Bus Regularity Evaluation using the Gini Index and the Lorenz Curve: A Case Study of New Delhi Bus Network

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Abstract: The ability of a public transport system to provide regular services is the main attraction for the system users. Assessing the regularity of the provided services from the user's perspective is thus crucial for stakeholders in order to establish actions for maintaining or improving their system reliability level and therefore increasing the number of the public transport users. The purpose of this paper is to reveal the pertinence of the Gini Index based on the Lorenz curve as headway and travel time regularity indicator and to carry out a case study of the reliability of a bus operator of the city of New Delhi. We began by reconstituting the missed data in the provided automatic vehicle location data using an approximate approach and then, using correlation coefficients, we studied the linear relationships, before and after data reconstruction, between Gini Index and some of the most used regularity measures; headway regularity, headway adherence, standard deviation and travel time variability. Results show that headway adherence and standard deviation are the two indicators that have the higher correlations with the Gini index and that Gini index is less influenced by missing data and errors.

1 INTRODUCTION

The literature is rich with indicators for public transport reliability measurement but most of the highly used ones are usually unsatisfactory for service regularity measures of high-frequency buses and are not immediately understandable for inexperienced stakeholders (Bhouri, 2016) and do not permit the comprehension of the entire issue. Moreover, the existing indicators cannot be used to compare between different routes, which is important for the stakeholders in order to perceive the ones in which more investments could be made.

This paper aims to study the relevance of the Gini index (GI) as both headway regularity and travel time regularity measures respecting both user's and operator's perspectives. For the headway regularity, we used GI based on the ratio between actual and scheduled headways in order to evaluate the adherence to the scheduled timetables. Unlike the previously reported measures, GI can be used to compare different routes in term of regularity and the associated Lorenz curve, which is the graphical representation of the distribution of the chosen criterion of GI, is a handy tool for revealing more information about the causes of irregularity that a numerical value cannot provide.

For this purpose, a correlation study is investigated between GI and previously reported indicators including headway regularity (HR), headway adherence (HA), standard deviation (STD) and travel time variability (TTV). A bus system of the city of New Delhi as a case study is selected to evaluate correlations and to study related reliability level of the operation. However, the provided Automatic vehicle location (AVL) data presents missing data which can lead to wrong conclusions.

To overcome this issue, an approximate data reconstitution had been realized.

Finally, the correlation results are encouraging for the use of GI as a versatile reliability measure and helped to show that it is less affected by the missing data and errors.

The paper is structured as follows; section 2 gives a literature review of the transport regularity indicators with a spotlight on papers proposing new ones, and a literature review of the use of the Gini index in the transport domain. In section 3 we define the used methods. In section 4 we analyze correlation results and study the reliability of the bus system of New Delhi and lastly, section 5 provides conclusions and perspectives.

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2 LITERATURE REVIEW

2.1 Literature Review on Transport Indicators

There are a considerable number of researches dealing with indicators that are used in the public transport regularity. (Gittens, 2015) give definitions and brief evaluations of 20 indicators sorted by their function (Travel time indicators, schedule adherence indicators, headway regularity indicators and wait time indicators). The paper takes interest in whether an indicator is "traveler-oriented" or not. According to (Gittens, 2015) the preferred indicators to use by bus operators are the percentage of buses running on time and excess waiting time. The authors also proposed a new composite indicator named Journey Time Buffer Indicator "JTBI". (Currie, 2012) review nine reliability indicators and give a comparison between them in terms of ease of understanding, accuracy measure, agency comparability and cost-efficiency, and give an overall rank for each one of them. (Trompet, 2011) benchmark 12 international bus benchmarking group (IBBG) bus operators with four regularity indicators and list the advantages and disadvantages of each one of the indicators regarding the ease of communication, objectivity, customer representation and the nature of inputs.

