Recursive Pedagogy: Automatic Question Generation using Real-time Learning Analytics

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Abstract: In this paper we introduce the notion of Recursive Pedagogy, which is a computer-supported approach to teaching and learning in which students solve problems assigned by an instructor using a Problem Solving Learning Environment, and their attempts to solve that problem are then used by the system to create new kinds of problems to help them build high level cognitive skills. The Recursive Pedagogy approach generalizes an approach we introduced earlier: the Solve-Then-Debug (STD) pedagogy to teach coding. In STD, students write a program given its description and, when their code passes all of the unit tests, they debug a sequence of incorrect programs submitted by their peers, starting with those with the most common errors. In this paper, we discuss two new Recursive Pedagogies that we have implemented in the Spinoza Python Tutor: Solve-Then-Critique-Correct-Solutions shows students correct programs from their peers and asks them to critique the code relative to a rubric; Solve-Then-Analyze-Unit-Tests shows students results of a set of unit tests and asks them to describe the probable error based on the unit test results. After students submit their Recursive Pedagogy analysis of a peer’s attempt, they are shown other students’ analyses of that attempt and are asked to select the best one. Recursive Pedagogies are designed to build skills in problem-solving, debugging, testing, and composing high quality solutions. Moreover, they can be used in a flipped class and they keep all students engaged in either problem solving or one of the other skills.

1 INTRODUCTION

There is a growing body of evidence demonstrating that active learning in STEM classes can improve learning outcomes (Freeman et al., 2014), increase retention (Rath et al., 2007) and increase diversity (Haak et al., 2011). One approach to incorporating active learning in a class is to use a Problem Solving Learning Environment (PSLE) which allows the instructor to pose problems in class for the students to solve. Two mathematics PSLEs are WebWork (Gage et al., 2002) and CalcTutor (Kime et al., 2015). There are many PSLEs for coding including Spinoza (Abu Deeb and Hickey, 2015), Codingbat (Parlante, 2007), CloudCoder (Andrei Papanecia, 2013), and many others.

Most PSLEs allow the students to submit multiple attempts and provide immediate feedback about the correctness of these attempts. They store student interaction with the system to allow the instructor to use these data to gauge a student’s understanding and mastery of specific activities and concepts.

In a typical class using a PSLE the instructor would use a Solve-Then-Debrief (STDB) pedagogy in which they introduce new content in lecture format and then test student understanding of the content by assigning a problem in the PSLE, then stopping the activity at the appropriate time, and moving into a debrief state where they discuss the most common errors and compare and contrast the variety of possible solutions.

If the PSLE is designed for classroom orchestra- tion (e.g. Spinoza (Abu Deeb and Hickey, 2017) or PCRS (Zingaro et al., 2013)), the instructor would be able to monitor the progress of the students in real-time and decide when to end the problem solving activity based on actionable real-time learning analytics.

One major issue that arises when using a PSLE as an in-class activity is that the students have different skill levels. Some students will complete the problem solving process very quickly and then have nothing to do, while others will struggle and never complete the...
problem, even if given the entire class time.

Partly motivated by this weakness of the Solve-Then-Debrief pedagogy using traditional PSLEs, we developed the Solve-Then-Debug Pedagogy (Abu Deeb et al., 2018). After students correctly solved a problem using the PSLE they were shown selected incorrect attempts from their classmates which represented the most common errors their class had made, and they were led through a seven step process to analyze the attempt and contrast their analyses with that of their peers.

The Solve-Then-Debug process can keep all students actively engaged and moreover can help them build valuable analytic skills. It does require that the PSLE maintain an equivalence relation on incorrect attempts so that it can find the most common mistakes and select actual student attempts from the largest of those equivalence classes.

After using Solve-Then-Debug in a large Python Programming class, we realized that this same approach could be used to help students develop a much wider range of important programming skills, including the ability to

- critique correct solutions for clarity, simplicity, efficiency, and programming style
- effectively analyze unit-test results,
- write and analyze program specifications
- create effective unit-tests specifying important corner cases

and many others. In this paper we formalize these observations and present a generalization of the Solve-Then-Debug approach which we call Recursive Pedagogy (RP) which can be applied to any Problem Solving Learning Environment which supports what we call Constructionist Problems. The fundamental idea is that whenever a student solves the original problem, the system analyzes all previous attempts made by their classmates, groups them into equivalence classes, and formulates a new kind of problem by selecting representatives from those equivalence classes.

