Accelerating Urban Modelling Algorithms with Artificial Intelligence
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Abstract: In this paper, we demonstrate that developments in computer hardware to support the increasingly complex artificial intelligence workflows for Deep Learning networks can be adapted for urban modelling and visualisation. The hypothesis here is that by leveraging the current practice of AI as a Service (AIaaS), then this enables Urban Modelling as a Service (UMaaS) to be developed. The starting point for this paper is a 3D visualisation of the Queen Elizabeth Olympic Park, developed using a web-based spatial interaction modelling system which calculates population metrics on the fly, capable of showing the results of interventions by urban planners in real-time. We take the web application that powers the interactive visualisation and use Google’s TensorFlow AI library to accelerate the matrix operations required to run the spatial interaction model, making the web application fast enough to be used interactively.

1 INTRODUCTION
This work takes our existing ‘QUANT’ website for spatial interaction modelling (figure 1) and uses its core modelling services in an interactive 3D planning scenario based around housing requirements for the Queen Elizabeth Olympic Park. Our stated aim is to use AI hardware acceleration to build an ‘urban modelling as a service’ system to provide urban planners with the ability to build their own applications, experiment and see immediate results. This then allows us to take a step further, and we test a number of neural spatial interaction models, asking the question about which mathematical model best fits the data.

While the links between artificial neural networks (ANNs) and spatial interaction models have been cited in the literature as far back as (Fischer and Sucharita, 1994), (Black, 1995) and (Openshaw, 1997), recent advances in specialised computing architectures designed to accelerate the matrix operations required to train a large-scale deep neural net have now become mainstream. As a consequence of this, open-source libraries such as TensorFlow (Abadi et al., 2016), Keras (Chollet, 2015), PyTorch (Paszke et al., 2017) and Caffe (Jia et al., 2014), can routinely deal with ANNs that are orders of magnitudes larger than a previous generation. These libraries are designed to work on conventional CPUs for small-scale work, in addition to utilising the multiple parallel cores of graphics processing units (GPUs) for large-scale networks. Add to this the TensorFlow Processing Units (TPUs), designed by Google for AI workflows at the level of warehouse scale computing, plus a myriad of other custom hardware for vision and speech processing ‘at the edge’ (e.g. MOVIDIUS Neural Compute Stick (Intel, 2019)) and the potential exists to make use of this high performance cloud and desktop processing capability. The natural question to ask is how to make a symbiotic link between the current advances in AI and existing urban models? This paper sets out to put urban modelling into the context of mainstream AI to both accelerate the current computer models and to suggest new avenues.
for future research. Existing applications of “AI as a Service” can be cited which relate to object and speech recognition, natural language processing and other artificially intelligent systems, but there is an absence of applications relating to urban modelling. While one discipline seeks to give computers human abilities, the other seeks to explain and predict human behaviour. A commonality exists where the pattern matching abilities of ANNs are compared and contrasted with the calibration and ‘model parameter fitting’ phase common in many types of urban modelling. In effect, both are seeking to minimise an error term and find the best fit for the data available.

The decision to focus on spatial interaction modelling is based on the already established theoretical links with ANNs, which are expanded upon in section 2. The initial work on QUANT developed a website for spatial interaction modelling using 7,201 zones for England and Wales. This is introduced in section 3 and provides the real-life urban planning scenarios used to test the algorithmic and architectural improvements. The urban datasets used in QUANT, for example the travel times between zones and trip matrices, are re-used in this paper, which extends the urban modelling system beyond the current capabilities of the QUANT system. This is a step-change in performance, as, while QUANT takes around 10 seconds to run a model, the system presented here takes just 0.28 seconds. The modular and web-based design of the system means that these improvements can be easily incorporated and used with both 2D and 3D visualisation workflows. As an application of the theory, we present the Queen Elizabeth Olympic Park (QEOP) urban planning example, which is a Unity application that requires the immediate urban modelling enabled by the work in this paper. Unity is a 3D game engine development environment, which is introduced in, “Unity 3D Game Development by Example” (Creighton, 2010). While this serves as a visual front-end for planners who use the system, by improving the performance of the spatial interaction modelling algorithms, we demonstrate a step in performance that makes the immediate urban modelling required by the 3D application possible.

