Optimal Design of Production Systems: Metaoptimization with Generalized De Novo Programming

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Abstract: Milan Zelený, in a number of papers, proposed and developed specially structured LP model called De Novo Programming. This approach uses, in an essential way, a transformation of the original problem to continuous knapsack problem, and it concerns only models with capacity constraints and some implicit assumptions about the problem data. Here we extend this methodology to cases involving not only capacity constraints but also requirement and balance constraints. This extension is based on the methodology of Zelený and uses some principles of the STEM methods. We present an example of an adaptation of De Novo approach for models with both capacity, requirement and balance constraints.

1 INTRODUCTION

Optimization of systems means finding "best available" values of some criterion given a defined input and output of this system. Optimal design of systems means "best available" setting of proposed system. Process of system design requires understanding the content of three system approach phases – reality, model and metamodel. In the first phase the inquiring system is used for description and understanding the real problem. If we do not understand the reality, we cannot solve its problems properly. In the second phase, inquiring system for creating the model of problem solved is important. The proper selection of the model is important for obtaining the good results. In the third phases the inquiring system of abstract process of model creation, metamodel is studied (Gigch, 1991).

Production system optimization, in business and marketing, is methodology for decision process, which leads to the optimal production mix under the defined criterion (criteria). Very often the mathematical programming, especially the linear optimization model is used to find the optimal solution. As Zelený (1986, 1990a, 1990b) emphasises, the already formulated model implicitly contains the optimal solution. Therefore, the decision is given by the set parameters of the model. The crucial problem is how to formulate this model correctly, respectively, how to choose the best input data? The question of the best design or formulation of the model can be seen as a metamodeling process.

A model is the first abstraction of the real-world problem, and then a metamodel can be seen as the second abstraction, highlighting and optimizing the properties of the model itself. Metamodeling typically involves studying the input, output relationships, and then fitting right models to represent that behaviour. Metamodeling identifies the underlying modelling process and provides tools and techniques for model development that will allow the proper application to real problems. In this process, Zelený (1990a, 1990b) suggests De Novo programming for optimal design of production systems described by the linear optimization model with the constraints of the "≤" type. This approach is used in practical applications for instance by Babic and Pavic, (1996), Huang, Tzeng, (2007) and Zhang et al., 2009. Fiala (2011) describes the future development and possible applications of various modifications of De Novo programming.

The main aim of this paper is to generalize the De Novo approach for finding of optimal design of

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production system so that more types of constraints are possible, in particular "≥" and "=".

2 DE NOVO PROGRAMING

To motivate the De Novo approach to optimal design of production systems, Zelený (1986, 1990a; 1990b, 2005, 2010) starts with considering the standard linear programming model for allocating given resources to possible activities in order to achieve a given economic objective. If only one criterion is considered and the objective is to maximize it, then we have the problem

$$c^T x \rightarrow Max \text{ s.t. } Ax \leq b, x \geq 0 \quad (1)$$

where A is a real (m, n)-matrix, b is a real m-vector, and c is a real n-vector.

When multiple criteria are involved, we have to solve the multiple objective problem

$$Cx \rightarrow Max \text{ s.t. } Ax \leq b, x \geq 0 \quad (2)$$

where C is a real (q, n)-matrix of coefficients of q objective functions.

An important question is what happens to the optimal solution if the resource allocation changes. Therefore, since the early days of linear programming, both practitioners and theorists have been interested in behaviour of solutions if coefficients of the problem vary. Such questions have led to the emergence of

- Sensitivity analysis (investigation of changes in the individual coefficients which cause an optimal solution to become non-optimal);
- Parametric programming (investigation of changes when some of the coefficients are functions of parameters);
- Robust optimization (investigation of solutions under uncertainty that is represented as deterministic variability in the value of the parameters);
- Inverse optimization (investigation of solutions with goal objective value when some of the coefficients are parameters);
- De Novo programming (investigation of budget allocation to individual resources which results in optimal system structure) (Zelený, 1990a, 1990b, 2005, 2010).

The De Novo methodology (Zelený, 1990b) allows for changes in some of the input data, particularly, with changes in the components b_i of the right-hand side vector b of model (1) or (2). Clearly, such changes describe changes in resources allocation; modify the system design, and, therefore, the set of feasible solutions, which may change the optimal solution. In contrast to the sensitivity analysis and parametric programming, De Novo programming similarly as Robust or Inverse optimization requires some additional exogenous data. De Novo programming requires specification of cost of resources and level of available budget.