For coding PSLEs in Computer Science courses, the original problem typically involves writing a program to solve a problem. This corresponds to the “Create” activity at the top of Bloom’s taxonomy of Cognitive Processes (Krathwohl, 2002). The additional Recursive Pedagogy problems usually involve additional high level activities such as the Analyze or Evaluate cognitive processes. They help students build their ability to analyze other students’ attempts along a number of dimensions. We will focus mostly on Problem Solving Learning Environments for Coding and we will describe in detail the Recursive Pedagogies available in Spinoza, but as we will see this approach is applicable in a wide-variety of fields.

The notion of Recursive Pedagogy can also be used in a collaborative learning context such as Pair Programming or POGIL, since group problem solving still is subject to the problem of some groups completing the work more quickly than others, especially in large classes.

2 RELATED WORK

Recursive Pedagogy is most similar to Contributing Student Pedagogy (CSP) (Herman, 2012; Falkner and Falkner, 2012) in which students are encouraged to contribute in a valuable way to the work of their peers. Peer Review is the best known example of CSP (Søndergaard and Mulder, 2012; Clarke et al., 2014). In Peer Review, students are asked to apply a rubric to grade the submissions of their peers. Research on Calibrated Peer Review (Walvoord et al., 2008) has provided evidence that if this is done carefully, the reviews can be nearly as accurate as those obtained using Teaching Assistants. Calibrated Peer Review requires all of the students to grade the same three sample assignments representing poor, average, and excellent work, and then it uses each student’s performance on these grading tasks to calibrate their skill as a grader. The calibration is then used to weight their grading of other students work and to select graders with at least one highly skilled grader for each group.

In recursive pedagogy, students analyze the work of their peers just as in Peer Review and their work helps other students learn the skills and concepts being taught as in Peer Review and CSP.

There are some fundamental differences though. The goal of Recursive Pedagogy is not to grade students work, especially since in this context students are typically making multiple attempts, getting feedback, and eventually getting the correct answer. Indeed, as originally conceived most students engaging in RP will not have their work directly analyzed by others, and those that do will usually not see the comments that others have made. Moreover, RP works best if the attempts can be automatically analyzed for correctness and automatically put into a natural equivalence class. Neither of these are features of traditional Peer Review or CSP. Finally, the RP approach provides a multi-step online process where students alternately perform their own analysis and then see and evaluate the analyses of their peers so they can learn from each other both to solve problems effectively and to critique attempted solutions.

Thus the main difference between our approach
and Peer Review is that Peer Review engages students in a reviewing dialog where the reviews directly contribute to improving that particular document being reviewed and every student has their work reviewed in this way. In our approach, the purpose of the reviews is for the reviewer to learn how to analyze attempted solutions and the person whose work is being analyzed won’t necessarily see those comments and hence couldn’t use them to improve their work.

3 RECURSIVE PEDAGOGIES FOR GENERAL PSLES

In this section we introduce the notion of Recursive Pedagogies for PSLEs in general. We present the properties that a PSLE must have to effectively support Recursive Pedagogies and we provide algorithms for picking the Recursive Pedagogy problems effectively and efficiently.

3.1 Constructionist Problems

Recursive pedagogies are applicable for any problems in which an attempted solution can be algorithmically analyzed for correctness and which can also be effectively critiqued by the teacher or a fellow student. Therefore, the optimal kind of problem for recursive pedagogy is one where the attempted solutions belong to a large space of solutions, ideally where the user constructs an answer using some formal language, e.g. coding, or mathematics. The PSLE tests these attempted solutions for correctness automatically using unit testing or some other uniform rubric.

Another important property for Recursive Pedagogies is that the domain must support a natural equivalence relation on the space of possible attempts, so that the system can identify the “most common mistakes.” These kinds of problems are often assigned in STEM courses in Mathematics, Computer Science, Physics, and Chemistry.

