Minimization of Attack Risk with Bayesian Detection Criteria

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Abstract: Strategic deterrence operates in and on a vast interstate network of rational actors seeking to minimize risk. Risk can be minimized by employing a likelihood ratio test (LRT) derived from Bayes' Theorem. The LRT is comprised of prior, detection, and false-alarm probabilities. The power-law, known for its applicability to complex systems, has been used to model the distribution of combat fatalities. However, it cannot be used as a Bayesian prior for war when its area is unbounded. Analytics applied to Correlates of War data reveals that combat fatalities follow a log-gamma or log-normal probability distribution depending on a state's escalation strategy. Results are used to show that nuclear war level fatalities pose increasing risk despite decreasing probability, that LRT-based decisions can minimize attack risk if an upper limit of impending fatalities is indicated by the detection system and commensurate with nominal false-alarm maximum, and that only successful defensive strategies are stable.

1 INTRODUCTION

Reflecting on how much the world and warfare have changed, famed political scientist Sir Lawrence Freedman observed that "there is no longer a dominant model for future war, but instead a blurred concept and a range of speculative possibilities" (Freedman, 2017). Strategists and politicians have proven unimpressive in predicting the circumstances and outcomes of wars, and the international arena has only become more complex. With the aim of maintaining peace, scholars and practitioners will over time narrow the possibilities and bring the concept of war into focus. Meanwhile, some truths will remain invariant. Among them, nations make decisions based on likelihoods derived from past experiences, not on mere reasoned possibilities. And despite the differences between, say, the Russo-Japanese War and a future nuclear war, they will be inextricably linked by at least one quantitative measure: combat fatalities.

War is by no means a solely rational endeavour. Nevertheless, when faced with the prospect of expending resources and lives, possibly risking its very existence, nation states will attempt to weigh the consequences of action and inaction in order to minimize risk. Given the uncertainties described by Freedman, the immense potential death toll of nuclear war, and human propensity for error under duress, understanding risk when considering evidence that an attack is imminent or underway is essential to sustain the two "grounds for making peace: the first is the improbability of victory; the second is its unacceptable cost" (von Clausewitz, 1976)

The power-law of statistics, known for its applicability to complex systems (Sornette, 2007), has been used since the 1950s to study violent conflict (Richardson, 1960). A phenomenon may be probabilistically distributed according to the power-law if the logarithm of the exceedance probability $P(S > s)$ plotted against the logarithm of severity s appears as a straight line with a negative slope $-q$. Intuitively this means that the probability of exponentially increasing consequences is decreasing exponentially. However, researchers consistently report that the power-law's exponential parameter q for the severity of war measured in deaths is less than one, indicating that the exceedance probability decreases slower than the increase in number of deaths. Given that risk is probability multiplied by consequences, this means that the risk of war forever increases for increasing fatalities. It is a condition that makes the power-law invalid as a probability distribution because the area under the $P(S > s)$ curve is unbounded and the mean is divergent.

Military deterrence is a function of rational actors seeking to minimize military risk within a vast and adversarial international system. These actors can minimize risk by applying a likelihood ratio test (LRT), derived from a dichotomous form of Bayes' Theorem, to a series of hypothesis tests weighing the risk of action versus inaction. Before applying an LRT, however, there must be a probability on which to base the test. For war, the power-law cannot be used given that $q < 1$. The aim of this research is to identify a valid probability for the severity of war that could be used in a Bayesian-derived LRT, and then draw conclusions advancing the field of strategic deterrence, with particular focus on detection and false-alarm probabilities in the context of attack warning.

2 RISK-INFORMED DECISIONS

Risk-informed decision-making requires the ability to prioritize decisions according to their quantitative risks. The field of probabilistic risk assessment has over the years led to a standard definition of risk, which is the expected cost of an event equal to the sum of the products of the consequences multiplied by their probabilities (Advisory Committee on Reactor Safeguards, 2000). The simplest risk-informed decision involves dichotomous outcomes, where the risk of two mutually exclusive choices are weighed against each other and the lower risk of the two is selected (i.e. dichotomous hypothesis testing).