According to Zelený (1990a, 1990b), if p denotes a given m-vector of unit cost of resources and B denotes a given available budget, then De Novo approach gives to b the freedom to vary freely in the region given by

$$p^T b \leq B \text{ and } b \geq 0. \quad (3)$$

To indicate that the components of b are now real variables we change the notation and use the letter y instead of b. Now it should be clear that instead considering the general linear programming problem (1), (2) resp., we deal with a special linear programming problem with one objective function

$$\begin{aligned} & c^T x \rightarrow Max \\ \text{s.t. } & Ax - y \leq 0 \\ & p^T y \leq B \\ & x \geq 0, y \geq 0. \end{aligned} \quad (4)$$

resp. with multiple objective functions

$$\begin{aligned} & Cx \rightarrow Max \\ \text{s.t. } & Ax - y \leq 0 \\ & p^T y \leq B \\ & x \geq 0, y \geq 0. \end{aligned} \quad (5)$$

To refer to the special structure of these problems, we say that we are considering (De Novo) optimal design problems with single, resp. multiple objective functions.

Originally, the De Novo approach employs the fact that, for each feasible solution (x, y) of problem (4), x is also a feasible solution of the problem

$$\begin{aligned} & c^T x \rightarrow Max \\ \text{s.t. } & Vx \leq B \\ & x \geq 0 \end{aligned} \quad (6)$$

where V stands for the n-vector $p^T A$.

This is a continuous linear knapsack problem whose optimal solution $\hat{x}$ we can easily obtain by the following procedure, provided all components of c and V are positive: Let k be such that

$$c_k / v_k = max\{c_1/v_1, c_2/v_2, \dots, c_n/v_n\}.$$

Then the components of the optimal solution $\hat{x}$ are given by $\hat{x}_k = B/v_k$, and $\hat{x}_i = 0$ otherwise. Using $\hat{x}$, we set $\hat{y} = A\hat{x}$ and $\hat{B} = V\hat{x}$. The resulting triple

$(\hat{x}, \hat{y}, \hat{B})$ is called the *optimal system design* for De Novo problem (4). It is clear that, in this simple case with one objective function, we have $\hat{B} = B$.

For the decision making under multiple criteria, the De Novo approach first proceeds by solving q single objective optimization problems (replacing c_k by $C, k = 1, \ldots, q$). Let Z^* be the q-vector of optimal values of individual objective functions over the set of feasible solutions, which is the same as in problem (4). Then Z^* is used to define an auxiliary problem, called the *meta-optimization problem*, which is formulated as the minimization problem (Zelený 1990b)

$$\begin{aligned} & Vx \to Min \\ \text{s.t. } & Cx \geq Z^* \\ & x \geq 0. \end{aligned} \tag{7}$$

Let x^* be an optimal solution of this problem and define y^* and B^* by $y^* = Ax^*$ and $B^* = Vx^*$. It is easy to see that $B^* \geq B$ and that the value B^* is the minimum budget for obtaining at least Z^* by using $x^*$and y^*. The model (7) is often (Zelený, 1986, 2005) defined using equations in the form

$$\begin{aligned} & Vx \to Min \\ \text{s.t. } & Cx = Z^* \\ & x \geq 0. \end{aligned} \tag{8}$$

The fraction $r^* = \frac{B}{B^*}$ is called *the optimum path ratio* (Shi (1995), Zelený (1990b)) for approaching Z^* with respect to given budget B, and the ordered triple $(r^*x^*; r^*y^*; r^*Z^*)$ is called the *optimal system design* for the De Novo problem (5).

Shi (1995) proposes some variations of Zelený's approach, and introduces several (formally uncountable many) optimum path ratios for enforcing different budget levels of resources, which leads to alternative optimal system designs. However, it turns out that most of the proposed alternatives are not real alternatives. To see it clearly let us consider Shi's proposal in more detail.