In CS1 classes, a programming problem that asks students to write code to satisfy specified constraints belongs to this type of problem as the students could have many ways to construct attempted solutions. The correctness of these attempts can be tested against an instructor-designed suite of unit tests. The attempted solutions can be grouped together in equivalence classes in many different ways (e.g. by the results of the unit tests).

In a Mathematics class, the instructor could ask the student to calculate the derivative of some function, and expect the student to write the result, not choose it from a multiple choice answer. The student could submit the attempt they made and get immediate feedback about correctness. Similar solutions produced by all the students are grouped together in the same category of attempts. In this case different categories of attempts will correspond to different errors that could signal common misconceptions in forming the derivative. The teacher or students could analyze or critique an attempted solution by trying to point out possible misconceptions that would have resulted in that error. They could also suggest ways of simplifying the answer to make it more clear.

In a problem where a STEM course is taught using programming, we can assume that students will be writing programs with relatively simple algorithms and that they have enough knowledge about coding to do so easily. So we expect most errors would be due to not having full mastery of the domain knowledge of the problem. Student could critique each other’s attempted solutions by pointing out these mistakes in the domain knowledge.

There are many PSLEs that support this kind of problem, for example for Mathematics classes we have WebWork (Gage et al., 2002), CalcTutor (Kime et al., 2015), and Provelt (Ferraro, 2010). In programming, we have Spinoza (Abu Deeb and Hickey, 2015), codingbat (Parlante, ), cloudcoder (Andrei Panic). There are many others.

3.2 Problem Solving Markov Models for RP

In any PSLE which supports constructionist problems a Problem Solving Markov Model (PSMM) can be created (Abu Deeb et al., 2016). A PSMM is a graph where each node represents an equivalent class of attempted solutions and each edge represents consecutive attempts from a member of one class to a member of the second class. The size of the node represents the frequency of the attempt and the color represents its correctness. The edges are labelled with the probability that a student will make that transition. The PSMM is used in Recursive Pedagogy to select the most common attempts of some particular type (e.g. logic errors, syntax errors, correct solutions). We will provide examples in the next section.

3.3 A Strategy for Choosing the Attempts in Recursive Pedagogy

Some Recursive Pedagogies rely on choosing attempts from the most common PSMM nodes. This is a little subtle though, because the ranking of the sizes of the PSMM nodes changes as time progresses. The precise algorithm we have implemented in Spinoza is
described in detail in (Abu Deeb and Hickey, 2017) and it proceeds by taking the chronologically first attempt in the largest equivalence class that the student has not yet seen. Typically the Recursive Pedagogy will restrict the nodes of interest (e.g. only incorrect attempts, or only attempts made by at least 5% of the class, etc.), and students will do 2-10 of these RP problems for each problem they solve.

3.4 Scalability for Recursive Pedagogy

Since all students comment on the same Recursive Pedagogy problems, often in the same order, the initial problems will get comments from large numbers of students. A student who is in the second phase may not be able to read through all of those comments to select the best one, especially in a class of over 100 students. Thus, there is a need to algorithmically limit the number of comments to be shown to the students. One possible approach is to only keep some of the most popular and some of the most recent comments. Another approach is to only show the first 20 comments or so, as the first ones are coming from the fastest students who most likely know how to solve the problem and are more likely to provide better comments.

Another use of Recursive Pedagogy is to generate hints for students who are still working on the initial problem. If their most recent attempt is in the equivalence class of a problem that has been selected as an RP problem, then the instructor can choose to allow them to see the critiques of that problem by other students in the RP phase. The same issue of information overload arises with hint generation as with RP comments. One needs to decide which of the RP comments to show the still struggling student. One possible solution is to ask the Teaching Assistants (TAs) to provide hints and only show hints from those TAs. We could also show just the most popular comments as hints.

4 RECURSIVE PEDAGOGIES IN SPINOZA

In this section we present the Recursive Pedagogies that are currently implemented in Spinoza, describe the pedagogy for each of these problems, and provide some justification for the choices made in designing these pedagogies.

4.1 Solve-Then-Debug

Any PSLE that supports a Constructionist problem solving activity, can be extended to include the Solve-Then-Debug (STD) pedagogy which adds another layer to the Solve-Then-Debrief (STDB) activity. It helps engage the fastest students in the learning process and can also provide additional hint generation for the slower students. It was first introduced in (Abu Deeb et al., 2018).