2 RELATED WORK

In his 1997 book, “Artificial Intelligence in Geography” (Openshaw, 1997), Openshaw demonstrates the application of an ANN to a spatial interaction modelling problem involving flows of people. Following on from this, in “Building an Automated Modeling System to Explore a Universe of Spatial Interaction Models” (Openshaw, 1998a), a ‘universe’ of modelling equations is fitted to calibration data in an attempt to find the equation that describes the data with the lowest error. The interesting result here is that the traditional gravity model equations do not give the best fit with the training set. In all of these examples, the inputs to the models are the origin and destination attractions, derived from the flows, and the distance between zones. For the purposes of the following discussion, a singly constrained spatial interaction, or gravity model, is defined by the following equation:

\[ T_{ij} = A_i O_i D_j e^{-\beta c_{ij}} \]  \hspace{1cm} (1)

\[ A_i = \frac{1}{\sum_j D_j e^{-\beta c_{ij}}} \]  \hspace{1cm} (2)

\[ i, j = \text{origin and destination zone numbers}, \]
\[ c_{ij} = \text{cost of travel between them}, \]
\[ \beta = \text{parameter for willingness to travel given cost}, \]
\[ O_i = \text{origin attraction}, \]
\[ D_j = \text{destination attraction}, \]
\[ A_i = \text{balancing factor} \]

An important distinction to make concerns the number of degrees of freedom of the model and the method of training or calibration. In a spatial interaction model, the calibration involves fitting a parameter, \( \beta \), that defines how far people are willing to travel. In Openshaw’s neural spatial interaction model, the net has been trained to predict flow from origin \((O_i)\) and destination \((D_j)\) attractions at the input layer. The \( \beta \) parameter is learnt by the network as a ‘black box’, which could explain why neural spatial interaction modelling techniques are not widely adopted. However, the point being made here is one of equivalence and some of the beliefs of neural nets as being black boxes that defy explanation are challenged in the remainder of this paper. Here we argue that black boxes are not the only option, advocating a different approach which leads to “Urban Modelling as a Service” for the exploration and explanation of different scenarios. By way of comparison with equation 1, a simple ‘feed forward’ neural network is defined as follows:

\[ v_k = \sum_j w_{kj} h_j \]  \hspace{1cm} (3)

\[ y_k = \varphi(v_k) + \theta_k \]  \hspace{1cm} (4)

\[ v_k = \text{neuron k internal state}, \]
\[ w_{kj} = \text{weight between input j and neuron k}, \]
\[ h_j = \text{input to neuron j}, \]
\[ y_k = \text{neuron output}, \]
\[ \varphi = \text{activation function}, \]
\[ \theta_k = \text{bias value} \]
In “Spatial Interaction Models: From the Gravity to the Neural Network Approach” (Fischer and Reggiani, 2005), Fischer and Reggiani discuss different forms of spatial interaction models, from singly and doubly constrained through to a singly constrained neural spatial interaction model, including the choice of activation function ($\phi(v_k)$ in equation 3). Their conclusion states that the classic gravity model equations (equation 1) are the preferred choice because of the “simplicity of their mathematical form and the theoretical nature of their underlying assumptions”. However, they go on to say that the neural network approach is attractive in “data-rich environments” and where “little is known about the form of the spatial interaction function to be approximated”. In “Why does deep and cheap learning work so well?” (Lin et al., 2017), Lin shows how a three layer continuous input feed forward network can be made to approximate a gate multiplier to any degree of accuracy. Their analysis of the types of functions that deep neural networks can be made to approximate is important in the context of modelling when the function underlying the observed data is under investigation. This final point fits with our statement in the introduction about discovering new avenues of research. The initial research with our previous QUANT model was to see whether a large-scale model covering the whole of England and Wales was possible. One of the difficulties with spatial interaction modelling is that the algorithm is of order $O(n^3)$, while the number of zones, $n$, can be in the thousands in order to combat edge effects and demonstrate the finer grain detail of the model. It should be noted that the QUANT system currently runs equation 1 in 1.23 seconds on an 8 core CPU using parallel optimisations\(^1\), while a test using OpenCL achieved the same result, but taking only 0.25 seconds. This test was using dual NVIDIA GTX 980 Ti GPUs, each with 640 cores clocked and a quoted speed of 1306 GFLOPS. This evidence forms our basis for believing that urban modelling can be influenced by the current pace of AI research. One thing stands out about equation 1 above all else. It is ‘pure parallel’, in that there are no loop dependencies between different $i$ or $j$ indexes, so every $T_{ij}$ can be calculated independently without having to wait for a prior dependency. This fact, which is backed by the evidence in the following results section, is the cornerstone of our argument. The re-formulation of the spatial interaction model equations into a parallel form enables maximum usage of all available processing cores on any massively parallel architecture.

