Improved Relay Feedback Identification using Shifting Method

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Abstract: This paper presents a new method for estimation of a static gain and remaining parameters of a second order time delayed model by relay feedback identification. For this purpose, it uses a recently published method called shifting method which enables to estimate two points of frequency characteristic from a single relay feedback test. These two frequency response points are determined without any assumptions about a model transfer function and they can be used for fitting parameters of a process transfer function with various structures. For the first time the shifting method is used for a static gain estimation. This unique solution is even applicable under constant load disturbance. A great advantage for practical use is the comprehensibility and computational simplicity of the method. This identification method is primarily proposed for automatic tuning of controllers. The method is demonstrated on a simulated example and a laboratory apparatus "Air Aggregate".

1 INTRODUCTION

There are many methods for automatic controller tuning but only some of them are really used in practice. Some of existing tuning rules for controllers rely on a model of the process.

The relay feedback test belongs to autotuning methods which are successfully applied in industry. This approach for parameter identification and autotuning PID controller was suggested by Åstrom and Hägglund (1984). For this purpose, they suggested the use of an ideal relay to generate a sustained oscillation in the closed loop. A closed loop where a process is under a relay control is illustrated by the block diagram in Fig. 1, where *w* denotes the desired variable, *y* the controlled variable, *u* the manipulated variable, *d* the disturbance variable and *e* the control error. This relay feedback approach enables to calculate the ultimate gain and the ultimate frequency like the Ziegler-Nichols method (Ziegler and Nichols, 1943) but without *a priory* information about the process, in a shorter time and in a controlled manner.

The relay feedback test belongs among the most popular methods in engineering applications for a closed-loop identification. The main advantage of the relay feedback test is to prevent the process drift away from its set point. There are many relay-based parametric estimation methods for single-input-single output (SISO) systems. These methods can be categorized into three groups, namely, describing function method, curve fitting approach, and use of frequency response estimation for model fitting (Liu, Wang and Huang, 2013). There are several overview publications dedicated to the relay feedback identification, e.g. Yu (1999), Liu and Gao (2012), Liu, Wang and Huang (2013), Chidambaram and Sathe (2014), Kalpana and Thyagarajan (2018), Ruderman (2019). The presented relay identification methods are devoted mainly to the identification of linear low-order time delayed models. Fortunately, PID controllers tuned according to low-order models of the processes can control most industrial processes

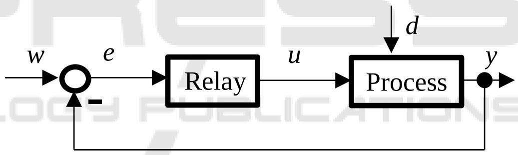

Figure 1: Block diagram of a process under relay feedback.

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sufficiently. Therefore, models with low number of parameters are predominantly used for modelling. Mostly it is the first order time delayed model called the FOTD model or the second order time delayed model called the SOTD model, which are sufficient for modelling of many industrial processes. But only a few presented relay methods are able to obtain all model parameters using one relay test without *a prior* information. Furthermore, some relay identification methods do not consider problems with the influence from load disturbance, measurement noise and nonzero initial process conditions that are in practical applications often encountered.

The paper presents a new method of determining process static gain and the remaining parameters of the SOTD model from a single relay feedback test. The obtained results are demonstrated on both a simulation model and a real device.

2 RELAY IDENTIFICATION BY SHIFTING METHOD

2.1 Specifications

Consider a process which operates in the neighbourhood of the operating point. Assume that this process can be described by a linear model in this neighbourhood. The process variable *y* should be kept near the operating point by a controller. The task is to determine process model which can be used for controller tuning by the relay feedback test.

2.2 Shifting Method

A recently published method called "shifting method" can be used for fitting a linear model (Hofreiter, 2016). This approach is based on the assumptions that in the relay feedback experiment there is a stable oscillation with the period T_p ($T_p=T_1+T_2$, $T_1 \neq T_2$, see Fig. 2), the identified process is time invariant and in the proximity of operating point has linear properties. The block diagram for the relay feedback test is slightly modified, see Fig. 3. Here, the additional integrator or alternatively the transport delay D are inserted in the closed loop (Hofreiter, 2018) and s is the complex variable in L-transform. The shifting method uses an asymmetrical relay with a hysteresis (see Fig. 4) to reduce the influence of noisy environment and for the model parameter estimation.