The Solve-Then-Debug activity begins with the instructor asking the students to solve a problem in a PSLE. The students solve the problem all at the same time, and they are allowed to submit multiple attempts until they get the correct result.

After a student solves a problem correctly, they can move to the next phase which we call the debug phase; this guarantees that the debugger knows at least one way to solve the problem, which could make the debugging more accurate.

In the debug phase, the PSLE creates a problem solving Markov Model for this problem, and the student is given a problem from the largest incorrect node in the Markov Model that they have not yet debugged. We choose the largest node to allow students to recognize the most common mistakes, and also to increase the possibility that other students, who are still in the solve phase, could be helped by these debugging comments.

The student will see the first attempt that belongs to that category of error, and then is asked to classify that error (syntax, logic, incomplete) and give a comment describing why that attempt is incorrect. All students who debug that node, will see the same attempt and hence that attempt will acquire very many comments. This was designed to insure that students are benefiting from each other’s comments, but as mentioned above, this can create a problem if hundreds of students are commenting on the same problem. We discussed some remedies to this over-commenting issue in the previous section.

The student in the debug phase can press the “run” button and see the results of the unit tests on the debug problem code, but our system doesn’t allow them to edit the buggy code. We make this restriction, because we do not want the students to write their solution to the problem; we want them to think critically about the error and imagine the feedback they will get using their fix.

After the student submits their comment, they can then see other students’ comments on the same incorrect attempt, and the restriction on editing the attempt is removed, but we also add a restriction to prevent the student from editing their original comment. Stu-
Figure 1: This view of the student debugging comments from the Solve-Then-Debug activity has been annotated with X’s for incorrect comments and checkmarks for correct comments, with a letter indicating which of the two main errors the comment is referring to: (a) a logic error where all centuries are incorrectly counted as leap years and (b) a style issue since the code can be written in one line, e.g. return y%4==0 and y%100!=0 or y%400==0.

dents are not allowed to change their comments since otherwise they might be tempted to just copy other students’ comments blindly in the next step without trying first, and not engage in critical thinking. At this point, the student can then edit the buggy code and get feedback of correctness or can use other students’ comments to try to fix the buggy attempt. This step allows the student to test their hypothesis, either by the Spinoza unit-test feedback on their edited attempt, or from looking at the other students’ comments.

The student must then pick the comment which they think is most accurate, this is to encourage them to read all the comments as it might help them gain insights that they missed in their own debugging analysis, and also help develop their skills in critiquing other students analyses. After choosing the most accurate comment they can then move to the next debug problem with the second highest number of attempts and the cycle repeats until the instructor stops the activity.

In a large class, it is rarely the case that the fastest students will end up solving all the generated debug practice problems before the instructor stops the activity. However, in case a student does finish all the major common misconceptions, Spinoza allows the student to view all the subsequent versions in each node, until all the versions are commented by this student. Lastly if the student finishes all the debug practice problems, they can view their previous comments and other students comments to all the debug practice problems they have completed.

If the instructor chooses to enable hints in this activity, then a struggling student who is still in the solve phase, and gets an error that is in the same equivalence class of a debug problem already solved by other students, will be able to choose to see the comments on that related attempt. This will allow slower students to engage in critical thinking, as they would need to examine the different suggestions their peers provided and pick the best one to try. Since these hints were written by fellow students, they may contain both insights and misleading suggestions.

Below is an incorrect attempt at writing a python program to return true if the argument \(y\) is a leap year and false otherwise. Fig. 1 shows an “other students comments” pane for this attempt where we have annotated the comments to show our analysis of their comments.

```python
def is_leap_year(y):
    if y%400==0:
        return True
    elif y%4==0:
        return True
    elif y%100!=0:
        return True
    else:
        return False
```

This problem was debugged by 10 students and 7 out of 10 gave correct comments. Two students indicated that they do not know how to debug the problem. The last student just gave a completely wrong comment on the coding style.