Unlike the Zelený procedure, which is based on $(x^*; y^*; Z^*)$, Shi's uses triple $(x^{**}; y^{**}; Z^{**})$. The solution x^{**} is defined by the non-zero components of the q^* $(q^* \leq q \leq n)$ different single objective optimal solutions $\hat{x}^k, k = 1, \ldots, q^*$. Without loss of generality, for each solution $\hat{x}^k$ we suppose $\hat{x}_k^k = B/v_k$, and $\hat{x}_j^k = 0$, otherwise. Hereupon Shi defines synthetic solution $x^{**} = \left(\hat{x}_1^1, \hat{x}_2^2, \ldots, \hat{x}_{q^*}^{q^*}, 0, \ldots, 0\right)$, and respective values $y^{**} = Ax^{**}$and $Z^{**} = Cx^{**}$ Then $(x^{**}; y^{**}; Z^{**})$ is used to define the following optimum-path ratios.

$$\begin{gathered} r^1 = \frac{B^*}{B^{**}}, \quad r^2 = \frac{B}{B^{**}}, \quad r^3(\lambda) = \frac{\sum_{k=1}^{q^*} \lambda_k B^k}{B^{**}}, \\ r^4 = \frac{B}{B^*}, \quad r^5(\lambda) = \frac{\sum_{k=1}^{q^*} \lambda_k B^k}{B^*}, \\ r^6(\lambda) = \frac{\sum_{k=1}^{q^*} \lambda_k B^k}{B}, \end{gathered} \tag{9}$$

where $B^{**} = (pA)x^{**}$, $B^k = (pA)\hat{x}^k$, $0 \leq \lambda_k \leq 1$, and $\sum_{k=1}^{q} \lambda_k = 1$.

However, simple computations show that all B^k are equal to B. Thus, for each λ, $r^4 = r^5(\lambda)$ are equal (Zelený's ratio). The following equalities also apply $r^2 = r^3(\lambda)$; $r^4 = r^5(\lambda)$; $r^6(\lambda)$=1.

The question of solvability of problem (4) by transforming it into a knapsack problem is mentioned only in Zelený (1990b). Almost none of other published articles mentions the prerequisites for using the classical De Novo Programming approach. Some of the usually tacitly assumed conditions are discussed in Vlach and Brožová, (2018). Let us noticed that:

- The model construction supposes only the constraints of type $Ax \leq b$, so called the capacity constraints, which ensure compliance with resource capacity.
- The transformation of the model (4) to the knapsack problem (6) requires the positivity of components of $p^T A$. The positivity of $p^T A$ is guaranteed if matrix A is nonnegative and has no zero-column and all components of vector p are positive.
- It is also necessary to find out whether the system of equations $Cx = Z^*$ or $Cx \geq Z^*$ is solvable, as required in Zelený (1990b).
- This approach can easily be extended to situations with upper bounds on the components of x.

3 DESIGN OPTIMIZATION OF GENERAL PRODUCTION SYSTEM

The system design optimization using a linear optimization model should be based on several tasks:

- The optimal choice of the type of constraints and their number;
- The optimal choice of criterion or criteria;
- The optimal choice of the model data values.

The De Novo standard procedure supposes only constraints of type $a^T x \leq b$, called the capacity constraints. The objective of these conditions is to maintain the consumption of resources below the given limits. De Novo also supposes, that the unit cost of these resources is known and total cost of these resources is known, also.

In the linear optimization models, often other types of constraints appear. For example, the constraints of type $a^T x = b$, so called the definitional or binding constraints, serve to meet a particular demand. Or, the so-called balance constraints, that is, the constraints of type $a^{+T} x^{+} + a^{-T} x^{-} + S - L = 0$, ($a^{+}$ are positive values and a^{-} are negative values) assure the balance between production and consumption perhaps with a surplus S or lack L allowed. Moreover, the constraints of type $a^T x \geq b$, so called the requirements constraints, guarantee the required amount of production for sale. In practical applications, it is often necessary to assume the cost of such requirements (the cost of the contract signed).

The typical multiple objective linear optimization model with the all types of mentioned constraints can be written as follows

$$\begin{aligned} & C^{Max} x \rightarrow Max \\ & C^{Min} x \rightarrow Min \\ \text{s.t. } & A^{\leq} x \leq b^{\leq} \\ & A^{=} x = b^{=} \\ & A^{\geq} x \geq b^{\geq} \\ & x \geq 0 \end{aligned} \tag{10}$$

where $A^{\leq}, b^{\leq}$ are coefficients from the capacity constraints,
$A^{=}, b^{=}$ are coefficients from constraints in the equational form,
$A^{\geq}, b^{\geq}$ are coefficients from the requirements constraints, and
C^{Max}, C^{Min} are coefficients of objective functions.

