MATE-BOOSTER: Design of an e-Learning Course to Boost Mathematical Competence

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Abstract: In the transition from lower to upper secondary education, Italian students are expected to have achieved a level of competence which allows them to use knowledge and abilities to model and to understand scientific and technical disciplines. Gaps or misunderstandings in basic knowledge can hinder the effort of students who attend technical high schools, where the core subjects are based on Mathematics. This paper deals with the design of a project conceived to strengthen mathematical competences of students attending the first year of a technical upper secondary school through an online course named “MATE-BOOSTER”. The online activities on the web-based platform have been developed using didactic methodologies founded on constructivist assumptions, as problem posing and problem solving, collaborative learning, learning by doing, automatic and adaptive formative assessment. In this work the process of design of MATE-BOOSTER is shown, the methodologies chosen are discussed, and the online activities are analysed from a constructivist perspective.

1 INTRODUCTION

Italian students completing lower secondary education – which in Italy ends at 8th grade – are supposed to have developed a positive attitude towards Mathematics and to understand how mathematical tools can be useful in many situations to operate in the real world (MIUR, 2012). INVALSI is the national institute in charge of verifying that the learning outcomes are achieved: it administers surveys and standardized tests in order to guarantee the quality of Italian instruction and to make it possible to be compared at international level. The results of INVALSI surveys highlight how, at all stages, but at the end of 8\textsuperscript{th} grade of instruction in particular, there are still difficulties in the deep understanding of basic mathematical concepts, in the ability of applying knowledge to solve problems in real contexts and, above all, in the process of argumentation, which shows the difficulty in formalizing the intuitive knowledge (INVALSI, 2017). These gaps increase in importance when students enrol to upper secondary school and they have to approach scientific and technical subjects, whose understanding relies upon their basic mathematical competence. This problem is particularly evident in technical upper secondary schools, where specialized disciplines are studied at an advanced theoretical level, though students’ average mathematical competence is lower than in Lyceums, as the national surveys show (INVALSI, 2017). The ability to use mathematical thinking to solve problems related to the real experience or to other disciplines – in other words, mathematical competence (MIUR, 2010; Pellerey, 2004) – thus acquires relevance in the delicate period of transition that young people go through when they enrol to upper secondary school, when school successes and failures are deeply interlaced with the shaping of their characters (Debnam et al., 2014; Mariani, 2006).

The Head Teacher of the Technical Upper Secondary School “Eugenio Bona” of Biella, together with her team of Mathematics teachers, designed a project aimed to strengthen the basic mathematical...
competences of first year students with the support of an e-learning platform and digital materials. The project, called “MATE-BOOSTER”, has been implemented in collaboration with the department of Mathematics of the University of Turin, which has a long experience in the development of virtual environments for learning Mathematics, especially to prevent school failure (Barana et al., 2017b; Barana and Marchisio, 2015) and to support students in the transition from lower to upper secondary school (Barana et al., 2016). The project started in September 2018 and it is currently developing.

This paper focuses on the design of the project; the methodologies chosen in relation to the students’ needs are deeply discussed and the process which led to the realization of innovative digital materials is shown and exemplified.

2 STATE OF THE ART

2.1 Web based Constructivist Learning Environments

The choice of the methodologies for developing the learning materials has been made on constructivist assumptions, according to which knowledge is situated, being a product of the activity, context and culture in which it has been developed and used (Brown et al., 1989). Learning is seen as a lifelong active process of knowledge building mediated by experiences and relations with the environment and the community (von Glasersfeld, 1989); thus constructivist learning environments should provide authentic activities and real world problems which can engage students. In Mathematics education this theme has been investigated by many researchers, as Schoenfeld who suggests that Mathematical thinking should be a tool to interpret quantitative phenomena of the outside world and it should be developed at school through meaningful modelling activities (Schoenfeld, 1992).

One of the main implications of the constructivist idea of the learner creating his or her own knowledge is the shift from a teacher-centred to a student-centred approach. If students become the protagonists, the teachers need to leave the stage and move aside, changing their role from leaders to mentors, and their task from knowledge transmission to the creation of a suitable environment for learning (Cornelius-White, 2007).

The community where the learner is integrated in is a core element as well. The sharing of opinions opens the mind and favours the process of knowledge building. Thus a constructivist learning environment should facilitate collaboration and activities should require discussion and interaction among peers (Lave, 1991).