The basic idea of the shifting method consists in determination of the time courses of the auxiliary variables $u_a(t)$ and $y_a(t)$ calculated according to (1) and (2).

$$u_a(t) = u(t) + u\left(t - \frac{T_p}{2}\right) \tag{1}$$

$$y_a(t) = y(t) + y\left(t - \frac{T_p}{2}\right) \tag{2}$$

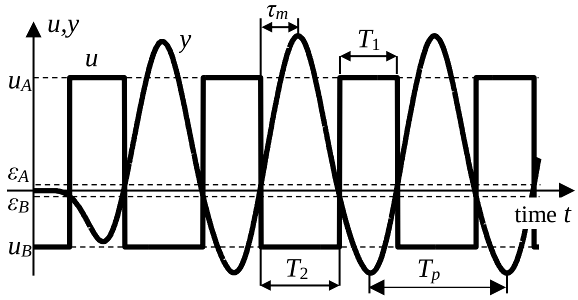

Figure 2: The time courses *u* and *y*.

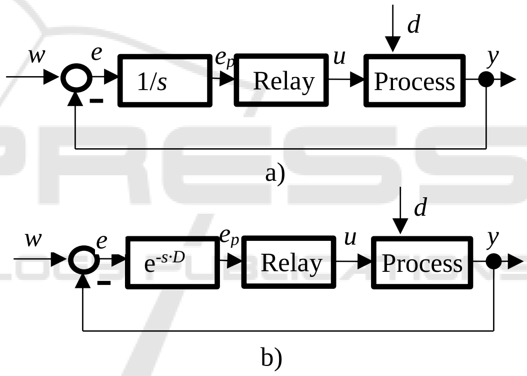

Figure 3: Block diagram of a process under relay feedback with a) integrator b) delay *D*.

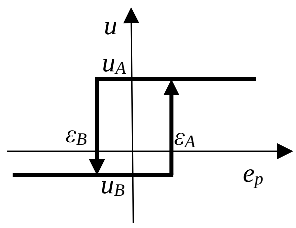

Figure 4: The static characteristic of an asymmetrical relay with hysteresis.

If there are sustained oscillations in the relay feedback test after the time t_L then frequency points $G(\mathrm{j}\omega_1)$ and $G(\mathrm{j}\omega_2)$ of a system can be estimated for angular frequencies

$$\omega_1 = \frac{2\pi}{T_p},\ \omega_2 = \frac{4\pi}{T_p} \tag{3}$$

where T_p is the period of a stable oscillation using the following formulas computed by a numerical integration

$$G(j\omega_1) = \frac{\int_t^{t+T_p} y(\tau) e^{-j\omega_1 \tau} d\tau}{\int_t^{t+T_p} u(\tau) e^{-j\omega_1 \tau} d\tau},\ t > t_L \tag{4}$$

$$G(j\omega_2) = \frac{\int_t^{t+T_p} y_a(\tau) e^{-j\omega_2 \tau} d\tau}{\int_t^{t+T_p} u_a(\tau) e^{-j\omega_2 \tau} d\tau},\ t > t_L \tag{5}$$

where *G(jω)* is the process frequency transfer function.

The use of a transport delay or an integrator allows to place the points $G(j\omega_2)$ and $G(j\omega_1)$ to the 3rd and 4th quadrant (see Fig. 5). These positions are more suitable for model fitting.

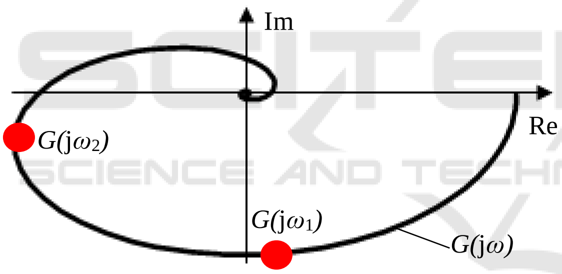

Figure 5: The Nyquist frequency characteristic of a process and the points obtained by the shifting method.

