V} &= \xi^T(t) (A^T P + PA) \xi - 2 \beta^T P \eta \xi + \\
&+ 2 \xi^T P \left( (\alpha + \beta) B \eta \xi \right) + 2 \xi^T P L \eta \xi + \\
&+ 2 \xi^T P \left( (\alpha + \beta) (W + D) \eta \xi \right) + 2 \xi^T P L \eta \xi + \\
&+ 2 \xi^T P \left( \theta S \Gamma(1 + \phi) \eta \xi \right) + 2 \xi^T P L \eta \xi + \\
&+ 2 \xi^T P \left( \phi \eta \xi \right) + 2 \xi^T P L \eta \xi,
\end{align*}
\]

where \( \psi \) is a small positive number.

Right terms of (14) are bounded by inequalities

\[
\begin{align*}
-2 \beta^T P \eta \xi \leq \beta \psi \xi^T + (\alpha + \beta) \eta \xi^T, \\
2 \xi^T P \left( (\alpha + \beta) B \eta \xi \right) \leq \psi \xi^T + (\alpha + \beta) \eta \xi^T, \\
2 \xi^T P \left( (\alpha + \beta) (W + D) \eta \xi \right) \leq \psi \xi^T + (\alpha + \beta) \eta \xi^T, \\
2 \xi^T P \left( \theta S \Gamma(1 + \phi) \eta \xi \right) \leq \psi \xi^T + (\alpha + \beta) \eta \xi^T, \\
2 \xi^T P \left( \phi \eta \xi \right) \leq \psi \xi^T + (\alpha + \beta) \eta \xi^T,
\end{align*}
\]

Taking into account (15) bound derivative of Lyapunov function

\[
\dot{V} \leq -\xi^T R_1 \xi + \psi \xi^T + \eta \xi^T + \theta,
\]

where \( R_1 = \Phi_1 - 2 P (W + D) - \beta \psi - \psi P \Gamma^T H P - \psi \theta S \Gamma(1 + \phi) \eta \xi^T - \psi \eta (1 + \phi) \eta \xi^T + \theta \).

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Let us propose an algorithm to solve this problem.

**Step 1.** On the base of known bounded set \( \Xi \) define set of Kharitonov polynomials (Kharitonov, 1978) for open-loop systems

\[
\begin{align*}
P_1 &= q_0 + q_1 s + q_2 s^2 + q_3 s^3 + \ldots, \\
P_2 &= q_0 + q_1 s + q_2 s^2 + q_3 s^3 + \ldots, \\
P_3 &= q_0 + q_1 s + q_2 s^2 + q_3 s^3 + \ldots, \\
P_4 &= q_0 + q_1 s + q_2 s^2 + q_3 s^3 + \ldots.
\end{align*}
\]

**Step 2.** Use consecutive compensator method and tuning algorithm (21) with zero initial conditions of controller coefficients for stabilization of each polynomial.

\[
\dot{\tilde{k}}_i = \int_0^t \chi_i(s) ds,
\]

where \( \chi_i(t) = \begin{cases} 0, |e| < \delta, \\
|e_0|, |e| > \delta,
\end{cases} \)

\[
\chi_0(t) = \begin{cases} 0, e < \delta, \\
|e_0|, e > \delta,
\end{cases}
\]

\[
\sigma_i = \sigma_0 k_i^2,
\]

and \( i = 1, 2, \ldots, 4 \).

**Step 3.** Choose the maximum values of the Kharitonov polynomials coefficients of the regulators as the initial values of the plant controller. In this case adaptive tuning algorithm takes the form

\[
\dot{\tilde{k}} = \max(\tilde{k}) + \int_0^t \chi(s) ds,
\]

where \( \chi_0 = \alpha + \beta \), \( \sigma_0 = \sigma_0 k^2 \).