\(^1\)The gravity model formula in equation 1 needs to execute $n \times n$ times to compute every value in the matrix where $n = 7201$ zones, giving rise to $\approx 52$ million iterations.

The final point to note regards the number of zones in the model. While Black’s example (Black, 1995) had 9 zones, Openshaw’s example of Durham had 73 zones. The example here has 7,201 zones for the whole of England and Wales, so the goodness of fit of a neural spatial interaction model of this size is one metric to be investigated, as it is possible that the network might not train on a model of this size. Also, the observed matrix contains 51,854,401 training samples.

In the following sections we demonstrate our current work on building a web-based urban modelling framework. The key consideration influencing the design is for immediate results, rather than having to wait hours for a simulation to run, as was the case in the past. This focuses our work on urban modelling for experimentation and exploration.

### 3 OVERVIEW

Previously, it has not been possible to create models with the number of zones we require and that also run fast enough on the hardware to make “Urban Modelling as a Service” (UMaaS) a viable proposition. Our method enables this through a set of web services acting as a “model view controller” and providing the client with a REST API for the urban modelling functions. As this is a real-time modelling service, the main requirement is to provide the user with model outputs fast enough to be interactive. In order to achieve this, we use the software and hardware improvements being driven by “AI as a Service” to power “Urban Modelling as a Service”.

While equation 1 introduced the basic gravity model equation, there are four variants that are defined as follows: unconstrained, singly constrained origin, singly constrained destination and doubly constrained. A complete description can be found in “A Family of Spatial Interaction Models” (Wilson, 1971). For simplicity, the results presented here focus on the singly constrained origin model, but the results are directly applicable to all four variants.

Of paramount importance to our case for ‘urban modelling as a service’ is the quantity and variety of data required to build a real urban model, as opposed to a theoretical example. The data required for the model are: zone boundaries, zone centroids, network graphs for each mode of travel (e.g. road, rail, bus), cost function between zones per mode of travel ($c_{ij}$), resident population, workplace population, travel to work flow matrix per mode ($T_{ij}$), land use (building constraints on space), green belt, parks and protected areas.
Figure 2: Single Source Shortest Path runs on random networks with varying numbers of vertices. The graph on the right is for a sparse network \((e = 0.9v)\), while the left graph is more highly connected \((e = 5v)\). The relative positions of the rail, bus and road networks are shown on the graph to give a sense of scale.

The cost function is required for every zone pair and mode, yielding an \(n \times n\) matrix that matches the dimensions of the flow matrix. This is significant undertaking computationally when \(n = 7201 \implies 51,854,401\) trips, where each trip requires running a shortest path algorithm to obtain the minimum trip time on a network graph containing 8 million edges. Figure 3 shows the three transport networks behind how the cost matrices are calculated. The two graphs in figure 2 show the application of Nvidia’s nvGraph library to this problem. While the cost matrices used in this paper use the real transport network for England and Wales (figure 3), the tests in figure 2 are for randomised networks with two variations on the ratio of the number of edges to the number of vertices.