(Eboli, 2011) develop a methodology that implements the objective (quantitative) and subjective (results of surveys) aspects of an indicator by implementing them to a single composite one. The final indicator is obtained by solving an optimization problem. The methodology has been tested in a case study for several types of indicators, among others, timetable adherence indicators. (De Ona, 2016) suggest a remodeling of this methodology by improving the optimization formulation and by the use of cluster analyses (CA) for the surveys.

(Jensen, 2014) review six types of timetable reliability indicators used in railways and compare them in terms of the information provided, the applicability domain (lines, stations, aggregated) and the necessary inputs for each one. In order to evaluate indicators robustness, a comparison between results of microscopic simulation and the ones of the indicators has been carried out in this study. (Fan, 2016) propose an indicator named The Reported Waiting Time which predicts the waiting time sensed by a traveler, this indicator allows bus operators to better understand the concept of waiting time from the customer's point of view.

(Teng, 2015) propose a new formulation of bus running indicator (BRI) based on bus planning travel time (BPTT) which is also proposed by authors.

The existing indicators are however unsatisfactory for high-frequency bus services (Bhouri, 2016) and can't answer the questions that a transit manager would ask, such as: how regular a bus route is? Among different routes, which one is the most regular? What are the causes of irregularity? The answers to these questions can be given by the GI which gives an easy-understanding and interpretable value even for inexperienced stakeholders, and since it is a normalized measure it can be used to compare different routes. In addition, the associated Lorenz curve helps to extract more information of the causes of irregularity.

2.2 Background on the Gini Index in the Transport Field

The Gini index (also called the Gini ratio or the Gini coefficient) is a measure of statistical distribution introduced by the Italian statistician and sociologist Gini Corridor; it is used to represent the income distribution of a country's residents.

Although it is used originally in economics, Gini index had been used in other fields to measure inequality; In the transport sector, we find a good number of papers using GI; (Delbosc, 2011) adapted the Gini index and Lorenz curve to assess public transport horizontal equity (Horizontal equity means that all population must have equal transit service regardless to the variability of transit needs within population groups.) for Australian city Melbourne.

Departing from this study (Delbosc, 2011) use also the index to measure horizontal equity for another Australian city and compares the results with ones obtained from Melbourne, (Ricciardi, 2015) also compares the public transport vertical equity, using Gini index, between 3 vulnerable groups: elderly residents, no-income households, and no-car households. (Delbosc, 2011) state that the existing measures of transit equity may be complex and not expressed by a single value; the use of GI in this subject is thus interesting because it yields an easy-understanding single value. GI has been largely used in the evaluation of public transport equity, in addition to these articles readers are referred to (Jang, 2017) and (Pavkova, 2015).

To the best of our knowledge, there are only three articles that use the GI for regularity evaluation: (Lee, 2017) propose the use of GI as an evaluation of travel time in order to assess its evenness among road users. GI is calculated in a case study of roads in Korea and is compared with standard deviation, speed, buffer time and buffer index to evaluate the significance of this measure; results show that the Buffer index has the higher positive correlation with the GI in this study.

(Henderson, 1991) assess headway regularity using GI. Along with wait time indicator, headway regularity based on GI was applied for several bus routes of New York City and Manhattan before being tested on a huge number of sets of random headways in order to study their behaviors and rate of change. (Bhouri, 2016) evaluate the adherence of actual headways to the scheduled headways by applying GI on the distribution of the ratio actual headway to scheduled headway. Regularity is one of the most important and relevant measures of public transport reliability, regularity consists in that successive vehicles depart, pass and arrive at a predefined point with predefined time intervals and with equal headways (Rudnicki, 1997). Regularity accordingly means, in a perfect case, delivering a service with equal waiting times and travel times for all the riders.

3 METHOD

3.1 Formulation of the Gini Index as Headway and Travel Time Regularity Indicator

GI is based on the Lorenz curve (figure 1), it varies from 0 to 1 with 0 indicating absolute equality and 1 indicating complete non-equality. The GI value corresponds to the area of the shaded surface on the Lorenz illustration.