A great advantage of the above procedure is that the location of the points $G(j\omega_1)$ and $G(j\omega_2)$ was determined on the basis of the relay experiment without assuming any model structure. Therefore this approach can be applied to models with more parameters and different structures. The newly acquired point $G(j\omega_2)$ determined by the shifting method allows the estimation of two other parameters of the model from a single relay test. It is possible due to the use of the second order harmonic of the relay oscillations. This follows from the relationships (1) and (2) which describe the filter with the frequency transfer function

$$G_F(j\omega) = 1 + e^{-j\omega \cdot \frac{T_p}{2}} \tag{6}$$

Applying the filter, all odd harmonic frequencies including the fundamental harmonic frequency ω_1 are filtered out. At the same time, the even harmonic frequencies including ω_2 are amplified twice (see Fig. 6).

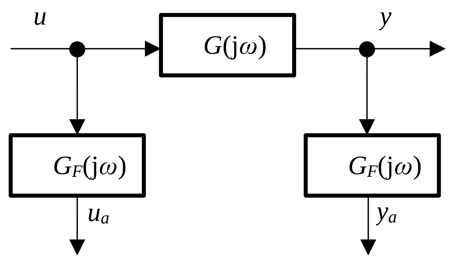

Figure 6: The block scheme of the shifting method.

The next advantage of this approach is that the presence of a static load disturbance with a magnitude of d_A does not have any influence on the calculation $G(j\omega_1)$ and $G(j\omega_2)$ as it holds

$$\int_t^{t+T_p} d_A \cdot e^{-j\omega_i} d\tau = d_A \int_t^{t+T_p} e^{-j\omega_i} d\tau = 0,\ i = 1,2 \tag{7}$$

2.3 Static Gain

The static gain is often assumed to be known *a priory* for estimating model parameters of proportional systems by the relay feedback identification, e.g. Luyben (1987) or more relay tests are necessary, e.g. Li, Eskinat and Luyben (1991). As well, the static gain is separately derived on the basis of the shape of response from the relay feedback test, see Yu (1999). Shen,Wu and Yu (1996) proposed to use an asymmetrical relay for the static gain estimation. In this approach the system is considered at equilibrium at the operating point (u_0, y_0). If the relay feedback test is applied on a proportional system, the static gain K can be determined by the following formula computed by a numerical integration

$$K = G(0) = \frac{\int_t^{t+T_p} (y(\tau) - y_0) d\tau}{\int_t^{t+T_p} (u(\tau) - u_0) d\tau},\ t > t_L \tag{8}$$

Thus, using the formulas (1), (2), (3), (4) and (5), we obtain the three points $G(0)$, $G(j\omega_1)$ and $G(j\omega_2)$ of the Nyquist frequency characteristic; see Fig. 7, which can be used for fitting the model.

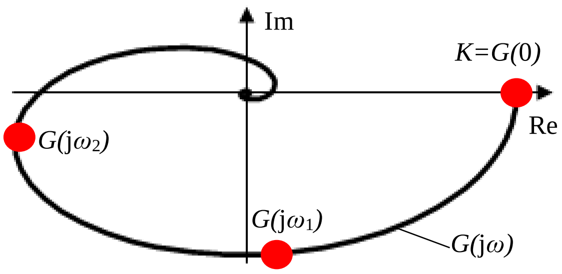

Figure 7: The Nyquist frequency characteristic of a process and the found points $G(0)$, $G(j\omega_1)$ and $G(j\omega_2)$.

We can use this solution in case that we know exactly the values u_0 and y_0. But if we do not know them exactly, e.g. due to a static load disturbance, we cannot use formula (8).

In case that we cannot use formula (8), the static gain may be estimated from the found points $G(j\omega_1)$ and $G(j\omega_2)$ obtained by the shifting method. It will be shown in the next section.

2.4 SOTD Model

Most industrial processes can be described near the operating point using the SOTD model with the transfer function

$$M(j\omega) = \frac{K \cdot e^{-s\cdot\tau_u}}{a_2 s + a_1 s + 1} \tag{9}$$

This model can be used for both oscillatory and aperiodic systems. Additionally, it is also possible to use this model to describe time delayed systems. Hofreiter (2017) derived the following explicit formulas for parameter estimation of the SOTD model from determined values ω_1, ω_2, $G(0)$, $G(j\omega_1)$ and $G(j\omega_2)$.