Decisions about dichotomous events "A" and "$\bar{A}$" can be made by comparing risks $R(A) = C_A P(A)$ and $R(\bar{A}) = C_{\bar{A}} P(\bar{A})$, respectively, where C_A is the cost of not countering A and $C_{\bar{A}}$ is the cost of countering $\bar{A}$. In this analysis, negative risks (i.e. profit, gain, etc.) are not considered. We call these "prior risks" because they rely on the prior probability $P(A)$. And as there are only two choices, $P(\bar{A}) = 1 - P(A)$. Event A could be nearly anything. In this paper it represents an "attack" and $\bar{A}$ represents "no attack". One chooses to believe an attack is the outcome if $R(A) > R(\bar{A})$, also written as follows:

$$C_A P(A) > C_{\bar{A}} P(\bar{A}) \qquad (1)$$

This formulation indicates when it is favourable to attack without detection or intelligence. Health insurers, for example, set premiums based solely on prior probability when they are not allowed to consider an individual's specific pre-existing conditions that is normally detected by a test (Sox et al., 2013). Given the multitude of detection capabilities fielded by most states today, use of equation (1) in isolation is not realistic. However, the computation of these risks represents a necessary step leading to decisions that take into account detection systems. The next step in this progression leads to Eq. (2), which is a dichotomous form of Bayes' theorem that includes the probability of detection $P(d|A)$ and the probability of false-alarm $P(d|\bar{A})$:

$$\frac{P(A|d)}{P(\bar{A}|d)} = \frac{P(d|A)}{P(d|\bar{A})}\frac{P(A)}{P(\bar{A})} \qquad (2)$$

The left-hand side of Eq. (2) is the ratio of the posteriors. Given datum d, attack is more likely than not if the right-hand side of Eq. (2) is greater than one. Notwithstanding the costs, $P(d|A)/P(d|\bar{A})$ must be greater than $P(\bar{A})/P(A)$ and also greater than one. Costs are factored in by replacing the prior probabilities with the prior risks as in Eq. (3), leading to a likelihood ratio test (LRT):

$$\frac{P(d|A)}{P(d|\bar{A})} > \frac{R(\bar{A})}{R(A)} \qquad (3)$$

We call the left-hand side of Eq. (3) the likelihood ratio, L, and the right-hand side the critical likelihood ratio, $L*$. Risk is minimized when the decision is made in accordance with the LRT. Specifically, if the LRT is true, then L is greater than $L*$, and one takes action based on the belief that an attack is real. Otherwise, no action is taken. Equations (1) and (3) are thus our models for rational behaviour, with and without detection, respectively.

3 DATA ANALYTICS

Risks in equations (1) and (3) will be per year per target state or alliance as derived from the Correlates of War (COW) Project historic war datasets (Sarkees, 2010). For illustrative purposes, North Atlantic Treaty Organization (NATO) states are arbitrarily chosen to be the collective target of attack.

3.1 Dataset Typology

The COW Project has published a traditional and expanded typology of war. We use the latter. COW's Inter-State classification of wars is based upon the status of territorial entities, focusing on those that are classified as members of the state system. This dataset encompassing wars that took place between or among recognized states where there are at least 1,000 fatalities. COW's Extra-State classification of wars involves imperial and colonial wars. The Intra-State classification of wars encompasses different kinds of wars that take place predominantly within the recognized territory of a state. The last category, Non-State wars, involve non-state territory or across state borders. COW war data exists as rows of named wars that include start and end dates, combat deaths, outcome, and little else. A state's population during a war, for example, is in a different dataset. We analyse only the data available in the COW war datasets.

The focus of this study is strategic war, which requires a level of resources achievable only by states. Therefore, only inter- and/or intra-state war data seem applicable, so the other datasets are not used. From 1816-2007 there are 91 and 199 Inter- and Intra-State wars, respectively.

3.2 Temporal Prior Probability

The short treatment of temporal probability that follows is simplistic. We use it, nevertheless, because of its illustrative value and because it quickly becomes apparent that the severity of war is much more important to arriving at risk-informed decisions than is temporal probability.

For the prior probability of attack, $P(A)$, we begin by using the temporal statistics of the COW Inter-State war dataset. The time between wars is exponentially distributed where there is on average about one interstate war every two years, yielding an exponential distribution parameter $\lambda = 0.5\ wars/yr$. The exponential distribution is fit to the data in Figure 1. The fit has an r^2 value equal to 0.93, indicating a good fit. Equivalently, the probability of there being one or more wars per year follows the Poisson distribution with the same parameter. Thus, in any given year there is a 31% chance that one war will occur somewhere in the world.