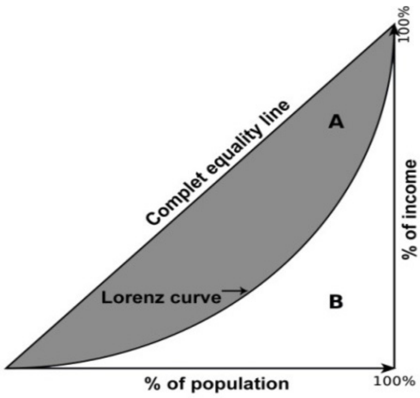

Figure 1: The Lorenz curve.

In our study we calculate the Gini value using trapezoids formula given by (1):

$$GI = 1 - \sum_{k=1}^{n} \left(X_k - X_{k-1} \right)\left(Y_k - Y_{k-1} \right) \qquad (1)$$

Where n is the number of observations, X_k is the k^{th} percentile of the cumulative proportion of the population and Y_k is the k^{th} percentile of the cumulative proportion of the income. The population in our case is the number of the trips, and the incomes are either the cumulative ratios actual to scheduled headways when dealing with headway regularity, or the cumulative travel times when dealing with travel time regularity. The Gini index is already a normalized measure but since we apply it for a ratio between two variables (when dealing with headway regularity), each ratio must be renormalized in order to compare between different bus routes as given by the formula:

$$New\_Ratio = 1 + a \, \frac{\left(actual\ headway - scheduled\ headway \right)}{\left(scheduled\ headway \right)} \qquad (2)$$

With this modification, a same delay (say 5 minutes) has the same effect on the ratio (thus on the Gini index) for lines with different frequencies.

$$\alpha = \frac{Nmin * Rline}{Nline * Rmin} \qquad (3)$$

Where *Nline* is the number of intervals for the studied line, *Rline* is the timetable range of the studied line *Nmin* and *Rmin* refer to the number of intervals and the timetable range of the line “min” such that *Nmin* /*Rmin* is the minimum of the quantities Nline/Rline, whatever the line (this implies $\alpha \leq 1$).

This leads to a new Gini index (related to α) named N_GI

3.2 Correlation Coefficient

The correlation coefficient between two measures is a dimensionless value which varies from -1 to 1; it determines the degree and the direction of the linear relationship between their movements. 1 indicates total positive correlation while -1 indicates total negative correlation, a correlation coefficient equal to 0 means that the two measures are not linearly related. The more it approaches 1 or -1 the stronger the measures are related. We use the correlation coefficient to compare the relations between the Gini index and each of the presented indicators and see how they change in order to better understand the behaviors of the Gini index.

3.3 Regularity Indicators

We present in what follows the highly used indicators that will be adopted for our study.

3.3.1 Standard Deviation

The standard deviation is a statistical measure of the dispersion of a dataset from its average.

3.3.2 Headways Adherence

HA is defined as the standard deviation of the observed headways from the scheduled ones divided by the average scheduled headways as given by the formula:

$$HA = \frac{\sqrt{\frac{1}{N}\sum_{1}^{n}\left(AH_i - M\right)^2}}{\frac{1}{N}\sum_{i}^{N}SH} \tag{4}$$

Where *AH* is the actual headway, *SH* is the scheduled headway and M is the mean actual headways.

3.3.3 Headway Regularity

HR has been used by the New York transit authority (Cramer, 2009), it provides the percentage of trips having acceptable headways.

Since we will be using GI based on the ratio R=(Actual Headway)/(Scheduled Headway), we adapted *HR* to compute the number of trips with acceptable ratios.