The (criteria) optimization means to find the optimal values of objective functions. Using De Novo approach, the system design optimization means to find the optimal values of capacities and requirements under the given budget.

Consider now the values of all capacities and requirements (right hand side values) as variables and reformulate constraints into form of equations as follows:

- capacity constraints with unknown capacities

$$A^{\leq} x - y^{\leq} = 0 \tag{11}$$

- equational constraints with unknown definitional value

$$A^{=} x - y^{=} = 0 \tag{12}$$

- requirements constraints with unknown requirements

$$A^{\geq} x - y^{\geq} = 0 \tag{13}$$

- cost of necessary capacities and possible requirements has to be less than or equal to the given budget B

$$p^{\leq T} y^{\leq} + p^{=T} y^{=} + p^{\geq T} y^{\geq} \leq B \tag{14}$$

where $y = (y^{\leq}, y^{=}, y^{\geq})$ are unknown values of capacities and requirements, $p^{\leq}, p^{=}, p^{\geq}$ are the cost of capacities and requirements and B is the available budget.

Optimal system design means optimal budget allocation and it means looking for optimal necessary capacities and possible requirements. If these values are known, the constraints of type (11), (12) and (13) can be seen as the equations (Zelený, 1986). Into this relaxed model, the budget constraint (14) has to be added. In order to ensure finding of a non-trivial solution, it is necessary to assume that the entire budget will be used, i.e. the condition (14) will be in the form of an equation. New model formulation will be

$$\begin{aligned} & C^{Max} x \rightarrow Max \\ & C^{Min} x \rightarrow Min \\ \text{s.t. } & Ax - y = 0 \\ & p^{\leq T} y^{\leq} + p^{=T} y^{=} + p^{\geq T} y^{\geq} = B \\ & x, y \geq 0 \end{aligned} \tag{15}$$

The feasible solution exists if $p^T A \neq 0$. The optimal solutions of model (15) are found individually for each objective function and these ideal values of all objective functions create the ideal vector

$$Z^I = \left(z_1^I, \dots, z_q^I\right) \tag{16}$$

The problem of optimal system design is now to find the values of capacities and requirements under the minimal necessary budget that guarantee at least the ideal values of objective functions. The general formulation of this meta-optimum model should be (Zhuang and Hocine, 2018)

$$\begin{aligned} & p^{\leq T} y^{\leq} + p^{=T} y^{=} + p^{\geq T} y^{\geq} \rightarrow Min \\ \text{s.t. } & Ax - y = 0 \\ & C^{Max} x \geq Z^{Imax} \\ & C^{Min} x \leq Z^{Imin} \\ & x, y \geq 0 \end{aligned} \tag{17}$$

After solving model (17), the minimal budget B^* for achieving at least ideal objective functions values, the optimal solution (x^*, y^*) and the corresponding values of objective functions $Z^* = Cx^*$ are received. Generally, this minimal budget B^* can be either smaller or larger than available budget B. The optimum path ratio for achieving the best performance Z for a given budget B can be defined using $r = \frac{B}{B^*}$. By using optimum path ratio r, the following data for optimal system design can be received:

Optimal right-hand side values $b = ry^*$

Optimal values of variables $x = rx^*$

Optimal values of objective functions $Z = rCx^*$

The optimal system design is done by equations

$$A^{\leq}x = b^{\leq}, A^{=}x = b^{=}, A^{\geq}x = b^{\geq} \tag{18}$$

Unfortunately, this approach cannot be used for all problems because there is no guarantee that there exists a solution of the system of criterial constraints as inequalities

$$\begin{aligned} C^{Max}x \geq Z^{Imax} \\ C^{Min}x \leq Z^{Imin} \end{aligned} \tag{19}$$

or a solution of the system of criterial constraints as equations

$$\begin{aligned} C^{Max}x = Z^{Imax} \\ C^{Min}x = Z^{Imin} \end{aligned} \tag{20}$$

Therefore, some principles of the method STEM (Benayoun et al., 1971, Roostaee et al., 2012) for multiple objective optimization can be utilized. We suggest not to solve inequalities (19) or equations (20) but to find minimal weighted deviations d from the goal values)

$$\begin{aligned} (Z_k^{Imax} - C_k^{max}x)w_k = d, k = 1, \dots, q \\ \left(C_k^{min}x - Z_k^{Imin}\right)w_k = d, k = 1, \dots, q \end{aligned} \tag{21}$$

where the additional signs min or max mean the type of objective optimization.