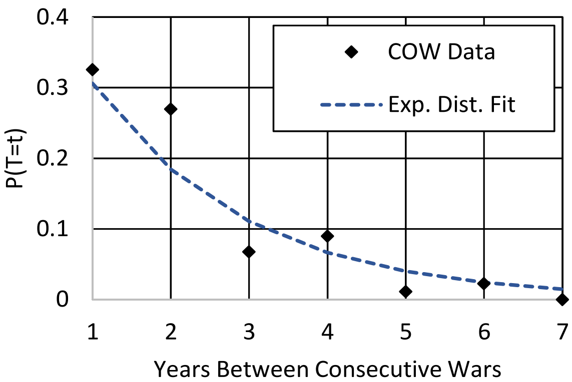

Figure 1: Exponential distribution fit (λ=0.51 wars/yr) to COW Inter-State war data where fit goodness r^2=0.93.

About 62% of the states are defensive in the wars (see Table 1). And, because 46% of the wars comprising this data involved countries that are today part of NATO, there is approximately a $0.31 \times 0.62 \times 0.46 = 0.088$ probability each year that NATO will be attacked once. The average deaths and number of states participating in wars has remained nearly constant in the last 200 years. Thus, additional temporal changes across datasets do not warrant further consideration.

3.3 Severity Probability

The severity of war is needed to estimate C_A, which is necessary to compute the risks in equations (1) and (3). Previous research suggests that severity is probabilistic, in which case C_A is equal to severity s times the probability that a war of severity S is equal to s, conditional on if an attack A has occurred. This is written as $sP(S = s|A)$. Severity has also been modelled as s multiplied by the exceedance probability, which we write here as $sP(S > s|A)$.

3.3.1 Power-law

Lewis Fry Richardson was the first to plot the logarithm of the frequency of deaths in violent conflict against the logarithm of their severity, revealing a straight line with a negative slope, suggesting the applicability of power-law statistics (Richardson, 1960). Exceedance probabilities are obtained simply by dividing the frequencies by the total number of conflicts. Cederman affirmed Richardson's work using COW data and reports a slope of negative 0.41 (Cederman, 2003). However, Cederman's log-log plot displays a slight curvature in the vicinity of 1,000 and 10,000 deaths. This curvature may indicate that the power-law is not the best distribution to be applied, that there is insufficient data, that the wrong kind of data has been used, or that a combination of these errors applies.

Pursuing a hunch that more data is needed to obtain a valid power-law result, we combined the COW Inter-State and Intra-State datasets, obtaining in this case a value of q=*0.70* with an r^2 of 0.99. This result is shown as the dashed orange line in Figure 2. The red-dashed line is for q=*1*, indicating the smallest valid q value. Above this line, the power-law is invalid as a probability distribution. Consistently, researchers report values that are less than one, ours included. Having proved the hunch incorrect, we sought to apply another probability distribution.

3.3.2 Log-normal Distribution

The curvature seen in the power-law fits suggest that the log-normal distribution might be better suited to model the data (Benguigui and Marinov, 2015).

However, because the COW Inter-State data only includes wars having a minimum of 1000 deaths, it is also necessary to combine Inter- and Intra-State dataset, thus providing statistics below this minimum.

A log-normal ($\mu = 3.6$, $\sigma = 0.81$) density function $P(S = s)$ is seen to closely follow the data. This is indicated by the blue-dashed, bell-shaped curve in Figure 2. COW data is indicated by the black line with black triangles. The r^2 obtained by comparing these two curves is *0.99*, indicating an excellent fit.

To highlight the small differences in cases of wars exceeding one million deaths, an expected result in any nuclear conflict, exceedance probability curves, $P(S > s)$, are included. In Figure 2 the log-normal exceedance probability is the blue-dashed curve above the probability density functions following the logarithmic scale on the right side of the graph. COW data is indicated by a black curve with black squares. The log-normal fit fails to match the COW data for high death totals. This result is in contrast to the extremely good fit provided by the power-law.