Moreover, we don't know whether a ratio is acceptable or not, we propose then another formulation of *HR* using a confidence interval which is given by:

$$HR = \frac{\text{Number of trips having a ratio} \in \text{CI}}{\text{Number of all trips}} \times 100 \tag{5}$$

Where *CI* is the confidence interval with a length of 6 sigmas:

$$CI = [\bar{x} - 3\times\frac{\sigma}{\sqrt{n}}\ ; \bar{x} + 3\times\frac{\sigma}{\sqrt{n}}] \tag{6}$$

Where $\bar{X}$ is the ideal case, i.e. when the actual headway is equal to the scheduled one, which yields to $\bar{X}=R_{ideal}=1$. When a ratio R_i belongs to the CI, the trip i is considered as having an acceptable ratio. σ is the standard deviation of the ratios of a given set and *n* is the number of the trips.

3.3.4 Travel Time Variability

Also known as buffer index, it is defined as the extra time a traveler should add to arrive on time 95% of the time.

$$TTV = \frac{TT95 - MTT}{MTT} \tag{7}$$

Where TT_{95} is the 95th percentile of the travel time and MTT is the mean travel time.

3.4 AVL and Missed Data Reconstitution Methodology

The main problem with the provided data is that we do not have the time of a bus passage at all the stops; these lost data cause discords between actual and scheduled headways which lead to distorted headways ratios. To overcome this issue and make reliable conclusions, we added the missing data with an approximate reconstitution method which utilizes the distance between stops and the speed of the bus; the approach consists, for a given missing, in adding the amount of time T_i=Distance$_i$/Speed$_i$ to the previous detected time, if it exists, if there is no previous detected time we subtract the amount from the posterior detected time and then from the added time and so on until refilling all gaps. It is important to mention that we might get some incoherencies due to using the mean speed in the absence of information on the real speed of a bus; in this case, the reconstituted time is deleted to avoid reproducing false data.

Our study is limited to 8 routes of the New Delhi bus operator consisting of 4 high-frequency and 4 low-frequency routes, within the 30 days of September 2016.

As mentioned, due to the number of missed data that would distort the results, the reconstitution model is applied to provide more accurate reliability measurement.

We acquired 30 files of AVL data (each one corresponding to a bus line for a day of September 2016) for all the routes stops that include actual and scheduled times along with actual and scheduled speed. We also got provided with a file that contains data for only the departure stop and the terminus for all the routes. These data are used to give a first overview of correlations between GI and the other indicators and also is used for the comparison between the routes. Information on bus routes is given in table 1.

Table 1: Information on the bus lines.

Length (frequency)	Route N°	Average Scheduled Headway (minute)	Average Observed Headway (minute)
Short (Low)	403CLUP	23.3	31.05
	403CLDOWN	24.46	33.28
Long (Low)	185UP	32.75	51.67
	185DOWN	32.94	44.34
Short (High)	507CLUP	18.51	24.03
	507CLDOWN	18.3	22.82
Long (High)	165UP	11.67	15.44
	165DOWN	11.75	16.91

4 RESULTS

In this section, we show the graphs of the different indicators drawn for the route N° 403CLUP before presenting and discussing the correlations results between GI and the other measures.

At the end of the section, we study the reliability of the bus services of the city.

4.1 A Visual Comparison between Gini and the Regularity Indicators

To have a first look on the behaviors of the indicators, we draw their charts for the bus route N° 403CLUP within the 30 days, figure 2 shows the graph of GI as a travel time indicator with the graphs of STD of travel times and TTV while figure 3 shows the graph of GI for the headway ratios along with the other headway regularity indicators.

We can notice from a first sight that GI concurs more with the indicators of travel times and that it has higher similarity with STD of travel times than with the STD of headway ratios which demonstrates already that resemblance between two given indicators is not always the same.