Moreover, activities should be rooted in assessment with a formative value in order to inform both teachers and students about progresses (Scriven, 1966). Assessment and metacognition are deeply interlaced: frequent and well-structured feedback helps learners understand where they are going and how they are going, giving information not only about how the task has been performed (task level), but also about the process that should have been mastered (process level), and enabling self-regulation and self-monitoring of actions (self-regulation level) (Hattie and Timperley, 2007).

Strategies as formative assessment, collaborative learning and relevant problem solving are also indicated by several researches as useful enablers of learners’ engagement, which is related to high learning achievements (Ng et al., 2018). Improving engagement is particularly important in students with challenging backgrounds, learning difficulties or low scholastic performances; in these contexts, interventions that only focus on the reinforcement of basic knowledge are often little effective, if they don’t rely on approaches which promote interest, motivation and self-efficacy (Haberman, 2010).

Technology can support the creation of constructivist digital environments, as it can provide computer mediated communication, computer supported collaborative work, case based learning environments, computer supported cognitive tools (Jonassen et al., 1995), as well as instruments for self and peer assessment (Kearns, 2012) and for automatic evaluation (Barana et al., 2015).

The analysis of the implementation of web based constructivist learning environments has involved many authors in literature in the last twenty years and several models have been designed to engage students of different school levels, in e-learning or blended modality, in learning several disciplines (Alonso et al., 2005; Czerekowski and Lyman, 2016; Lefoe, 1998; Sangsawang, 2015). Their results mainly deal with the relations between strategies, media and tool used and processes activated. Constructivist instructional designers generally accept as a valid and well-established framework for building learning environments the seven learning goals devised by Cunningham, Duffy and Knuth in 1993 and illustrated by Honebein (1996); they are:

1. to provide experience with the knowledge construction process;
2. to provide experience in and appreciation of multiple perspectives;
3. to embed learning in realistic and relevant contexts;
4. to encourage ownership and voice in the learning process;
5. to embed learning in social experience;
6. to encourage the use of multiple modes of representation; and
7. to encourage self-awareness in the knowledge construction process.

2.1 Automatic Formative Assessment

In a virtual learning environment, formative assessment can be easily automatized in order to provide students immediate and personalized feedback. There are several Automatic Assessment Systems (AAS) that allow the creation of questions for STEM (Science, Technology, Engineering and Mathematics); those which are based on a Computer Algebra System (CAS) support the creation of automatically graded open Mathematical answers, such as formulas and equations, but also sets, vectors and graphs, which are accepted for their meaning, not only for their form.

These tools can be usefully adopted to enhance learning, master problem solving strategies, improving metacognition, facilitate adaptive teaching strategies and support teachers’ work (Barana et al., in press).

Using Moebius AAS (Moebius Assessment, 2018), the Department of Mathematics of the University of Turin has designed a model for the formative automatic assessment for Mathematics, based on the following principles (Barana et al., 2018):

1. availability of the assignments to the students, who can work at their own pace;
2. algorithm-based questions and answers, so that at every attempt the students are expected to repeat solving processes on different values;
3. open-ended answers, going beyond the multiple-choice modality;
4. immediate feedback, returned to the students at a moment that is useful to identify and correct mistakes;
5. contextualization of problems in the real world, to make tasks relevant to students;
6. interactive feedback, which appears when students give the wrong answer to a problem. It has the form of a step-by-step guided resolution which interactively shows a possible process for solving the task.

The last one consists of a step-by-step approach to problem solving with automatic assessment, but it is conceptualized in terms of feedback, highlighting the formative function that the sub-questions fulfill for a student who failed the main task. The interactive nature of this feedback and its immediacy prevent students from not processing it, a risk well-known in literature which causes formative feedback to lose all of its powerful effects (Sadler, 1989). Moreover, students are rewarded with partial grading, which improves motivation.

This model relies on other models of online assessment and feedback developed in literature, such as Nicol and Macfarlane Dick’s principles for the development of self-regulated learning (Nicol and Macfarlane Dick, 2006) and Hattie’s model of feedback to enhance learning (Hattie and Timperley, 2007).