Despite the overall excellent fit of the log-normal, we are motivated to seek an alternative method that better fits the high severity data. The excellent fit to this data by power-law, even in the case of combining the Inter- and Intra-State datasets, indicates that there is an underlying phenomena favouring higher severity. The log-normal distribution is a symmetric distribution that does not favour upper statistics. Use of the log-gamma distribution, however, may solve this problem as it is an asymmetric distribution that favours the higher range (Halliwell, 2018).

3.3.3 The Effect of Alliances

Combining Inter- and Intra-State datasets creates a dataset with deaths below 1000, enabling log-normal fit with a high r^2 value, but it fails to enable a fit the high-magnitude war data points. Mixing these data sets may add error to the analysis.

Figure 3 is a plot of the base-10 logarithm of deaths versus the number of allies in the named interstate wars. It appears that part of the interstate war data correlates with the number of alliances. However, all of the data cannot be satisfactorily fit using a single exponential line. The best fit to all of the data is poor (r^2=0.1943). The best fit to the wars involving five or less participants, the blue dots, is extremely poor (r^2=0.0552). The best fit to wars having greater than five participants, marked with red dots, begins to show some correlation (r^2=0.5672). A partial correlation can only adversely affect the statistics and obfuscate a more applicable probability distribution. Therefore, we ungroup the Inter-State dataset so that deaths are not the total of named wars for all allies. Ungrouping also creates a larger dataset that includes deaths below 1,000. The number of wars also increases from 91 to 319 and more than half of the wars have less than 10,000 fatalities.

Removing the named-war grouping helps with data analysis, but its potential significance is also worth discussing. Jackson and Nei reported that there were ten times fewer wars between 1950 and 2000 as a result of political, military, and economic alliances (Jackson and Nei, 2015). In other words, peace and war are at least partly the result of a network phenomenon.

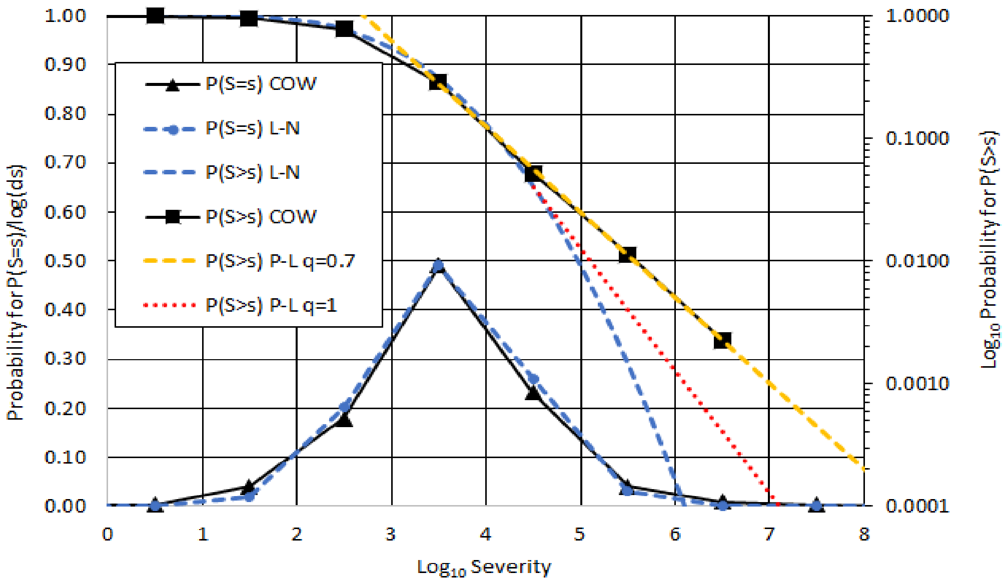

Figure 2: Power-law (P-L) and log-normal (L-N) fit to COW Inter- and Intra-State war datasets.