$$K = G(0) \tag{10}$$

$$a_2 = \frac{1}{2\cdot\omega_1^2}\sqrt{\frac{1}{3}\left(\frac{K^2}{|G(j\omega_2)|^2} - \frac{4K^2}{|G(j\omega_1)|^2} + 3\right)} \tag{11}$$

$$a_1 = \frac{1}{\omega_1}\sqrt{\frac{K^2}{|G(j\omega_1)|^2} - \left(1 - a_2\cdot\omega_1^2\right)^2} \tag{12}$$

$$\tau_u = \frac{1}{2}\sum_{l=1}^{2}\frac{\measuredangle\left(\frac{1}{a_2(j\omega_l)^2 + a_1\cdot j\omega_l + 1}\right) - \measuredangle G(j\omega_l)}{\omega_l} \tag{13}$$

However, this solution can be applied only in case we know *a priory* the static gain K or we can estimate K from a single relay feedback test by formula (8). If the static sensitivity cannot be determined by the above mentioned procedure we can determine K and parameters a_2, a_1, τ_u using the chosen model structure (9) and knowledge of the values ω_1, ω_2, $G(j\omega_1)$ and $G(j\omega_2)$ obtained by the shifting method from a single relay feedback test. For this purpose we can use the following criterion

$$Kr(K, a_2, a_1, \tau_u) = \sum_{i=1}^{2}\left|G(j\omega_i) - M(j\omega_i)\right|^2 \tag{14}$$

where $M(j\omega)$ is the frequency transfer function of model (9).

The value of the criterion Kr depends on the values of K, a_2, a_1 and τ_u. For more compact notation we introduce the vector

$$\theta = \begin{bmatrix} K & a_2 & a_1 & \tau_u \end{bmatrix}^T \tag{15}$$

containing the unknown values of the parameter K, a_2, a_1 and τ_u of the SOTD model (9). For a stable system, the value of the vector θ that minimises the criterion (15) can be determined by

$$\theta = \arg\min_{\theta\in D} Kr(\theta) \tag{16}$$

where $D=\{(K,a_2,a_1,\tau_u)$: $K>0$, $a_2>0$, $a_1>0$, $\tau\in\langle 0,\tau_m\rangle\}$ and τ_m see Fig. 2.

Denote the real and imaginary part of the complex values $G(j\omega_1)$ and $G(j\omega_2)$

$$G(j\omega_i) = R_i + j\cdot I_i \quad \text{for } i=1,2 \tag{17}$$

then

$$\hat{\theta} = \arg\min_{\substack{\tau\in\langle 0,\tau_m\rangle \\ K,a_2,a_1>0}} Kr\left(\begin{bmatrix}\left(Z^T Z\right)^{-1}\cdot Z^T p \\ \tau_u\end{bmatrix}\right) \tag{18}$$

where

$$Z = \begin{bmatrix} \cos\omega_1\tau_u & R_1\omega_1^2 & I_1\omega_1 \\ -\sin\omega_1\tau_u & I_1\omega_1^2 & -R_1\omega_1 \\ \cos\omega_2\tau_u & R_2\omega_2^2 & I_2\omega_2 \\ -\sin\omega_2\tau_u & I_2\omega_2^2 & -R_2\omega_2 \end{bmatrix}, \; p = \begin{bmatrix} R_1 \\ I_1 \\ R_2 \\ I_2 \end{bmatrix} \tag{19}$$

3 SIMULATED EXAMPLE

The introduced relay identification method is demonstrated on an aperiodic proportional process which is taken from Berner, Hägglund and Åström (2016). This process is described by the following transfer function

$$P_1(s) = \frac{1}{(s+1)(0.1s+1)(0.01s+1)(0.001s+1)} \tag{20}$$

where s is the complex variable in L-transform. We assume that the process can be described by a SOTD model in the form (9), the relay feedback experiment is with integrator (see Fig. 3a) and the asymmetrical relay is with a hysteresis having the following parameters (see Fig. 4)

$$u_A = 2, u_B = -1, \varepsilon_A = 0.1, \varepsilon_B = -0.1 \quad (21)$$

We will estimate the model parameters without using the formula (8) only from the values ω_1, ω_2, $G(j\omega_1)$ and $G(j\omega_2)$ obtained by a single relay feedback test and using the criterion (14). The time courses of the manipulated variable u and the controlled variable y are shown in Fig. 8.