3 METHODOLOGY

The MATE-BOOSTER project was conceived with the aim of strengthening basic mathematical competence of first-year students of a technical upper secondary school, acting with methodologies and tools able to activate students’ motivation and engagement, in order to prevent failures in scientific, technological and economic subjects which are at the core of their curriculum. The main feature of the project involves the creation of a web-based course in a virtual learning environment where students can revise the contents in a self-paced way or under their teachers’ guide, both in the classroom and at home. Materials have been created according to didactic methodologies which are in line with the theories of constructivism and formative assessment outlined in the previous paragraph.

The project involves 202 students of nine classes with their seven teachers of Mathematics, plus one teacher in charge of coordinating the works from inside the school.

MATE-BOOSTER has been developed following a model of learning design of “ASSURE” (Heinich et al., 1999), which includes the following steps:

1. Analyse the learners;
2. State objectives;
3. Select methods, media and materials;
4. Utilize media and materials;
5. Require learner participation;
6. Evaluate and revise.

The whole design process has been conducted by researchers from the Department of Mathematics of the University of Turin in close collaboration with the
teachers of Mathematics of the nine classes involved. In fact, it has been considered essential that teachers share the instructional strategies, approve the didactic materials and are consulted at each step of the design; otherwise they couldn’t present the project to their students in a convincing way that make them take part in the online activities.

4 DESIGN OF THE COURSE

4.1 Analysis of the Learners

The analysis of the learning needs, preceding the development of the course, was carried out with two different aims:
- to examine students’ competence in Mathematics, and the gaps in their knowledge;
- to inquire about students’ motivations to the study in general and to the study of Mathematics in particular.

Two different tools have thus been chosen for these objectives: an entry test to assess the initial competence and a questionnaire to understand their motivations.

The entry test was composed of 20 multiple choice questions to be answered in 45 minutes. For each correct answer students got 5 points, 0 for incorrect or ungiven answers. It has been administered online with an automatic assessment system. All students took the test on the same day (8th October 2018); some settings were added to the test to prevent students from cheating: the questions and the choice of the answers were shuffled, there were some random numeric parameters, there was only one attempt available with an automatically set time limit, so that the test automatically quitted after 45 minutes. Few days before the date of the test, students were given the log in data to access the platform where the test would take place; there, they could find a sample test with the instructions to navigate through the questions.

Questions were distributed among the core topics studied in the lower secondary school, in proportion to the time generally dedicated to each one. Each question referred to one of the main content areas of the curriculum (numbers, space and shapes, functions and relations, data and predictions), moreover there were two questions about simple logic reasonings. More details are shown in Table 1.

Questions were built in order to verify the comprehension of particular concepts or processes, not just to check the memorization of rules or formulas.

The results of the test have been statistically treated using the difficulty index, which corresponds to the ratio between the number of correct answers and the sample size, and the discrimination index, which is the difference between the difficulty indexes of the best performing group and the worst performing one, where the two groups are equal sized and cover the whole sample (Ebel, 1954). The test reliability has been assessed through the Cronbach Alpha.

Results of the entry tests were not particularly good, with an average score of 41/100, meaning that the level of difficulty was quite high, at least for the students of this school. Nobody scored more than 80 out of 100, while the lowest registered score was 5/100. If aggregated by classes, the average score varied significantly, from a minimum of 34/100 to a maximum of 54/100; the belonging to a specific class explains the 18.5% of the variance of test results (square eta = 0.185, p<0.0001). It can be noticed that the best performers attend a curriculum which is more rooted in Mathematics than the worst performers. Results aggregated by content areas show the same trend as INVALSI tests: space and shapes turned out to be the most difficult area, with an average index of difficulty of 0.27; it was followed by logic reasonings (0.36), whilst data and predictions was the easiest one (0.60). Results for numbers and relations hung around 0.40.

The difficulty of questions ranged from 0.22 to 0.87; only 4 out of 20 questions can be considered "easy", reaching more than 50% of correct answers. The majority of questions can be considered coherent with the general test, since the discrimination index is greater than 0.25 for the 75% of the questions. Questions with a low discrimination index have been qualitatively analysed: they include frequent

| Table 1: Content areas and topics of the questions. |
|----------------|----------------|----------------|
| Content area   | Number of questions | Topics                          |
| Number         | 8                     | Rational numbers, number estimation, fractions, percentages, powers |
| Space and shapes| 4                     | Perimeter and area of plane figures. |
| Functions and relations | 4                       | Symbolic computations, equations, proportions. |
| Data and predictions | 2                   | Tabular and graphic representation of frequencies. |
| Logic reasonings | 2                     | Simple logic reasonings involving order relations and set theory. |
misunderstandings among the incorrect options or high-level reasonings that caused also the most skilled students to make mistakes (Tristan Lopez, 1998). The test Cronbach Alpha was 0.65; it was negatively influenced by these questions which hindered the students. Our claim is that this test is quite efficient for grade 9 students, but in general students who enrol to a technical secondary school like this one have low-level competence in Mathematics, that the test highlighted.