4 APPLICATION

The current release of QUANT, simulates the impact of changes in population, employment and travel costs in UK Cities, using a simple model of how workers choose where to live in respect to the attractiveness of those places and the cost of traveling to their workplaces. By building this as a set of web services, the core modelling component can be re-used...
in other scenarios. The following sections describe how the QUANT urban modelling services are used in a 3D interactive planning application for the Queen Elizabeth Olympic Park (figure 4). The application tests scenarios online by assigning proposed increase in employment, and by calculating flows of population based on a singly constrained spatial interaction model. These are based on three means of transportation: tube, buses and trains. As such it can calculate inter-city flows that come from not only within London, but from all of the UK.

This is very useful for the evaluation of master-plans for areas in metropolitan cities such as London. For example, The London Legacy Development Corporation (LLDC) is now using the opportunity of the London 2012 Games and the opening of Queen Elizabeth Olympic Park (QEOP) to create “a dynamic new metropolitan centre for London, and develop an inspiring and innovative place where people want – and can afford – to live, work and visit” (LLDC, 2011). The plans include large scale residential developments and supralocal commercial centres such as Westfield London, that are programmed to attract people from all over London and beyond. As such it cannot be studied in isolation, and would be useful to include a regional model that calculates flows of employment and population in order to program the capacities needed for transportation, housing and services.

We will use QUANT MSOA zones as the regional zones (I,J), whose modelling outputs can be accessed real time using requests from the web server using QUANT’s REST API, and develop a fine scale planning zone system for the Olympic Park in the Unity 3D graphics package. Essentially, we develop a system, in which we design new developments, these will be translated into floorspace, and then to population. This means, that it is possible to create scenarios by calculating the total of employment and population from all planning zones within each MSOA in the Olympic Park and then send it back to QUANT to calculate flows on a regional level. This loop will ensure the stability of the planning scenarios, and allow QUANT and the planning model to communicate by exchanging information back and forth.

To create the fine-scale planning zones we use the official planning policies of the QEOP as set by LLDC. These indicators will essentially inform the design scenarios with permissions set by the different urban authorities. We can then develop a calibrated urban planning model as an internal pre-model which will take the role of assigning population flows from
the QUANT regional model to the designed planning zones. This follows the initial work visualising complex urban models in procedural 3D environments (Roumpani et al., 2013), (Hudson-Smith, 2014). In this case we have linked building floorspace and land uses with population figures. e.g. A housing unit of 300m² hosts maximum 10 people. The result of this work, is then a composite interactive 3D plan of the Olympic Park, which links floorspace as designed in Unity3D with population projections from QUANT. Starting with the existing conditions of the Park and adding the projected developments of the LCS, we set out to show how the assigned development types would affect growth over the projected period, and to compare alternative development strategies. We have developed this as an android application, which uses the 3D model as an interface (figure 4). This will allow the user to experiment with different policies and test scenarios within a 3D model of the Olympic park and use QUANT to evaluate these scenarios in terms of population distribution.

5 IMPLEMENTATION

In this section we present our architecture for running urban modelling experiments, followed by an analysis of the achieved performance in the ‘Results’ section. At the top level, this consists of a collection of web services using a ‘model view controller’ pattern. The structure of this can be seen in figure 5. Below the web services layer are the libraries that perform the actual modelling functions. This enables us to separate the functionality from the web protocol framework and test the performance of the urban modelling library in isolation. By exposing the modelling functions as a set of web services, different clients can be used for exploration and visualisation. In the process of our testing we used both conventional 2D maps and a 3D client written using the Unity framework. For the purposes of this paper, though, only the “QUANT Model” and “Web Services” boxes are considered as these deal with the calibration and the running of the model.

The rest of this section describes the implementation of the modelling library as follows:

1. CPU based spatial interaction model
2. GPU based spatial interaction model (OpenCL)
3. AI optimised spatial interaction model (TensorFlow)
4. AI optimised neural spatial interaction model (Keras/Tensorflow)
5. AI alternative model structure (radial basis, generative adversarial networks, neurodynamic models (Milton et al., 2018))

This methodology allows 1, 2 and 3 to be compared for speed as they are functionally identical, while 4 and 5 are interesting from a research point of view as they explore modelling equations themselves. The implementation is using Python 3.6 as it allows for a TensorFlow implementation with the trained model able to be run on the CPU or GPU for benchmarking.