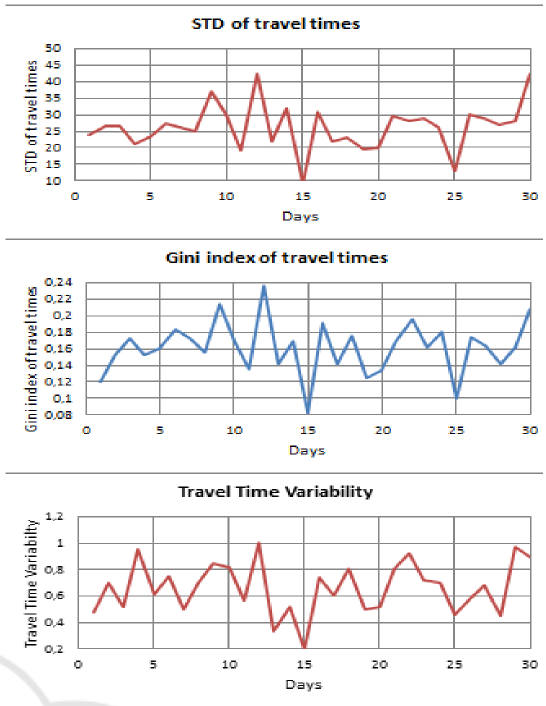

Figure 2: Graphs of travel time regularity indicators for the bus route 403CLUP within 30 days.

To better understand the relationships between GI and the other measures; we use the correlation coefficient because it is more efficient and faster than the visual inspection of the charts.

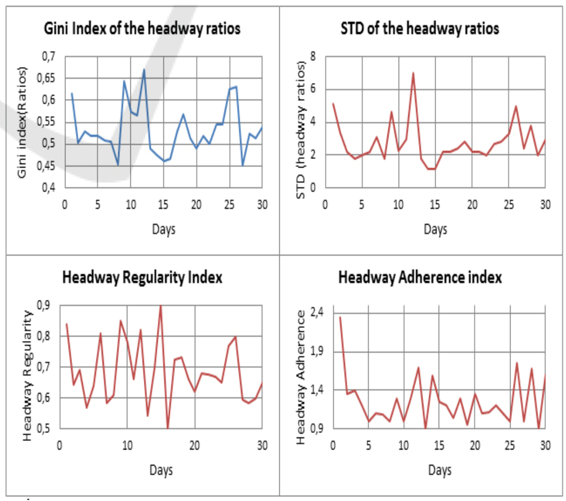

Figure 3: Graphs of headway regularity indicators for the bus route 403CLUP within the 30 days.

4.2 Correlation Coefficients between Gini and the Other Regularity Indicators

Table 2 show the correlation coefficients obtained for all the bus routes between GI and the used indicators.

One can notice from table 2 that STD has the higher correlation coefficients with GI; this is expected since GI is based on STD, we also notice that GI presents a good correlation with TTV and HR which is encouraging for using the indicator for both headway and travel time regularity but, it is important to mention that these correlation results would variate according to the nature of the data; in fact, in a set of data which contains values that are largely deviated from the mean, STD and TTV are highly influenced by these values (LEE, 2017), especially TTV because it takes into account only the deviance of the 95th percentile from the mean , hence it shows larger deviations, while GI would assess the reliability from the perspective of evenness and may not show the same behavior as STD and TTV, correlation would be less good in this case while it would be excellent in the opposite case.

In order to show the influence of data characteristics on correlation, we compare the correlation coefficient between GI and STD before and after the data reconstitution for the bus route N° 165DOWN, figure 4 gives the correlation coefficient values and the charts of GI and STD for day 1 before and after.

We notice that correlations have increased from 0.84831 before reconstitution to 0.95621 after; this is due to the fact that when adding the missing data for this day we actually decreased the relative 'huge variations' as figure 4 shows; before reconstitution (the left side of the figure) there are a considerable number of values that have large deflections from the average which influenced the correlation coefficient negatively, after the reconstitution the data show fewer variations which clarify the increment of correlation between GI and STD.