The questionnaire was composed of 33 statements where students were asked to state their level of agreement with Likert scale from 1 to 4 (completely disagree – completely agree) or from 1 to 5 (insufficient – excellent). It was administered online on the same platform where the entry test took place. The questionnaire is inspired by the student questionnaire of 2012 PISA survey, when Mathematics was the main focus (OECD, 2013). It was aimed to measure attitudes and behaviours towards school and Mathematics, in particular intrinsic motivation (shown by students that study mathematics because they like it), instrumental motivation (shown by students that study Mathematics because it will be useful for their future), perseverance, openness to problem solving, perceived control over success in Mathematics, ethic and respect of school roles, mathematical activities outside school. Moreover, it was asked if students have an internet connection and a device (tablet/computer) at their home. It emerged that students’ intrinsic motivation is not so high (the average is 2.6 in a scale from 1 to 4), although it varies widely (standard deviation: 0.9), while instrumental motivation is higher (the average is 3.1 in a scale from 1 to 4, standard deviation: 0.5). All students have the possibility to use a computer with internet connection for large part of their time at home. Deeper analysis on the answers to the questionnaire will be carried out later in the project; the information gained will be used to better interpret the outcomes.

4.2 Statement of the Objectives

In the light of the results of the entry test and of the questionnaire, during a focus group researchers and teachers listed the learning outcomes of the course. The choice of the topic that the course should cover was made considering the contents needed to understand the scientific courses of the first years (Mathematics, Computer Science, Economy, Science, Physics). They are the following:

- fractions (operating with rational numbers);
- proportions (calculating the unknow term of a proportion, to solve problems involving direct and inverse proportionality in real contexts);
- percentages (calculating percentages in real contexts);
- powers (knowing the meaning of exponentiation and applying the properties of powers);
- mathematical formulas and functions (working with symbols and formulas and with their graphical representations);
- equations (reading and building equations, solving linear equations in one unknown);
- plane geometrical shapes (knowing and calculating measures of angles, triangles and squares);
- statistics and probability (managing data, descriptive statistics indexes and graphical representations, calculating elementary probabilities in real contexts);
- mathematical language (understanding and using different registers of representation: verbal, symbolic, graphical, geometrical, numerical);
- logics (managing simple logic reasonings using Boolean operators).

4.3 Selection of Methods, Media and Materials

The choice to create an online course which students can use at home in a self-paced modality has been validated by their availability of technological devices to access the material, expressed in the questionnaire. Moreover, in all the classrooms of the school there is an Interactive White Board (IWB) that teachers can use to show students the platform and to complete the activities together; the school has three computer labs and several tablets that allow students to work with the course activities even at school. As a Virtual Learning Environment, an integrated Moodle platform has been adopted, managed by the ICT services of the Department of Computer Science of the University of Turin, in collaboration with the Department of Mathematics, the same platform where the entry test and the questionnaire have been delivered. MATE-BOOSTER has been inserted on an instance of the Moodle platform that the University of Turin commonly adopts for e-learning and that often hosts school teachers and students for educational projects (Barana et al., 2017a, 2017c; Barana and Marchisio, 2016a; Giraudo et al., 2014; Marchisio et al., 2017). It is integrated with an Advanced Computing Environment (Maple) for the
creation of interactive materials, and with Moebius Assessment for automatically graded assignments.

The didactic methodologies for the development of the contents have been selected on the base of the constructivist framework and of the evidence gained during previous experiences of e-learning courses (Barana and Marchisio, 2016b). They are the following:

- Problem posing and problem solving: assuming the social-constructive insight of problem solving, problems are considered as learning environments where mathematical knowledge is created in a collaborative discussion starting from a problem. The top-down order traditionally used to study Mathematics is inverted: from the analysis of a real-world situation, paths to the solutions are drawn, in a constructive approach toward the discipline. Afterward, the solving steps are synthetized and generalized, introducing the typical rigor of Mathematics. Learning technologies are used both for online cooperation and as a mean of representation of the solving process: freed from the burden of calculations, students can focus on the solving strategy, find relationships and better understand the solutions (Brancaccio et al., 2015).