This idea generally could allow to decision-maker to change the goal values, which have to be reached.

4 GENERALIZED DE NOVO PROGRAMMING

To solve the general linear optimization model (10) to optimize the system design we suggest the Generalized De Novo optimization approach. This methodology of system design consists of the following four steps.

Suppose now we look for a solution of the model (10) under the possibility to change the values of b with $p^{\leq}, p^{=}, p^{\geq}$ as the cost of capacities and requirements while respecting budget B.

1) Model Reformulation.
To allow the change the values of b, the model reformulation (15) with unknown variables y represents unknown values of capacities and requirements while respecting budget B will be used.

2) Partial Optimization.
Model (15) is now solved separately for the individual objective functions. Received solutions are filled into the decision matrix containing all values of individual objective functions for q single optimal solutions of model (15)

$$\begin{array}{c|c} & \begin{matrix} c_1 & c_2 & \cdots & c_q \end{matrix} \\ \hline \begin{matrix} x_1^{opt} \\ x_2^{opt} \\ \vdots \\ x_q^{opt} \end{matrix} & \begin{pmatrix} z_1^1 & z_1^2 & & z_1^q \\ z_2^1 & z_2^2 & & z_2^q \\ & & \ddots & \\ z_q^1 & z_q^2 & & z_q^q \end{pmatrix} \end{array} \tag{22}$$

Besides the vector of ideal values Z^I (16) which contains the best values from each column in the decision matrix (22), the nadir vector is created

$$Z^N = \left(z_1^N, \dots, z_q^N\right) \tag{23}$$

which contains the worst values of each objective function in the decision matrix (22).

3) Metaoptimization.
The solution with the minimal deviations from the ideal values of the criteria is found by solving the following single objective model.

$$\begin{aligned} d \rightarrow Min \\ \text{s.t. } Ax - y = 0 \\ (Z_k^{Imax} - C_k^{max}x)w_k = d, k = 1, \dots, q \\ \left(C_k^{min}x - Z_k^{Imin}\right)w_k = d, k = 1, \dots, q \\ x, y, d \geq 0 \end{aligned} \tag{24}$$

Weights w_k are calculated based on the ideal and nadir values as normalized values

$$w_k = \frac{z_k^I + z_k^N 2}{2} \cdot \frac{1}{\sum_{j=1}^{q} \frac{1}{2}(z_j^I + z_j^N)} \tag{25}$$

Such values of weights allow comparisons of the deviations from ideal values of objective functions without affecting their size.

The solution of this problem is (x^*, y^*) and achieved values of objective functions $Z^* = Cx^*$

Remarks: Similarly as in the STEP method, the decision maker could change the required goal values and repeat the metamodel optimization.

4) Metametaoptimization.
Minimal necessary budget is found by solving the following optimization model with one objective function

$$\begin{aligned} p^{\leq T}y^{\leq} + p^{=T}y^{=} + p^{\geq T}y^{\geq} \rightarrow Min \\ \text{s.t. } Ax - y = 0 \\ C^{Max}x = Z^* \\ C^{Min}x = Z^* \\ x, y \geq 0 \end{aligned} \tag{26}$$

Solution of the model (26) exists, because the solution of the model (24) exists.

Let the solution of problem (26) is (x^{**}, y^{**}), values of objective functions $Z^{**} = Z^{*}$ and minimal necessary budget is B^{**}.