4.2 Solve-Then-Analyze-Unit-Tests

“Solve-Then-Analyze-Unit-Tests” is a variant of Solve-Then-Debug. In this RP the students as before
will need to solve an instructor specified problem and after they solve the problem correctly, they can advance to "Analyze-Unit-Tests" phase. The "Analyze-Unit-Test" phase is different from the debug phase of STD in that the student will only see the result of the unit tests. The code that generates these unit tests will be hidden as in Figure 2. After the student submits a comment describing how to fix the program to meet the description of the problem, the code will be shown to them along with other students’ comments for the same version of the program. The student can edit the code at this point to confirm their analysis or consider other students’ comments. To advance to the next "Analyze-Unit-Tests" problem, the student must select the best comment that describes the bug based on the unit tests.

We speculate that "Solve-Then-Analyze-Unit-Tests" (STAUC) would be better for providing a general hint for the students who are still in the Solve phase, as the students in the analyze phase will not be presented with the code at that point so they will not give too specific comment that suits only one version of that class of attempt. For example, with hints generated by the Solve-Then-Debug pedagogy, students often refer to some specific variable by name or to an error on a particular line number. A problem in the same equivalence class is unlikely to have a variable with the same name or an error on that same line number, though they may well have the same error.

4.3 Solve-Then-Critique-Correct-Solution

Another recursive pedagogical activity that we have implemented in Spinoza is Solve-Then-Critique-Correct-Solution. In this activity, all students are first asked to solve a problem using a PSLE, as usual. After a student solves the problem correctly, the student moves to the critique phase. In the critique phase the PSLE selects a correct problem for all students to critique. Ideally it would follow the same Markov Model approach and use some finer equivalence relation on the space of correct solutions to group them into nodes and pick the most common nodes first. This categorization could be based on the style or the simplicity of the solutions. Our current version in Spinoza does not do any finer analysis and simply moves through the solutions in chronological order.

Our current implementation for Solve-Then-Critique-Correct-Solution has two main steps. After the student solves the problem, they are presented with a correct solution by one of their classmates and a rubric. After they submit their critique they can see other students’ evaluations of the same attempt.
and they must select the best critique before they can move on to the next problem. For example, Fig. 3 shows the critiques of the student attempt shown below, which is a correct solution to a particular problem to calculate the cost of a movie ticket based on the patron’s age.

```python
def movieCost(age):
    if age < 18:
        return 6
    if age >= 65:
        return 6
    else:
        return 10
```

This step can help the student evaluate the quality of their own critique by comparing it to the critiques of their peers.

### 4.4 Preliminary Results

Spinoza was tested in a small Introduction to Python Programming class with 19 students in Fall 2018. Students were asked to solve Python programming problems both in class and as homework. They were also asked to complete six Recursive Pedagogy problems for each Spinoza programming problem they completed. At the end of the semester students were asked to complete a short survey about their impressions of using Spinoza with Recursive Pedagogy. 15 out of 19 completed the survey.

When asked which type of Recursive Pedagogy they liked the most, 53% selected the Solve-Then-Debug (STD), 33% chose Solve-Then-Analyze-Unit-Tests (STAUT), and 13% chose Solve-Then-Critique-Correct-Solution (STCCS). When asked which was most challenging, the results were 47% for STAUT, 33% for STCCS, and 20% for STD. A majority preferred having a mixture of different kinds of Recursive Pedagogies (67%) rather than just one type (33%). A plurality (40%) preferred to use Spinoza as an in-class activity only, while (33%) preferred to use it both as an in-class and a homework activity. Only 13% wanted to use it as homework only, and another 13% didn’t want to use Recursive Pedagogy at all.

### 5 CONCLUSIONS AND FINAL REMARKS

This paper presented the idea of Recursive Pedagogy which is a computer supported pedagogy designed specifically for a class of students with varying skills and speeds that are using a problem solving learning environment to support active learning. It provides some initial evidence that students found this pedagogy useful and interesting, but larger trial are needed to determine its efficacy.

In the future we intend to implement more examples of RP in Spinoza and to explore adding Recursive Pedagogy to other PSL.Es such as CalcTutor. We also
plan to explore the use of different equivalence relations that look into the structure of the programs and not just the behavior under unit tests.

REFERENCES


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