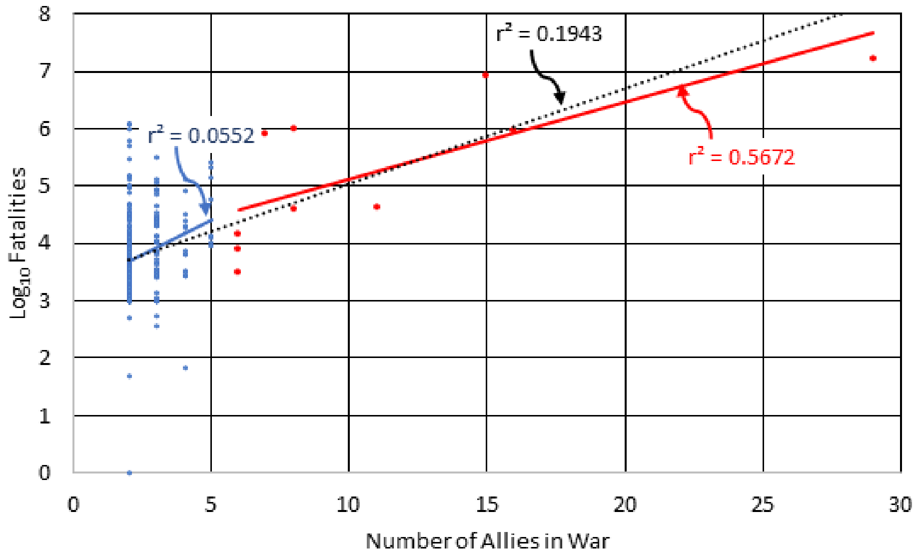

Figure 3: Inter- and Intra-State COW dataset deaths versus the number of states indicate an inconsistent effect caused by alliances, where r^2 for 2 to 5 state wars is very weak (0.055, blue dots and blue line), moderately good for 6 – 29 states (0.57, red dots and red line), and weak for the combined data (0.19, black dashed line).

In the instant case, however, removing the effect of alliances helps better understand the state individually as a rational actor.

3.3.4 Log-gamma Distribution

As with the log-normal, the log-gamma is a distribution of the log of datum. Probability density functions derived from both COW data and a log-gamma distribution ($\alpha = 9.0$, $\beta = 0.39$) are indicated in Figure 4. The fatalities used to derive the curves are from individual rows in the Intra-State dataset, not the sum of fatalities for respective named wars involving multiple states. The r^2 of the log-gamma fit compared to the COW data is 0.99, indicating an excellent fit. Equally important, the log-gamma fit holds for $s > 10^6$. This is indicated by the exceedance probability curve that follows the logarithmic scale on the right-side of Figure 4. A side effect of de-grouping named wars is that the maximum number of deaths experienced for a given war does not exceed 10^7. Furthermore, because the $P(S>s)$ curve follows the complement of the integral of $P(S=s)$, there is no data points for $P(S>10^7)$. To check the fit for these high values, we compare the average slope of the power-law and log-gamma curves between $P(S>10^4)$ and $P(S>10^8)$. We find them in good agreement (0.62 versus 0.55). Thus, the log-gamma distribution fits the entire range of severity covered by the COW data when the wars are analysed only by nation state. The slope of the log-gamma increases in negativity, however, so that the distribution is valid for higher death values. Specifically, the slope of the $P(S>s)$ curve between 10^8 and 10^9 is $\log(1.1{\times}10^{-3} - 9.5{\times}10^{-5}) = -3.0$. As this slope is less than negative one and decreasing, the fit is valid.

4 RISK MINIMIZATION

Keeping peace requires that states not take undue action while avoiding inaction that might invite attack. This delicate balance can be optimized by minimizing expected combat deaths, taking into account attack detection and false-alarm probabilities, which we can now do using prior probabilities that span both conventional and nuclear levels of fatalities. Exactly how and why becomes clearer in the presence of a game-theoretic model of war, which we provide first and then incorporate into a likelihood-ratio analysis. It is then reasonable and practical to assume that the probability of detection is exactly one. Most detection systems provide nearly this level of performance and the assumption leads to a single risk of inaction with and without detection, which makes more tractable an analysis of the impact of false-alarm probability on attack decisions.

4.1 Game-theoretic Analysis

Figure 5 shows the win-loss distribution of deaths for attackers and defenders from the un-grouped Inter-State dataset based on COW's assessment of what constitutes "win" and "lose".