5) Solution – Optimal System Design.
By using optimum-path ratio $r = \frac{B}{B^{**}}$, the following solution of the optimal system design is received:

Optimal right hand side values $b = ry^{**}$

Optimal values of variables $x = rx^{**}$

Optimal values of objective functions $Z = rCx^{**}$

The optimal system design

$$A^{\leq}x = b^{\leq}, A^{=}x = b^{=}, A^{\geq}x = b^{\geq} \tag{27}$$

Remark: It is possible to suppose that not all resources or requirements as RHS values (or corresponding constraints) are subject of optimization of system design. Such constraints of model (9) are not transformed for system design optimization. In such case the model (15) would have the following form

$$\begin{aligned} & C^{Max}x \rightarrow Max \\ & C^{Min}x \rightarrow Min \\ \text{s.t. } & A_1 x - y = 0 \\ & A_2^{\leq}x \leq b^{\leq} \\ & A_2^{=}x = b^{=} \\ & A_2^{\geq}x \geq b^{\geq} \\ & p^{\leq T}y^{\leq} + p^{=T}y^{=} + p^{\geq T}y^{\geq} = B \\ & x, y \geq 0 \end{aligned} \tag{28}$$

where A_1 consists of the coefficients from the constraints for which the optimal RHS have to be find and A_2 consists of the coefficients from the constraints with fixed RHS values b.

The steps of the suggested methodology then are used accordingly. However, the model (28) may not have a feasible solution with the budget constraints in equation or inequality form.

5 EXAMPLE

The question of this problem is how many pieces of three products have to be produced to fulfil the contracts and minimized the labour cost and maximized the profit under the production system constraints. Problem with three products P1, P2, and P3, two capacity constraints R1 and R2 (limited recourses), three requirements constraints C1, C2 and C3 (contracts with minimal supply), and two criteria (minimization of the labour cost, maximization of the profit) will be solved. The initial formulation of the model is in Table 1 together with the price of the resources and contracts and the total budget.

Table 1: Model of the three products problem.

	P1	P2	P3		RHS	Price	Total budget
R1	2	0	1	≤	25	5	206
R2	1	1	1	≤	20	3	
C1	1			≥	10	1	
C2		1		≥	3	1	
C3			1	≥	4	2	
L. costs	1	1	8		MIN		
Profit	2	3	6		MAX		

Ideal values of objective functions of this model are in the vector $Z = (65; 45)$.

1) Model Reformulation.
The model is reformulated using 5 unknown variables $y_1, \dots, y_5$ representing unknow values of capacities and requirements while respecting budget B. The new formulation of model is in Table 2.

Table 2: Reformulated model.

	P1	P2	P3	yR1	yR2	yC1	yC2	yC3		
R1	2	0	1	-1	0	0	0	0	=	0
R2	1	1	1	0	-1	0	0	0	=	0
C1	1	0	0	0	0	-1	0	0	=	0
C2	0	1	0	0	0	0	-1	0	=	0
C3	0	0	1	0	0	0	0	-1	=	0
Budget	0	0	0	5	3	1	1	2	=	206
L. cost	1	1	8	0	0	0	0	0		MIN
Profit	2	3	6	0	0	0	0	0		MAX

2) Partial Optimization.
Ideal solutions are found solving two single objective optimization models received by reformulation of the initial model according to the (15).

The solution of minimization of the labour cost is to produce only 14.71 pcs of the product P1 with necessary 29.43 units of resource 1 and 14.71 units of resource 2. This system design allows to closed only the first contract. The minimal labour cost is 14.71 thous. CZK and maximal profit is 29.43 thous. CZK.

The solution of maximization of the profit is to produce only 51.5 pcs of the product P2 with necessary 51.5 units of resource 2. This system design allows to closed only the second contract. The minimal labour cost is 154.5 thous. CZK and maximal profit is 51.5 thous. CZK.

Ideal values of objective functions under budget B are in the vector $Z^{Ideal} = (14.71; 154.5)$.

3) Metaoptimization.
Based on the ideal and nadir objective function values the following weights are used for calculation of metaoptimization model (Table 3)

Table 3: Normalized weights.

	Cost	Profit
Z^I	14.71	154.5
Z^N	51.5	29.43
Average	33.11	91.96
Normalized weights	**0.73**	**0.27**

The formulation of the metaoptimization model minimizing the deviation of ideal values is in Table 4.

Table 4: Formulation and solution of metaoptimization model.