- Collaborative learning: in a Virtual Learning Environment, collaboration can be fostered through activities for synchronous or asynchronous discussion; it enhances students’ comprehension of problems and of Mathematical concepts. Moreover, positive collaborations affect the quality of the environment and they are reflected on students’ motivation. Collaborative virtual learning environments force the shift of the teachers’ role, who let students create their own learning – but carefully monitoring it (Barana and Marchisio, 2017).

- Learning by doing: interactivity enhances students’ engagement and contributes to increase their motivation. Feedback that students get from activities help them control their learning and move forward (Gossen et al., 2018).

- Automatic formative assessment: implemented with an AAS specialized for STEM, it allows students to practice at their own pace and to obtain immediate feedback to acknowledge their own level of preparation. Questions and assignments can be enhanced by varying them in a random controlled form, inserting parts expressed in a special programming language. This allows a great variety of assessment modalities which strengthen reasoning until it is mastered: students can obtain different data or graphics at every new attempt, the system can adaptively suggest guided resolutions, feedback and questions can automatically be proposed on the base of previous answers (Barana et al., 2018).

The process of creation of the materials took place in a “Management course”, where school teachers could access and follow the work, propose ideas and suggestions, get in touch with the researchers.

The structure given to the course is modular, according to the general guidelines for the creation of e-learning course (Rogerson-Revell, 2007), each module corresponding to a different topic, to the purpose of addressing students through the course topics and to show at a glance the whole content. Figure 1 shows the course homepage with the 11 modules in a grid format, chosen for its graphical impact on the learner.

All the modules have a fixed structure, composed of submodules containing:

1. materials with theoretical explanation of the fundamental concepts in the form of e-book, that students can read online or download in pdf. Explanations begin with problems and are correlated with examples, graphics and images;

2. interactive materials for the exploration of the concepts illustrated in the e-book, which help students to put theory into practice, to visualize

![Figure 1: Homepage of the course with the modules in grid format.](image-url)
and analyse different representations of the same mathematical structures when parameters change;

3. automatically graded assignments to check the understanding of the concepts presented and of the related abilities.

At the end of every module there are:

4. one or more real-world problems which require the use of the contents of that unit to be solved;

5. a final test, automatically graded, to verify the achievement of the learning objectives expected for the module.

Figure 2 shows an example of course module.

Taking into account the methodologies chosen and the needs of the students, their frequent misunderstandings emerged both in the entry test and from teachers’ experience, the didactic materials have been created to populate the modules. As an example, in the entry tests one of the most difficult questions was about the properties of powers, in which students had to choose the only wrong answer between 4 equalities (difficulty index: 0.34). A set of questions were developed, focused on the most frequent mistakes in the applications of the properties of powers, on the scheme of the question shown in Figure 3. Initially, students are asked to decide whether an equality involving properties of powers was correct or incorrect. They can earn half the score if they answer correctly; after that, they are asked to fill two subsequent sections which refer to the general rule to apply, through which they can earn up to the remaining half of the score. The last two sections can have a double function: justifying the choice, if the student had answered correctly to the first part, or showing a reasoning process, if the student had given the wrong answer (or guessed by chance) to the first step. Once they finish the test, students can try it again and find questions with a similar structure but different examples of applications of the same and other power properties. This is an example of question with interactive feedback: after the first section the student receives a first feedback in a form of green tick or a red cross depending on whether he answered correctly or not; the following sections are a feedback about how he was supposed to develop his reasoning in order to reach the solution. The feedback is interactive, because the student has to complete step by step the sub-questions, following the proposed reasoning.

Figure 4 shows an example of problem solving question developed with the automatic assessment, related to the module about mathematical formulas and functions. A real-world problem is given and students can explore different solving strategies: a numerical solution through an interactive table; a symbolic solution through an open-ended response area which offers the possibility to enter formulas through a symbolic equation editor, and a graphic solution, made possible through the graph of the function entered by the students generated by the system. Students can compare the different mathematical representations of the real world situation and deepen their understanding of the involved concepts, namely functions and their zeros. When they try the question again, students will find different values that allow them to repeat the process and to acquire awareness of the meanings laying behind abstract mathematical objects.