We start with an origin constrained spatial interaction model of commuter flow in England and Wales using data from the 2011 Census. Data is available at the ‘Middle level Super Output Area (MSOA)’ geography, where there are 7201 zones in the model. 

\[
T_{ij} = O_i \frac{B_j D_{ij} e^{-|B_i|}}{\sum_{j} B_j D_{ij} e^{-|B_i|}} \tag{5}
\]

\[
O_i = \sum_{j \leq n} T_{ij}, \quad D_j = \sum_{i = 0}^{j \leq n} T_{ij} \tag{6}
\]

The variables in equation 5 follow the same pattern as equation 1, earlier, with the exception of \(B_j\), which is a constraints factor, used to prevent building in zones where none is possible, for example on green belt land, parks, or areas of outstanding natural beauty. In terms of the system diagram in figure 5, this fits into the box labelled ‘QUANT Model’.

While commuter flows have been used as the initial example (travel to work), other types of flow are also considered such as retail (travel to shop). As long as there is some type of flow matrix, \(T_{ij}\), then anything can be simulated. In addition to this, flows can be disaggregated by type, so, in the case of travel to work, the flows are broken down into three modes ‘road (mode=0)’, ‘bus and tube (mode=1)’ and ‘heavy rail (mode=2)’. This results in one order \(3 \times n\) matrix \(T_{ij}\) and one trip cost matrix \(c_{ij}\) for each of the three transport modes. The final result is then given by the following equation:

\[
T_{ij}^{total} = \sum_{m=0}^{m<3} T_{ij}^m, \{m = mode\} \tag{7}
\]

Learning in this respect is seen as fitting the \(\beta\) parameter to achieve the optimal goodness of fit measured over \(D_{ij}\) for singly constrained, or both \(O_i\) and \(D_{ij}\) for doubly constrained. The \(\beta\) parameter represents the willingness of a commuter to travel the distance between \(i\) and \(j\), which is where the functional difference between the conventional spatial interaction model and the neural spatial interaction model can be measured.
Figure 5: QUANT system diagram showing the server/client split. The box labelled “QUANT Model” contains the code outlined in this paper. This is the section that is critical to the speed of the entire system.

For the TensorFlow version of the conventional spatial interaction model, the operations required to run the model are defined by an execution graph. This graph is shown in figure 6, which shows the parallelism achievable in the flows of dependencies. The $C_{bar}$ calculation (mean trips) from $T_{ij}$ (trips matrix) is visible, along with the $O_i$ and $D_j$ row and column sum reductions. Other than the trips matrix, the only other inputs to the system are $C_{ij}$ (cost matrix) and the $\beta$ parameter which is the variable to be calibrated. This execution graph only shows a single run of the model equation, with no adaptation of the $\beta$ parameter to fit the data to the observed values ($T_{ij}$). The reason for this is that this is the execution graph that was used for the benchmark tests between the regular Python code and the TensorFlow code to measure both speed of execution and equivalence of numerical results. Adding the $\beta$ calibration to the graph is added in the production version of the class, while the simplification here is to make direct testing possible.

All the code for this paper can be found on GitHub under the following project: https://github.com/maptube/UMaaS.

5.1 Neural Model

The neural spatial interaction model follows the previous work of Openshaw (Openshaw, 1997, pp174), Black (Black, 1995) and (Fischer and Sucharita, 1994), using $[O_i,D_j,C_{ij}]$ as inputs to the model with $T_{ij}$ as the target. Openshaw included an additional “intrazonal flag”, used to indicate when $i = j$ and travel is within the same zone. This has not been used here, but one advantage of the neural model over the conventional spatial interaction model is that additional parameters can be added as inputs to explore their significance, as discussed in (Openshaw, 1998a). Another problem concerns the scaling of the inputs and outputs, which need to be in the range $[0..1]$ for the neural network type used here. The practice adopted in the literature is used, with the inputs and outputs being scaled linearly. This introduces a problem, as the maximum $O_i$ value is 24,061, resulting in a scaling factor of $\frac{1}{\max(O_{ij})} = 4.15\times10^{-5}$. This needs to be applied to the loss function, used to establish the model’s goodness of fit when training.