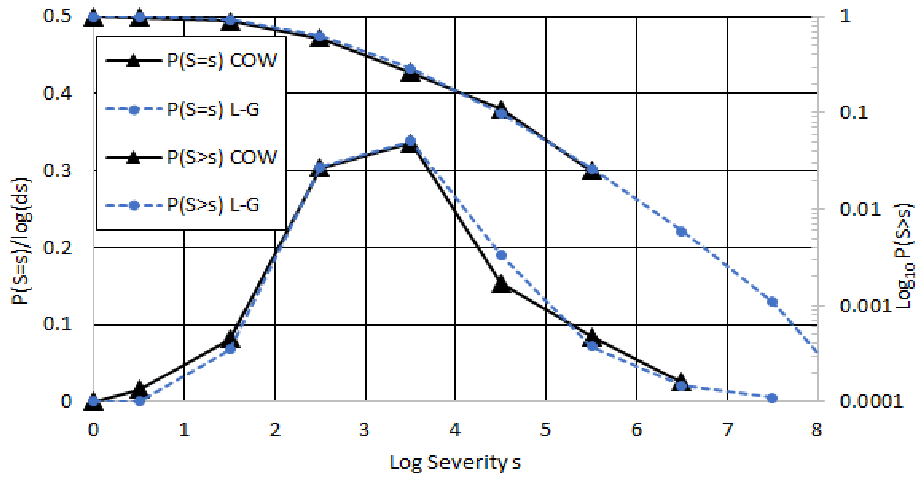

Figure 4: COW Inter-State war and log-gamma (L-G) probability density curves, *P(S=s)*, and exceedance probability curves, *P(S>s)*, where density curves track the left-side scale and the exceedance curves track the right-side scale.

Key information from this graph is summarized in Table 1. The "Other" category in Table 1 includes ties, transformations, and stalemates. The percentages are the number of wars in the category divided by the total number of wars, where the total for the six categories is 100%.

Table 2 reconciles the "max deaths" information in Table 1 with game-theoretic strategies. Given that an attack has already occurred, we hypothesize that attackers and defenders have two available strategies: "escalate" and "deescalate." Both take into account the strength of a state's motives and resources. Thus, use of the deescalate strategy may be the result of previous escalation having depleted the state's national will and resources. The maximum deaths experienced by a state is chosen to be the limit of losses a state would accept in a war for the particular strategy. For example, given that 7.5M is the maximum loss a state (U.S.S.R. in WWII) has endured by way of defensive escalation, this is taken to be the maximum loss for the strategy. Conversely, 3.5M is the maximum loss of an attacker (Germany in WWII) endured via offensive escalation. Maximum losses for other strategies are similarly derived.

The game-theoretic model of Table 2 leads to two Nash Equilibrium points (Nash, 1950), one at escalate-deescalate, the other at deescalate-escalate. These equilibrium points are consistent with the fact that wars, once started, normally escalate and result in high losses no matter if the attacker or defender is the winner. The escalate-escalate cell is not an equilibrium point because mutual escalation leads to losses that are greater than losses in adjacent cells. Eventually, conflicts move to escalate-deescalate or deescalate-escalate where a winner and loser eventually emerge. The deescalate-deescalate cell is normally unstable, which is why only 18% of the wars end in this state.

The strategies in Table 2 are also consistent with the distributions of war severity. Escalation or deescalation is a multiplicative increase or decrease in the expense of human resources. Where E_n is a random variable representing the fractional increase or decrease of combatants during each escalation or deescalation, severity is random variable equal to the products of these changes, $S_N \sim E_1 \times E_2 \times \ldots E_N$, resulting in the applicability of a logarithmic distribution. In other words, leaders escalate or deescalate based on the quantity of deaths already incurred. Relative increases or decreases follow a logarithmically distributed process (Ott, 1990). Noting that gamma and Poisson distributions are conjugates, what is more challenging to understand is why three of the win-lose distributions in Figure 5 appear to be log-gamma distributions (~L-G), but only the defend-win curve appears to follow a log-normal distribution (~L-N). The defend-win category is comprised of far greater losses than any other category (i.e. 17M versus 6.4M for defend-lose, 5.4M for attack-lose, and 0.8M for attack-win).

Symmetry of the defend-win distribution may indicate that there are underlying random variables that are not strictly positive numbers as are fatalities. Economy and infrastructure are examples of variables that could also be negative and whose effect might be in play. More likely, a defender who is escalating in response to an attacker who is escalating increasingly relies on the benefits of alliances as discussed in section 3.3.3. Indeed, most of the fatalities associated with the log-normal defend-win curve are from the many allied countries in WWII.