Within the online course there are also a forum of discussion for students, a progress bar, through which learners can visualize their learning steps, and a link to the gradebook, where all the assignments results are recorded.
4.4 Utilization of the Materials

Once the course was completed, it has been duplicated in 9 single courses, one for each class, so that teachers can easily control the progress of their own students and give them personalized support and advise.

Courses were opened to the students at the end of October 2018, and they are currently ongoing; they will be active until Spring 2019, even though students will be able to access the contents successively to the estimate end of the project.

Students received an e-mail at their institutional e-mail address with the indications to log in the platform and to access the materials. Interactive instructions about how to use the automatic assessment were provided to the students directly through the platform. The teachers were also asked to repeat the instructions to the classes at school and to show through the IWB how to use the materials. The learning materials can thus be used by students who need to revise basic skills at their own pace, but it is also suitable to class activities of different kind when teachers need to introduce new topics based on previous knowledge or assign differentiated activities to different group of students.

4.5 Requirement of Learner Participation

Aware that little motivated students won’t be too keen on autonomously doing online mathematical activities in their spare time, some expedients have been taken in order to assure their frequency to the course.

The main one is the certification: students who initially have low grades will be required to present, by the month of April, the certificate of completion of the course. The certificate can be automatically downloaded from the platform, at the condition that all the activities will figure as completed. So, they are forced to use the materials.

If the certification acts as “external” motivational lever, the learning methodologies chosen to develop the materials contribute to the development of intrinsic motivation. The real contexts, the immediate feedback, adaptivity and interactivity make all the materials engaging and useful to get prepared, so that students who try the activities can acknowledge their usefulness and go on with the modules. The interactive feedback provided through automatic assessment help them understand solving strategies and processes, contributing to the development of self-regulation. Through a progress bar they can be made aware of their position in the learning path and be motivated to complete it.

In addition, all teachers have been asked to present the course to their classes, to invite them to do the activities as homework and to recall the problems during lessons. In fact, students need to see the course as linked to their study and not as an external and additional duty; the more they are convinced of the usefulness of the online course for their learning, the more easily they will participate. The collaboration with the teachers could also have the positive effect to renew their teaching practices, introducing the use of the didactic methodologies and technologies adopted in the online course. As a consequence, not only the online course, but the whole school experience with Mathematics could be more engaging for the students, who can be facilitated in the development of interest for Mathematics.

4.6 Evaluation and Revision

In April 2019, when the time limit for the course completion will come, an evaluation of the course will be performed in several modalities.

The achievement of the learning outcomes will be assessed through a final test, similar to the entry one, for all the students. The appreciation of the course
will be evaluated via a questionnaire, which investigates the appreciation and perceived usefulness of the online activities to get a better understanding of the contents. Teachers will be interviewed to express their point of view about students’ performances.

Data collected through the two tests, the two questionnaires, platform usage and students’ scores and teachers’ interviews will be cross-checked in order to understand key strengths and limits of MATE-BOOSTER for future implementations of the project.

5 FIRST RESULTS AND DISCUSSION

The project is at a too early stage to get results from students. Nevertheless, the appreciation from the teachers involved is very high, since they perceive the online course as a valid support for their didactic, for reducing failures and motivating students.

The design of the course, made according to constructivist directions, actually respects the seven goals for building constructivist learning environments theorized by Cunningham, Duffy and Knuth.

1. Real-world problems offer students a learning environment in which to create mathematical knowledge starting from a specific case; exploration activities let students build and associate meanings to mathematical concepts; adaptive questions with step-by-step guided solutions help them manage a complex resolution following their own ideas. Thus, students get to experience the very knowledge construction process.

2. Interactive exploration materials show Mathematics from different points of view; the resolution of the problems is often discussed offering more than one solving process; peer discussions ask students to come to terms with different opinions and ways of understanding. These features can provide learners with experience in and appreciation of multiple perspectives.

3. Not only all the problems, but also great part of the automatically graded questions and of the interactive materials are contextualized in real-world situations, interesting and challenging for students. In this way learning is embedded in realistic and relevant contexts.