Having defined the inputs and outputs, the next problem concerns the network structure itself. While three inputs and one output is fixed, the number of hidden layers in between can be varied, as can the number of neurons in each layer. This introduces an optimisation problem where too few layers and neurons results in the model not fitting the observed data to a high enough level of accuracy, while too many will result in over-fitting the model. Black (Black, 1995) used a single hidden layer, obtaining results better than the conventional formulation of the model, which suggests that one layer is adequate. Fischer (Fischer and Sucharita, 1994) evaluated ten different network architectures, varying the number of hidden neurons in order to find the optimum for his telecommunications data. Our zonal model is an order of magnitude bigger than any in the literature (Black 9 zones, Openshaw 73 zones), so the structure of the network is under investigation. Finally, the activation function, $\varphi()$ in equation 3, needs to be chosen. In
“Spatial Interaction Models: From the Gravity to the Neural Network Approach” (Fischer and Reggiani, 2005), Fischer and Reggiani use the sigmoid, or logistic function, defined as follows:

\[ \varphi(x) = \frac{1}{1 + e^{-x}} \] (8)

The ANN model is implemented using the Keras library. While this could be implemented directly using TensorFlow, the advantage of using a higher level library over the lower level TensorFlow library is the ease with which different neural network architectures can be explored. In the older literature cited at the beginning of this section, three layer networks with a single hidden layer were used. Here, we investigate the effect of adding additional hidden layers on the prediction performance using a parameter sweep of number of layers and number of neurons. During training the error is taken as mean square error between output and target summed over the batch. A single epoch represents all of the trips data available being applied in batches.

The following data is an example from the training set, which contains 1,960,428 non-zero patterns from original census data, or 3.78% of the full 7201x7201 matrix.

\[
\begin{align*}
[ O_i, D_j, C_{i,j} ] &\rightarrow T_{i,j} \\
[ 24061.0, 165.0, 1.0 ] &\rightarrow 19.0 \\
[ 24061.0, 1006.0, 13.0 ] &\rightarrow 8.0 \\
[ 24061.0, 1645.0, 11.0 ] &\rightarrow 11.0 \\
[ 24061.0, 1248.0, 13.0 ] &\rightarrow 13.0 \\
[ 24061.0, 1419.0, 11.0 ] &\rightarrow 18.0 \\
[ 24061.0, 1241.0, 12.0 ] &\rightarrow 9.0
\end{align*}
\]

For the purposes of evaluating the performance of the neural model, six different network architectures were tested, as shown in Table 1.

<table>
<thead>
<tr>
<th>Model</th>
<th>Inputs</th>
<th>Hidden 1</th>
<th>Hidden 2</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>64</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>32</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>16</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>8</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The network architectures were chosen as follows: model 1 is larger than Openshaw’s test in (Openshaw, 1998b), which had a maximum of 50 hidden neurons, while model 5 is smaller than his minimum of 7. This is included as a test to allow a fast executing model to be trained for longer. Finally, model 6 is a deep network, included as a comparison with the single hidden layer architectures. While the conventional spatial interaction model can be approximated to any degree using a single hidden layer, the deep network is to test whether a more complex function can be discovered which fits the data more closely.

Benchmark tests were run on a cropped version of the England and Wales data, consisting of a 500 zone segment of London. This was necessary to reduce the training time and allow a parameter sweep of the number of hidden neurons, number of layers, batch size and number of training epochs. With the full England and Wales data, there are 51,854,401 in-
put/target training pairs compared to 250,000 for the smaller subset of London, so this represents a significant reduction in compute time. Training is based on minimisation of “mean square error” over the batch, with a batch size of 200,000 to fit inside GPU memory. Batches are presented for training in a random order, which is a technique designed to prevent overfitting the data.

Following the parameter sweep with the test model, experiments were then conducted using the full 7201 zone model for direct comparison with the conventional gravity model.