The characteristic polynomial of the closed-loop system has the form \( Q(\lambda) + (\alpha + \beta)R(\lambda)D(\lambda) \), where first term is non Hurwitz polynomial and second term is Hurwitz polynomial. Thus, the increasing of the controller coefficient suppresses an unstable component of a closed-loop system. Therefore, sufficiently large choice of coefficients provides stability of the Kharitonov polynomials, and hence the stability of control plant too.
5 AGENT MODEL SETUP

Let us describe the mathematical model of agent, which was created for multi-agent systems experiments. There is the photo of agent on the Fig.3.

The agent is a hand made robot with two wheels, computing boards and battery. It dimensions are \((0.070 \times 0.055 \times 0.050)\) m. Hardware are consists of microcontroller board, DC-motors driver board, IMU-module board and Wi-Fi module board. Main controller of agent is STM32F031K6. Wi-Fi board is ESP-01 module with own firmware.

On the outdoor robots coordination is defined by satellite navigation systems, but when experiments are made in laboratory which in the building, it is impossible to use navigation systems. It this case we use IMU-module consist of accelerometer on chip LIS331DLH, electronic compass on chip LIS3MDL, gyroscope on chip L3G4200D and barometer. On measuring from this module we calculate coordinates of robots.

For control all agents in system has been implementioned next structure - on PC make up Wi-Fi access point and start TCP server application. This application interacts with control system on Matlab. Each agent by ESP-01 application connect to Wi-Fi network, connect to TCP server and make a bridge between STM controller with own control system and between Matlab control system of all multi-agent stand. This schematic can be seen in Fig.4.

Describe robot motion as change coordinates of central point of robot in time. Robot schematically imaged on Fig.5. Value of longitudinal speed is defined as average between linear speed of each wheel

\[
V = \frac{\omega_l + \omega_r}{2} R_k
\]

where \(\omega_l, \omega_r\) angular speed of wheels, \(r_k\) radius of wheel. If angular speed of wheels is different, raised rotation moment

\[
\omega = \frac{\omega_l + \omega_r}{2} \frac{r_k}{R}
\]

where \(R\) long wheel base.

We can consider angular speed of each wheel

\[
\omega_l(s) = W_{D_l}(s)U_l(s)
\]

\[
\omega_r(s) = W_{D_r}(s)U_l(s)
\]

where \(U_l(s), U_r(s)\) are Laplace image of control voltage, \(W_{D_l}, W_{D_r}\) transfer function of motors in from

\[
W(s) = \frac{1}{k} \frac{1}{l_{m} l_{m} s^2 + l_{m} s + 1}
\]

Adding control low(4) designed in Section 4 written in transfer function form to robot model we get agent with controller. Structure of agent demonstrated by Fig.6.
6 EXPERIMENTS

We take 3 different models of robots with same motors and different wheels and wheels base. Parameters of the systems
1. Robot 1. \( r_k = 0.01m \), robot base 0.02m.
2. Robot 2. \( r_k = 0.05m \), robot base 0.02m.
3. Robot 3. \( r_k = 0.1m \), robot base 0.2m.

There are two experiments, the first is set speed of motion of robot \( V = 0.05m/s \) and angle \( \varphi = 0 \), the next is set the same speed, but with angle \( \varphi = 0.65 \text{rad} \). Result of experiments are on Fig.7 - 10.

According to the graphs in first experiment, take required speed it time about 2 seconds, but first robot with some over-regulation(Fig. 7). Control signals are higher on robot with small wheels, and in this case, coefficients of regulator high on this robot(Fig. 8). In the next experiment, when robots ride and turns, transition processes of speed are same as in first experiment. But in control signals we can see emergence at start of motion, after that, processes becomes simple(Fig. 9). Coefficients of regulator increase too(Fig. 10).

7 CONCLUSIONS

During this research, was designed control system for group of robots moving through crossroad. The proposed approach consist of external controller which define desired speed of robots and inner adaptive control system. Designed adaptive control law provide desired moving speed with necessary accuracy independently of parameters of agents. For analysis control multi-agent system own robots were made. Experiments on this robots show effectiveness of the developed system.

ACKNOWLEDGEMENTS

This work was supported by Goverment of Russian Federation (Grant 08-08).

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