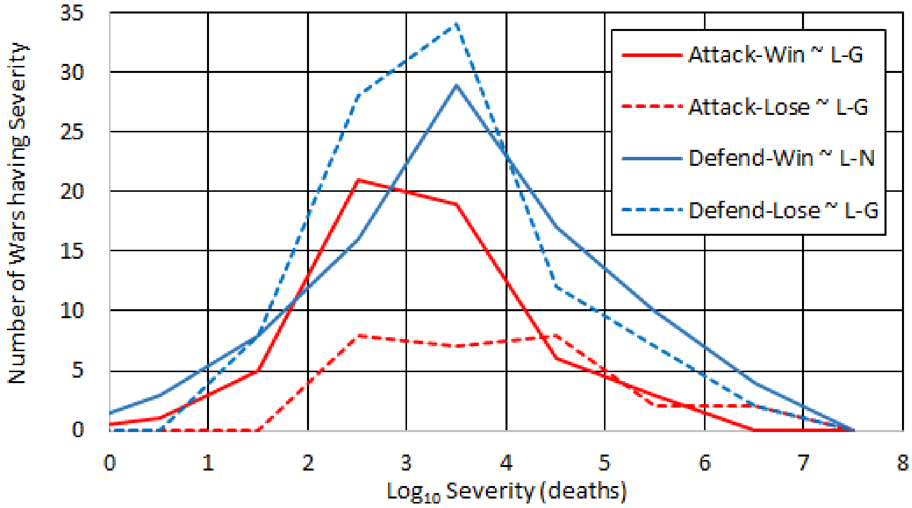

Figure 5: Attack-defend-win-lose distributions for wars as a function of the log of severity.

4.2 Minimization without Detection

Table 3 reports the annual risk of inaction for NATO based on probabilities in Table 1 and Figure 5. Risk is the severity probability in the row times the midpoint of the range of deaths. The result is then multiplied by 0.088, as estimated in section 3.2, to calculate the risk per year for NATO. Numbers are rounded to two significant figures. Two sets of probabilities and two sets of risks are provided, one from the COW data and the other based on the defend-lose log-gamma fit.

Table 1: Attack-Defend-Win-Lose statistics, based on COW definitions, and parametric fits to the data.

	Attack (38%)	Defend (62%)
Win (44%)	17%	27%
	0.25M max deaths	7.5M max deaths
	$\mathcal{L}$-$\mathcal{G}$(10,0.32) r^2=0.99	$\mathcal{L}$-$\mathcal{N}$(3.5,1.2) r^2=0.95
	μ=2.9, σ=0.99	μ=3.5, σ=1.2
Lose (38%)	9%	29%
	3.5M max deaths	1.8M max deaths
	$\mathcal{L}$-$\mathcal{G}$(8.0,0.48) r^2=0.88	$\mathcal{L}$-$\mathcal{G}$(11,0.31) r^2=0.97
	μ=3.9, σ=1.1	μ=3.4, σ=1.0
Other (18%)	12%	6%
	0.50M max deaths	0.75M max deaths
	$\mathcal{L}$-$\mathcal{G}$(17, 0.20) r^2=0.84	$\mathcal{L}$-$\mathcal{G}$(12, 0.24) r^2=0.96
	μ=2.8, σ=1.3	μ=3, σ=1.0

The rightmost columns are the "expected risk of inaction" because they are the average number of deaths that will result from war each year if nothing is done. Given the exponential scale, the data and model agree reasonably well until the data ends. Although we don't distinguish between deaths from conventional or nuclear weapons, the number of deaths resulting from nuclear weapons used in WWII suggest that 100K or more deaths are nuclear-war-level. What one can conclude from this table, then, is that the risk of war increases for increasing ranges of fatalities, making nuclear war the highest risk despite decreasing in probability. This is true whether from surprise attack or a slow build-up.

In the absence of a detection capability, risk-informed decision can be made based only on the prior probabilities in Table 3. A state may consider attacking its foe to pre-empt an attack that it thinks is probable. The predilection to attack, or the likelihood of being attacked, would depend on the risk of action weighed against inaction. Normally there will be many scenarios for action, and each must be weighed against inaction.

Table 2: Game model based on maximum win-loss deaths (millions) with Nash Equilibria indicated (circled).