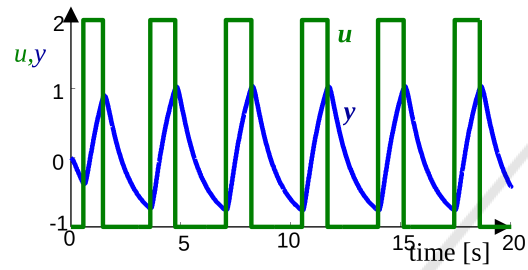

Figure 8: The time courses of the relay output u and the process output y obtained from the relay feedback experiment with integrator.

Solution:

The period of stable oscillation T_p and the values ω_1, ω_2, $G(j\omega_1)$ and $G(j\omega_2)$ can be determined from the stable time courses u and y (see Fig. 8) utilizing formulas (1), (2), (3), (4) and (5).

$$T_p = 3.465 \text{ s} \quad (22)$$

$$\omega_1 = \frac{2\pi}{T_p} = 1.8133 \text{ rad}\cdot\text{s}^{-1} \quad (23)$$

$$\omega_2 = \frac{4\pi}{T_p} = 3.6267 \text{ rad}\cdot\text{s}^{-1} \quad (24)$$

$$G(j\omega_1) = 0.1421 - 0.4533j \quad (25)$$

$$G(j\omega_2) = -0.0300 - 0.2479j\,. \quad (26)$$

The model transfer function $M_1(s)$ is obtained by minimizing the criterion (14) and the calculated values ω_1, ω_2, $G(j\omega_1)$ and $G(j\omega_2)$.

$$M_1(s) = \frac{1}{0.1s^2 + 1.1s + 1} e^{-0.011s}\,. \quad (27)$$

The position of the points $G(j\omega_1)$ and $G(j\omega_2)$ together with the Nyquist diagram of the transfer functions $P_1(s)$ and $M_1(s)$ are shown in Fig. 9. The step responses of the transfer functions $P_1(s)$ and $M_1(s)$ are shown in Fig. 10. Fig. 9 and Fig. 10 show a very good conformity between the process and its model.

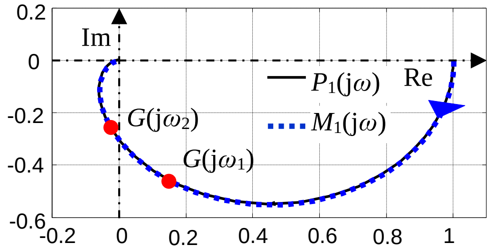

Figure 9: The Nyquist diagram for the transfer functions $P_1(s)$, $M_1(s)$ and the calculated points $G(j\omega_1)$ and $G(j\omega_2)$.

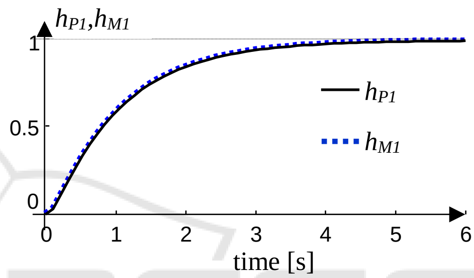

Figure 10: The step response h_{P1} of the process P_1 and the step response h_{M1} of the model M_1.

Next, consider the process with the transfer function (15) but now the relay feedback identification is realized under a constant load disturbance d where

$$d = 0.5 \quad (28)$$

The process is controlled by the asymmetrical relay with integrator, see Fig. 11. The time courses of the manipulated variable u and the controlled variable y are shown in Fig. 12. The goal is to approximate the process transfer function by the SOTD model.

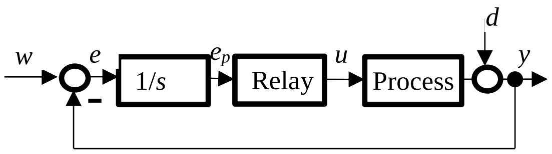

Figure 11: The process under the load disturbance d controlled by the asymmetrical relay with integrator.

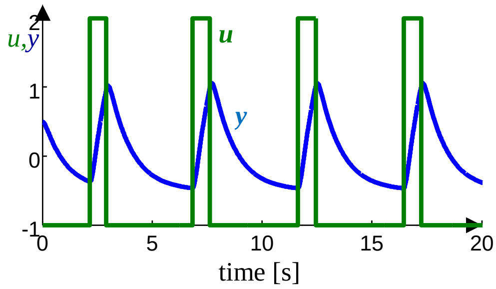

Figure 12: The time courses of the relay output u and the process output y obtained from the relay feedback experiment with integrator under the static load disturbance.