4. When opening the online course, students can choose their own path between the offered topics and materials. They are at the center of their own learning. All the activities do not flow automatically in front of students’ eyes: they have to autonomously get into each one and browse pages and questions with a click, thus enhancing their commitment. In this way ownership and voice in the learning process can be encouraged.

5. Students’ work, their problems and successes are not isolated: they can share them with other learners through the forum. Moreover, MATE-BOOSTER is inserted in a blended context, where students actually meet every morning at school and teachers are advised to discuss the activities during the lessons, to the purpose of embedding learning in social experience.

6. Exploring activities often present the same concept with different registers (in words, symbolic, graphic, tabular, and so on) and try to simplify its understanding via a shift of register. The same approach is applied in the automatically graded assignments and in the problems, in order to encourage students to the use of multiple modes of representation.

7. Immediate feedback facilitates students’ acknowledgement of their preparation; moreover, the tracking of activities and the progress bar offer them a visual insight of the learning path that they have undertaken. Automatically graded open answers and interactive feedback ask students to explain processes, not only to give results. Hence, the course activities pursue the goal to encourage self-awareness in the knowledge construction process.

Thus MATE-BOOSTER can be a suitable learning environment where students can reinforce their knowledge with a constructivist approach.

In the design of the course a special attention has been dedicated to feedback, considered a core element for promoting success. MATE-BOOSTER feedback works at three levels: at task level, when it informs students whether the task has been performed correctly or knowledge has been achieved; at process level, when it explains how the task should be performed; and at self-regulation level, when it helps learners monitor their own learning. Table 2 shows the MATE-BOOSTER features and activities which provide the three kinds of feedback.
6 CONCLUSIONS

In summary, MATE-BOOSTER has been conceived with the aim of supporting students in the transition from lower to upper secondary school by strengthening basic mathematical competences. The project has been managed using a design method of ASSURE kind (Analyse the learners; State objectives; Select methods, media and materials; Utilize media and materials; Require learner participation; Evaluate and revise). The core action of the project involves the implementation of an online course that students can use at their own pace as a support to their study. The design of the virtual learning environment has been carried out according to constructivist assumptions, and under the seven goals for building constructivist learning environments theorized by Cunningham, Duffy and Knuth (to provide experience with the knowledge construction process; to provide experience in and appreciation of multiple perspectives; to embed learning in realistic and relevant contexts; to encourage ownership and voice in the learning process; to embed learning in social experience; to encourage the use of multiple modes of representation; and to encourage self-awareness in the knowledge construction process). The learning methodologies used are problem posing and problem solving, collaborative learning, learning by doing and automatic formative assessment.

In the course design, the collaboration of the researchers with the school teachers of Mathematics is a key strategy to maximize learners’ participation, since the presentation of the course is filtered by the teachers’ voice.

The course is currently open to students and the results, in terms of teachers’ and students’ satisfaction and competence achieved, will be analysed as soon as they will be available and used for perfecting the course and proposing it again.

Table 2: Analysis of the feedback provided by MATE-BOOSTER.

<table>
<thead>
<tr>
<th>Level of feedback</th>
<th>MATE-BOOSTER features</th>
</tr>
</thead>
<tbody>
<tr>
<td>Task level</td>
<td>Immediate feedback from automatic grading; interactions with peers and teachers</td>
</tr>
<tr>
<td>Process level</td>
<td>Interactive feedback; resolutions of the problem; interaction with peers and teachers</td>
</tr>
<tr>
<td>Self-regulation level</td>
<td>Automatic assessment, tracking of activity completion, progress bar, gradebook, certification.</td>
</tr>
</tbody>
</table>

Since in Italy schools and teachers need to offer paths for the revision to students who get low marks, including individualized courses and further assessment, similar courses could have a double effect on the optimization of scholastic resources: firstly, they could reduce failures at their root, as they are often due to gaps in the basic knowledge that cause difficulties in learning new things; secondly, they can be used as part of the paths of content revision, because the topics included within the course are those which are required in advanced for understanding the first year course, and they are usually object of the revision courses. Thus, schools using online courses as MATE-BOOSTER could save human resources in delivering revision courses and collocate them elsewhere, such as in projects for the innovation of methodologies and curricula. This procedure could be even promoted by the Ministry of Education, maybe proposing a format that schools can customize. The project could be extended to other core disciplines, such as Italian and Foreign Languages, with the collaboration of experts in these disciplines.

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