We also catch from figure 4 that GI is able to detect huge variations but without amplifying them unlike STD and TTV (TTV amplifies the variations more since it computes the deviation between the 95th percentile and the mean, while STD computes the average deviation from the mean.) which leads to conclude that GI is less influenced by variations caused by the errors and misses in data, in fact, the correlation coefficient between STD before and after is 0.3536 whilst the correlation coefficient between GI before and after is 0.6379.

Table 2: correlation coefficients between GI and the other indicators for all the bus lines.

Gini (Ratios)	STD	HR	HA
Low-frequency routes			
403CLUP	0.8076	0.6083	0.4251
403CLDOWN	0.7624	0.5368	0.6157
185UP	0.7240	0.5873	0.4049
185DOWN	0.7626	0.6448	0.5593
High-frequency routes			
507CLUP	0.7730	0.6511	0.2909
507CLDOWN	0.7115	0.6860	0.3826
165UP	0.8188	0.6962	0.4617
165DOWN	0.7179	0.6454	0.5182
Gini (Travel times)	STD (Travel times)	TTV	
Low-frequency routes			
403CLUP	0.9017	0.7322	
403CLDOWN	0.9070	0.7825	
185UP	0.9364	0.7617	
185DOWN	0.9126	0.6221	
High-frequency routes			
507CLUP	0.9550	0.7445	
507CLDOWN	0.9375	0.6179	
165UP	0.9425	0.6259	
165DOWN	0.9679	0.5932	

If we draw the GINI index for all the buses at once, before and after data reestablishment, we would notice that GI values do not stir much, as figure 5 shows.

Although for day 1, 526 missed data is reconstituted (21.11% of the data) and 312 for day 26 (12.53 %), the curves before and after are not very different and the Gini values are barely changing, we noticed also that when drawing the Lorenz curve for each stop separately the Gini values still change slightly which enhances the hypothesis of the ability of GI to provide a reliable measurement, despite the data errors and misses.

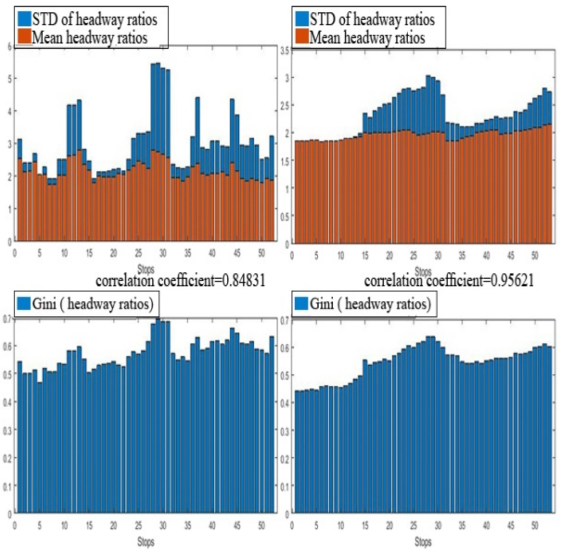

Figure 4: Correlation before (left) and after data reconstitution.

After the data reconstitution, GI shows really excellent correlations with HA, which means that using GI based on the ratio actual to scheduled headways is capable of assessing the adherence to the planned timetable; table 3 shows the results for data for several days.

Table 3: Correlation between GI and HA after data reconstitution.

Day	Correlation coefficient (GI, HA)
2	0.9514
5	0.9566
11	0.9767
26	0.8848

The poor correlations which were obtained before reconstitution are surely caused by the incoherence in the initial data. As to correlations between GI and Headway Regularity, the observed criterion that influences the correlation is the length of the confidence interval, for some datasets; giving a larger CI leads to better correlations. (The data is not normally distributed thus, the characteristics of CI are not the ones defined for the normal distributions).