Solution:
The period of stable oscillation Tp and the values ω_1, ω_2, $G(j\omega_1)$ and $G(j\omega_2)$ can be determined from the stable time courses u and y (see Fig. 12) utilizing formulas (1), (2), (3), (4) and (5).

$$T_p = 4.805\,\text{s} \tag{29}$$

$$\omega_1 = \frac{2\pi}{T_p} = 1.3076\,\text{rad}\cdot\text{s}^{-1} \tag{30}$$

$$\omega_2 = \frac{4\pi}{T_p} = 2.6153\ \text{rad}\cdot\text{s}^{-1} \tag{31}$$

$$G(j\omega_1) = 0.2928 - 0.5262\,j \tag{32}$$

$$G(j\omega_2) = 0.0276 - 0.3440\,j\,. \tag{33}$$

The model transfer function $M_5(s)$ is obtained by minimizing the criterion (14) and the calculated values ω_1, ω_2, $G(j\omega_1)$ and $G(j\omega_2)$.

$$M_2(s) = \frac{1}{0.1017s^2 + 1.102s + 1} e^{-0.00964s}\,. \tag{34}$$

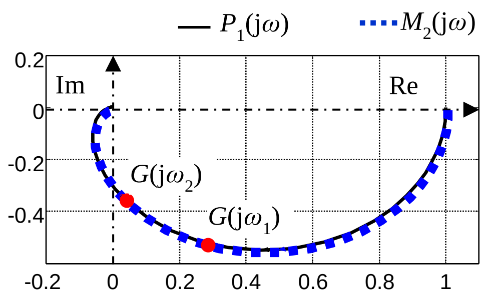

Figure 13: The Nyquist diagram for the transfer functions $P_1(s)$, $M_2(s)$ and the calculated points $G(j\omega_1)$ and $G(j\omega_2)$.

The position of the points $G(j\omega_1)$ and $G(j\omega_2)$ together with the Nyquist diagram of the transfer functions $P_1(s)$ and $M_2(s)$ are shown in Fig. 13. The step responses of the transfer functions $P_1(s)$ and $M_2(s)$ are shown in Fig. 14. Although the static load disturbance d affects the period of sustained oscillations (see Fig. 8 and Fig. 12 or relations (22) and (29)), its effect is eliminated when calculating $G(j\omega_1)$ and $G(j\omega_2)$ with respect to relation (7). This is a very important feature for practice.

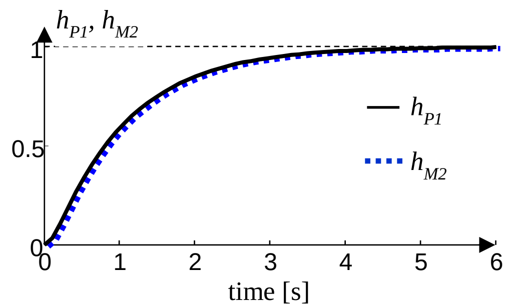

Figure 14: The step response h_{P1} of the process P_1 and the step response h_{M2} of the model M_2.

4 LABORATORY EXPERIMENT

The introduced method was also verified on a laboratory apparatus “Air Aggregate”, see Fig. 15. The apparatus consists of a ventilator and a flow rate meter located in a tunnel. The desired value of air flow is maintained by the asymmetrical relay with integrator. The manipulated variable (power to the ventilator) u and the controlled variable (air flow) y are provided via unified electrical signals (0-10 V). The time courses of the biased relay output u and the output y are shown in Fig. 16. The goal is to approximate the process transfer function by the SOTD model.

Solution:
The period of stable oscillation Tp and the values ω_1, ω_2, $G(j\omega_1)$ and $G(j\omega_2)$ can be determined from the stable time courses u and y (see Fig. 16) utilizing formulas (1), (2), (3), (4) and (5).

$$T_p = 46.5\,\text{s} \tag{35}$$

$$\omega_1 = \frac{2\pi}{T_p} = 0.1351\,\text{rad}\cdot\text{s}^{-1} \tag{36}$$

$$\omega_2 = \frac{4\pi}{T_p} = 0.2702\ \text{rad}\cdot\text{s}^{-1} \tag{37}$$

$$G(j\omega_1) = 0.2416 - 1.2674\,j \tag{38}$$

$$G(j\omega_2) = -0.4809 - 0.6787\,j\,. \tag{39}$$

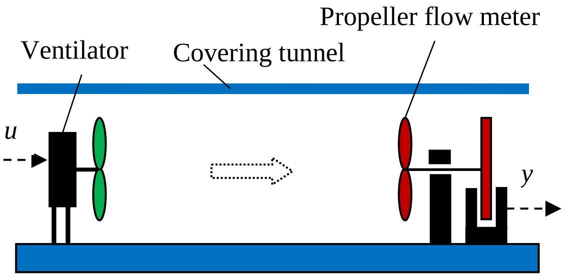

Figure 15: The laboratory apparatus “Air Aggregate”.