6 RESULTS

This section presents the results of testing the final evolution of the code optimisations. The computer used to do the tests is an Alienware Area 51 computer, i7 5960X @ 3GHz, 32GB RAM, dual NVIDIA GTX980 Ti GPUs. For the Google TensorFlow tests, this used TensorFlow version 1.12 with NVIDIA CUDA Toolkit 9.0. For the purposes of testing, figure 7 shows the results of executing the main spatial interaction model given by equation 5. This is the base equation used in both the calibration (training) phase and later for inference, or testing scenarios. Where a multi-modal model is concerned, each mode is computed separately using the same equation, so this serves as a representative test.

The TensorFlow model class $^2$ is used to build the graphs for the $O_i$ and $D_j$ inputs to the model equation, which are row and column sums computed using a reduction kernel. One of the problems with measuring GPU performance is that there is significant setup time compared to the CPU, the effects of which have been minimised by timing batches of 1000 runs of the code.

For comparison, a C# implementation was constructed to match the Python implementation. This runs the benchmark test on a matrix of size 7201 in 1.23 seconds, compared to the Python implementation presented here, which takes 1.54 seconds on the same test.

6.1 Neural Model Results

While Tensorflow and GPU hardware has been used to accelerate the evaluation of the conventional gravity model, this section presents the results of training an artificial neural network (ANN) on identical trips

$$C = \frac{\sum_{i,j} T_{ij} C_{ij}}{\sum_{i,j} T_{ij}}$$

(data). Firstly, the ANN has more degrees of freedom than the $\beta$ parameter of the gravity model. Here, the model has three inputs, one or more hidden layers and an output layer containing a single neuron. For a hidden layer with $h$ hidden neurons, there are $3h$ weights in the hidden layer and $h$ weights in the output layer. This makes calibration time longer than in the gravity model case as shown in table 2.

In order to measure the goodness of fit of the models, the $\bar{C}$ metric is used, which is a measure of the total trip time recorded in the original observation data. During the calibration phase of the conventional gravity model, the $\beta$ value is adjusted until the mean trip times factor, $\bar{C}$ is within 0.1 of the observed value. Equation 9 defines how $\bar{C}$ is calculated, with the values for each of the models shown in table 2.

$$\bar{C} = \frac{\sum_{i,j} T_{ij} C_{ij}}{\sum_{i,j} T_{ij}}$$

Training only needs to be performed once, though, while speed of inference is the main metric of interest. The test here is to see whether the mathematically complicated ANN can be used to infer the data for the entire $N \times N$ trips matrix in a time comparable to the gravity model by using the hardware acceleration provided by the GPU.

Table 2: Training time for 10,000 epochs and final error. NOTE: this is for the 500 x 500 test matrix.

<table>
<thead>
<tr>
<th>Model</th>
<th>Net</th>
<th>Training Time</th>
<th>Min Error</th>
<th>Error (\bar{C})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3-64-0-1</td>
<td>35min</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>2</td>
<td>3-32-0-1</td>
<td>33min</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>3</td>
<td>3-16-0-1</td>
<td>23min</td>
<td>0.18</td>
<td>0.18</td>
</tr>
<tr>
<td>4</td>
<td>3-8-0-1</td>
<td>19min</td>
<td>0.017</td>
<td>0.017</td>
</tr>
<tr>
<td>5</td>
<td>3-4-0-1</td>
<td>17min</td>
<td>0.19</td>
<td>0.19</td>
</tr>
<tr>
<td>6</td>
<td>3-4-4-1</td>
<td>19min</td>
<td>0.033</td>
<td>0.033</td>
</tr>
</tbody>
</table>

Then, as a secondary test, the results are compared with the conventional gravity model to see whether they are consistent. Table 3 shows the inference times for the full 7201 matrix data.