Finally, like any other indicator, GI has its unique vision of regularity which is the evenness of the distribution of the chosen criterion, it offers a new point of view of the reliability of the public transport. As the correlation study outcomes show, GI agrees with the other indicators under some

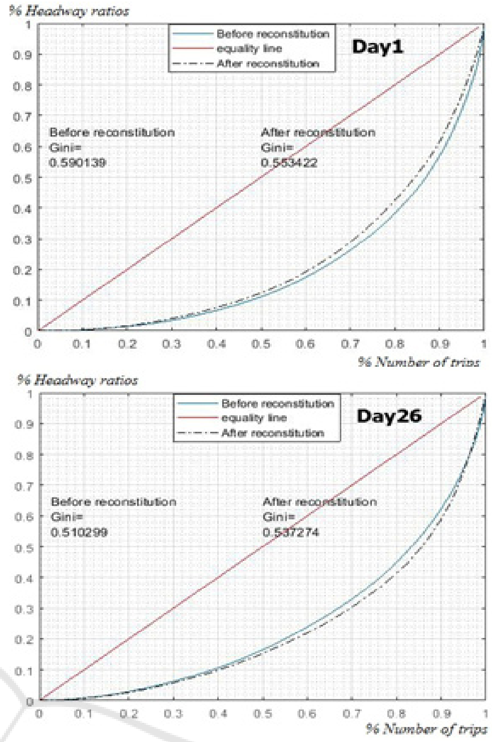

Figure 5: Lorenz curve for all the stops before and after data reconstitution.

conditions, outside these conditions it behaves differently, this is not to be seen as a failure, on the contrary, it shows another perspective from which a bus operator can see the reliability.

In the next section, we discuss the results of the case study of the reliability of the bus operator of New Delhi and show the utility of the Lorenz illustration.

4.3 Reliability of the Bus Services of New Delhi

As a reminder, GI is a value between 0 and 1, the value 0 indicates perfect equality while 1 indicates the non-equality of the distribution. In what follows we show and discuss results for the bus line 165DOWN using the new data. Figure 6 shows the Lorenz curves drawn for the ratio actual to scheduled headways for the 1st and the 26th September. For the first day, GI values show that the first 14 stops are more regular than the rest, that's why we see their curves approaching more to the perfect equality line, in the rest of the stops there are more bus bunches (the left side of the curves are more parallel to the horizontal axe) and more buses with headways that largely deviate from the scheduled ones, shown by the higher discards of the curves from the equality line at the right side.

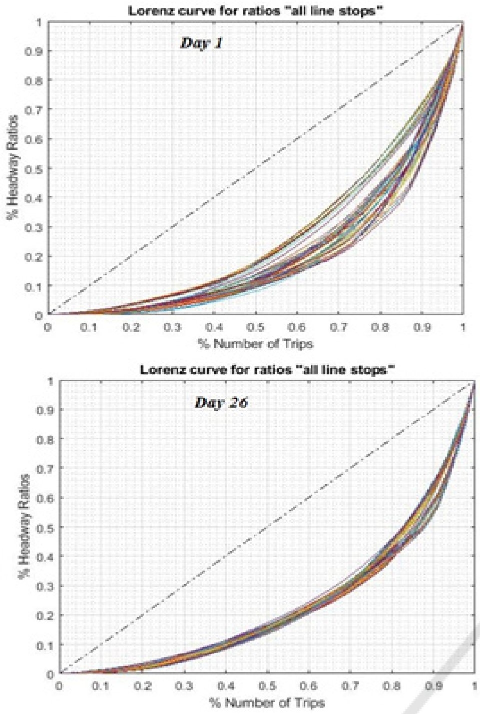

Figure 6: Lorenz curves for all 53 stops separately (headway ratios).