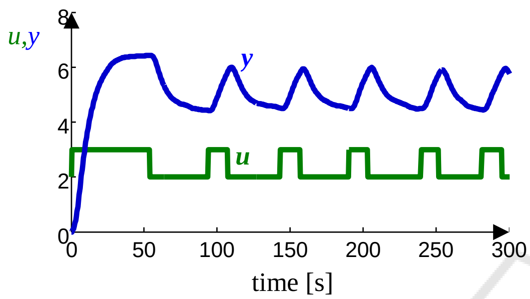

Figure 16: The time courses of the relay output u and the process output y obtained from the relay feedback experiment with integrator.

The model transfer function $M_T(s)$ is obtained by minimizing the criterion (14) and the calculated values ω_1, ω_2, $G(j\omega_1)$ and $G(j\omega_2)$.

$$M_T(s) = \frac{1.969}{0.0044s^2 + 8.315s + 1} e^{-3.89s}. \quad (40)$$

The position of the points $G(j\omega_1)$ and $G(j\omega_2)$ together with the Nyquist diagram of the transfer function $M_T(s)$ are shown in Fig. 17. The step responses of the identified process y and the model output y_M on the manipulated variable u are shown in Fig. 18. Fig. 17 and Fig. 18 show a very good conformity between the process and its model.

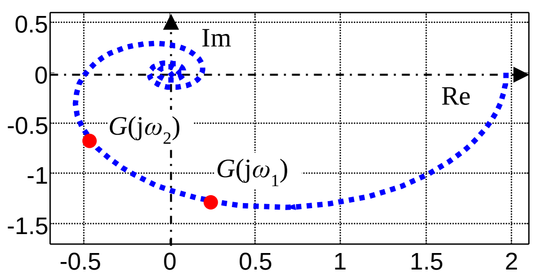

Figure 17: The Nyquist diagram for the transfer functions $M_T(s)$ and the calculated points $G(j\omega_1)$ and $G(j\omega_2)$.

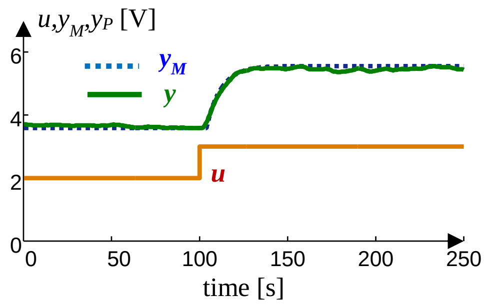

Figure 18: The time courses of the manipulated variable u, the model output y_M and the process output y.

5 CONCLUSIONS

The introduced relay identification method has the following important properties:

- The shifting method can be applied if in the relay feedback experiment there is a stable oscillation with the period T_p ($T_p = T_1 + T_2$, $T_1 \neq T_2$, see Fig. 1), the identified process is time invariant and in the proximity of operating point has linear properties.
- This approach enables to obtain two frequency response points $G(j\omega_1)$ and $G(j\omega_2)$ using a single relay test.
- The obtained frequency points $G(j\omega_1)$ and $G(j\omega_2)$ are determined without any assumption about a model.
- The constant load disturbance has no effect on the positions of the frequency points $G(j\omega_1)$ and $G(j\omega_2)$.
- The identification method is primarily proposed for automatic tuning of controllers.
- The method enables to estimate all the parameters of the SOTD model from a single relay feedback test.
- By using the SOTD model, it is possible to estimate the static gain even in the presence of a constant load disturbance.
- The shifting method can be used both for overdamped/underdamped systems and also for time delayed systems.
- Noisy environment is reduced by using the asymmetrical relay with a hysteresis.
- The calculation of relations (4) and (5) can be refined by integration over multiple periods T_p.

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