Table 3: Inference times for the full 7201 element matrix data on the six models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Net</th>
<th>Inference Time (secs)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GPU</td>
<td>CPU</td>
</tr>
<tr>
<td>1</td>
<td>3-64-1</td>
<td>1.156</td>
</tr>
<tr>
<td>2</td>
<td>3-32-1</td>
<td>1.025</td>
</tr>
<tr>
<td>3</td>
<td>3-16-1</td>
<td>1.214</td>
</tr>
<tr>
<td>4</td>
<td>3-8-1</td>
<td>0.900</td>
</tr>
<tr>
<td>5</td>
<td>3-4-1</td>
<td>0.929</td>
</tr>
<tr>
<td>6</td>
<td>3-4-4-1</td>
<td>0.996</td>
</tr>
</tbody>
</table>

Now, the goodness of fit between the ANN model and the observed data is measured. However, with the neural model, training uses the mean square error between predicted and target as the loss function, so the back-propagation algorithm seeks to minimise this factor instead. This marks an important difference of the neural model, as the $C$ calculation requires the entire predicted matrix for its evaluation, while the neural model operates at the level of batches of individual predicted/target pairs. This leads to the result that the neural model’s mean square error is not directly comparable with $C$, the usual measure of fit for a gravity model. The $C$ metric is trips weighted by travel time, while the ANN’s loss algorithm only optimises for the raw number of trips. The mean square error provides an upper bound on $C$, which can be seen in the data if $C$ is plotted against mean square error.

Looking at the training data in figure 8, any of the models with a final error below 0.1 would be acceptable in the context of the conventional gravity model, which is calibrated to this accuracy. In other words, we have a universe of models that provide a different way of fitting the observed flow data. While figure 8 shows the training on a test matrix of 500x500 for speed, this is still only a subset of the full data. The faster training speed on this smaller data set enables a parameter sweep which then translates into faster training on the full 7201x7201 data set, which is a factor of 207 times bigger. Although the smaller matrix size was used as a test, it still constitutes real data and could, in theory, be used to ‘seed’ the training with the full data. This has not been attempted here, with the full training performed starting from randomised weights. Using the graph in figure 8 and timings obtained from running the software, the “3-64-1” model was rejected on the grounds that training would need at least 10,000 epochs at 30 seconds each (83 hours), similarly for “3-32-1” and “1-16-1”. The “1-4-1” model was rejected for being too small to generalise the data, leaving the “1-8-1” and “1-4-4-1” (approx 0.5s per epoch) models. The “1-4-4-1” model would make for an interesting comparison being a two layer deep network, but, unfortunately, it proved too difficult to train, requiring over 40,000 epochs. Of the remaining “1-8-1” model, after 12,000 epochs of training the $C$ mean tips length is 23.8, when the observed data is 12.5, this is with a mean square error of $4.16\times10^{-6}$. The main lesson learned from the training exercise is to train with the full data and update the weights at the end of the epoch, rather than using small batches and updating more frequently.

7 CONCLUSIONS

In this paper we showed how the performance of a gravity model is speeded up using the TensorFlow artificial intelligence library and GPU hardware. We compared the CPU performance of the model with the GPU accelerated model and showed how large models approaching 10,000 zones can be handled at interactive speeds. We termed this, “Urban Modelling as a Service”, leveraging “AI as a Service” for urban modelling. The immediacy of the results is important here due to the interactive nature of the visualisation and exploration of modelling scenarios through a real-time, web-based, user interface. The power of this architecture is that it opens up the possibility of testing modelling scenarios to the general public. Data like the trips and cost matrices, zone boundaries, and constraints can be held and curated on the server, thereby providing all the data required for modelling in one place.

A secondary result is the comparison of a neural spatial interaction model with a conventional model. Here, the scaling of inputs to fit the sigmoid activation function and extended training times on the data are shown to be a problem. This is unfortunate, as the logical extension to this work is to continue with the comparison between the neural spatial interaction models and the conventional model and show that the neural network is not a “black box”. The same methodological testing can be applied to both types of model in order to assess their function. The future direction of this research is to investigate the ensemble of models provided by this method when applied to the same scenario.

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3The estimates of how long training would take are based on analysing the drop in error over time.
Figure 8: Number of training epochs and goodness of fit ($\bar{C}$), this is for 500 x 500 matrix test data.

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