	P1	P2	P3	yR1	yR2	yC1	yC2	yC3	*d*		
R1	2	0	1	-1	0	0	0	0	0	=	0
R2	1	1	1	0	-1	0	0	0	0	=	0
C1	1	0	0	0	0	-1	0	0	0	=	0
C2	0	1	0	0	0	0	-1	0	0	=	0
C3	0	0	1	0	0	0	0	-1	0	=	0
L. cost	1	1	8	0	0	0	0	0	-1.4	=	14.7
Profit	2	3	6	0	0	0	0	0	3.8	=	154.5
Dev.	0	0	0	0	0	0	0	0	1		MIN

Its optimal solution is to produce only 33.82 pcs of the product P2 with necessary 33.8 units of resource 2. This system design allows to closed only the second contract. The minimal labour cost is 33.82 thous. CZK and maximal profit is 101.44 thous. CZK. The vector Z^* is $(33.82; 101.44)$

4) Metametaoptimization.

With the best obtainable values of both criteria is solved the metametamodel to find the minimal necessary budget. Similarly, as in the STEP method, the decision maker could change these values and repeat the metamodel optimization from the previous step. Now the minimal budget is calculated with objective functions values of the optimal solution of metaoptimization model (Table 5).

Table 5: Metametaoptimization model.

	P1	P2	P3	yR1	yR2	yC1	yC2	yC3		
R1	2	0	1	-1	0	0	0	0	=	0
R2	1	1	1	0	-1	0	0	0	=	0
C1	1	0	0	0	0	-1	0	0	=	0
C2	0	1	0	0	0	0	-1	0	=	0
C3	0	0	1	0	0	0	0	-1	=	0
L. cost	1	1	8	0	0	0	0	0	≤	33.8
Profit	2	3	6	0	0	0	0	0	≥	101.4
Budget	0	0	0	5	3	1	1	2		MIN

The optimal solution of metametaoptimization model is equal to the solution of the previous metaoptimization model, so we receive $Z^{**} = Z^* = (33.82; 101.44)$. The minimal necessary budget B is 135.26 thous. CZK, what is less then we suppose to invest to production. So, it is possible to extend the production process.

4) Optimal System Design.

Optimal production structure under optimal design of production system and available budget allows expansion according to the optimal path ratio which is equal to $r = \frac{B}{B^*} = \frac{206}{135.26} = 1{,}523$.

The optimal system design (Table 6) allows producing 51.5 pcs of the product P2 with necessary 51.5 units of resource 2. This system design allows to closed only the second contract on 51.5 pcs of products of products sold. The minimal labour cost is 51.5 thous. CZK and maximal profit is 154.5 thous. CZK; $rZ^{**} = (51.5; 154.5)$. This means that there is no need for resource 1 but higher consumption of resource 2, e. g. 51.5 units. Also, there is only one optimal contract, the contract C2 with 51.5 pcs of product 2.

Table 6: Optimal system design.

	P1	P2	P3		
	0	**51.5**	**0**	Solution	
L. costs	1	1	8	**51.5**	
Revenue	2	3	6	**154.5**	
					Input data
Resource R1	2	0	1	**0**	25
Resource R2	1	1	1	**51.5**	20
Contracts C1	1	0	0	**0**	10
Contracts C2	0	1	0	**51.5**	3
Contracts C3	0	0	1	**0**	4

In the figure 1 the objective functions values of selected models are shown.

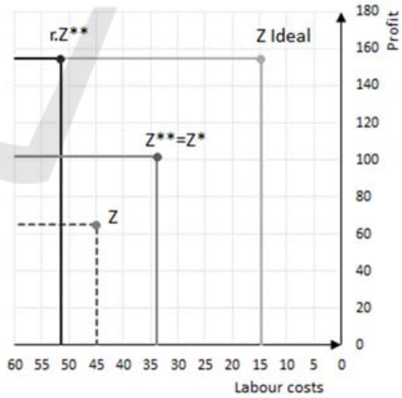

Figure 1: Values of labour costs and profit for selected solutions of three products problem.

It is possible to say, that the optimal system design has resulted in significantly higher profit but with the highest labour cost. Optimal system design results in necessity to product only one type of product and allocate the whole budget for the second resource and the contract for the optimal type of products.

6 CONCLUSIONS

In this paper, we continued our previous discussion of De Novo Programming; see Vlach and Brožová (2018). We briefly recalled the original approach of Zelený, rectified some oversights in the alternative proposal by Shi. Then we presented adaptation of De Novo methodology for models with capacity, requirement, and balance constraints, where the transformation to continuous knapsack problem is not possible.

Our proposal for Generalized De Novo Programing is a way to optimize the system design in more general settings. In particular, it is possible to deal with more types of constraints and more types of criteria.

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