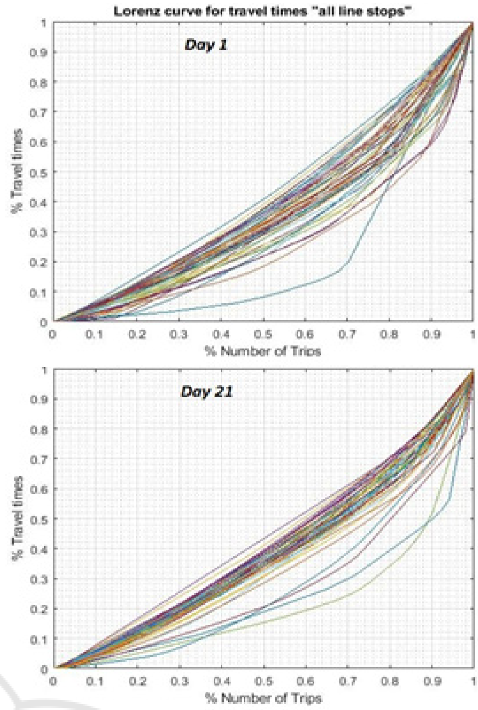

Figure 7: Lorenz curves drawn for travel times for day 1 and day 21.

For the day 26, the stops are showing approximately the same behavior, the mean GI and HA of this day are 0.5345 and 1.5175 which reveals the irregularity of the service due to bus bunching and disrespect to the scheduled timetable.

As an overall conclusion, the bus service for this route within the month of September is not so decent and suffers from bus bunching, which also leads to the appearance of large intervals, and deviations from the scheduled timetable, in addition, we noticed that the number of performed trips varies remarkably from a day to another which also is a real cause of unreliability.

In terms of travel time regularity, the bus operator seems to provide a correct service as can be seen in figure 7, we can see that most of the curves are near the equality line, but one particular curve deviates highly for all the days, it is the road between the stops “Libas pur GT ROAD “and “Sanjay Ghandi Transport Nagar”, which is, in fact, a highway highly influenced by traffic, otherwise for the rest of the trips, most of the users are provided with approximately equal travel times. In order to compare the bus routes in term of headway adherence, we apply the N_GI that was defined in the methodology section by equation (2). Table 4 gives the mean N_GI values for all the studied bus routes of the city of New Delhi.

Table 4: Normalized Gini values for all bus routes.

Frequency	Length	Routes	Mean N_GI
Low	Short	403CLUP	0.4752
		403CLDOWN	0.4713
	Long	185UP	0.4927
		185DOWN	0.5103
High	Short	507CLUP	0.4841
		507CLDOWN	0.5146
	Long	165UP	0.5226
		165DOWN	0.5215

The values of the N_GI on table 4 show that most of the low-frequency routes are the more regular, which is normal as the high-frequency routes are harder to manage, also we notice that the short length routes are more reliable comparing the high length routes, in addition, the GI values are all near 0.5 which indicates a mediocre service for all the routes in terms of headway adherence.

5 CONCLUSION

For the public transportation, regularity of the travel times and the respect to the scheduled timetables are the essential qualities that appeal the users, nevertheless, deviating from the planned program and from the expected travel time are inevitable.

Assessing the irregularities from the user's perspective is necessary for stakeholders in order to establish actions for maintaining or improving their system reliability level and to attract more users. In this paper, we highlighted the relevance of the Gini index based on the Lorenz curve as an indicator of the adherence of actual headways to the pre-established ones and as a travel time regularity indicator, by showing its relationship with some of the most used indicators: headways adherence, headway regularity, standard deviation and travel time variability.

Results show that headway adherence and standard deviation are the two indicators that have the higher correlations with the Gini index. We noticed also that GI remains approximately stable before and after data reconstitution and do not show huge differences unlike the other used indicators, which permitted to judge this indicator as less affected by errors and misses in data. After revealing the effectiveness of the presented measure, we studied and discussed the reliability of the bus services of the city using GI and the Lorenz curve. The results of this study show that the services are irregular in terms of headway adherence but on the other hand, the users are provided by regular trips in terms of travel time. An extension of our study would be to develop a better data reconstitution method, compare the Gini index with other indicators and using other methods of comparison to emphasize the relevance of the Gini index.

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