		Defender	
		Escalate	Deescalate
Attacker	Escalate	3.5 , 7.5	0.25 , 1.8
	Deescalate	1.8 , 0.25	0.5 , 0.75

Pre-emptively attacking may reduce the number of fatalities through destroying part of the enemy's attack capabilities. A rational NATO alliance would do so only if the risk of action is less than the corres-

Table 3: Annual risk of attack on NATO countries in expected deaths.

Severity s_n	Severity Range s_{n-1} < Dead ≤ s_n	Δs $= s_n - s_{n-1}$	Probability for Severity Range		Expected Annual Risk of Inaction	
			COW Defend-Lose	L-G Defend-lose	Based on Data	Based on Model
10	1 < Dead ≤ 10	9	0.00	0.000016	0	0
100	10 < Dead ≤100	90	0.088	0.041	1	0
1K	100 < Dead ≤1K	900	0.31	0.30	24	23
10K	1K < Dead ≤10K	9K	0.37	0.38	300	300
100K	10K < Dead ≤ 100K	90K	0.13	0.20	1000	1,600
1M	100K < Dead ≤ 1M	900K	0.077	0.066	6,100	5,300
10M	1M < Dead ≤ 10M	9M	0.022	0.016	17,000	12,000
100M	10M < Dead ≤ 100M	90M	No Data	0.0029	No Data	23,000
1B	100M < Dead ≤ 1B	900M	No Data	0.00044	No Data	35,000

ponding row in Table 3. In the case of the row designated "1M < Dead ≤ 10M", for example, the risk of the pre-emptive attack would need to be less than 17K, based on data (12K modelled). Solving Eq. (1), the maximum consequences of incorrect action is $C_{\bar{A}} = 17{,}000 \times (1 - 0.088)/0.088 = 180{,}000$. In other words, if NATO were confident that no more than 180K dead would result in a pre-emptive attack, then the attack in this case is rational from a strictly a fatalities perspective. Again, this scenario is appropriate only if the alliance expects an attack.

4.3 Minimization with Detection

A pre-emptive strike based solely on prior probability of combat deaths is not realistic given the many detection systems fielded. However, computing the risk of action and inaction per Eq. (1) is useful because these quantities are needed in the right-hand side of Eq. (3). Equation (4) below specifies the right-hand side of Eq. (3), the Bayesian detection criteria, using details from Table 1 (i.e. $\mathcal{L}$-$\mathcal{G}$, $\alpha = 11$ and $\beta = 0.31$) where the consequence of action $C_{\bar{A}}$ is the only unknown quantity. Modelled severity, rather than data, is used to enable a study of extreme conflict that might cause between 10M and 1B fatalities:

$$L^* = \frac{C_{\bar{A}} \times (1 - 0.088)}{\Delta s \times \mathcal{L}\text{-}\mathcal{G}(\log(s); 11, 0.31) \times 0.088} \tag{4}$$

L^* from Eq. (4) can be used in Eq. (3) to study possible decisions involving alert bomber forces, for example, that can be launched on warning of attack and recalled if there is a false-alarm. Alert forces must be supported by a continuously functioning attack detection system that is survivable through all foreseeable conflicts. If NATO policy is to launch its bombers upon warning of inbound ballistic missiles, similar to U.S. policy (U.S. Dept. of Defense, 2018), there are at least two possible outcomes with risks that are defined in terms of detection and false-alarm: the system fails to detect an actual attack, no action is taken, resulting in the loss of the alert force and fatalities proportional to the number of missiles; or the system reports a false-alarm, prompting the launch of the alert forces, causing the enemy to make its own launch-on-warning decision. These two outcomes are considered in turn using Eq. (4).

Being a number close to, but necessarily less than one, the detection probability proportionally reduces the threat that the NATO alert forces pose to an enemy. This proportionally increases the threat of attack by an opponent who has intelligence about the detection probability or is capable of reducing it through cyber- or information-operations. However, because detection probabilities approach one and false-alarm probabilities are normally much less than one, the ratio of detection over false-alarm probabilities is numerically dominated by the false-alarm value. For this reason, it is correct and practical to assign the probability of detection a perfect value of one and assume that only the false-alarm probability changes the likelihood ratio L.

Table 4 reports L^*as a function of C_A and $C_{\bar{A}}$. NATO should launch its bombers if the likelihood ratio of its detection system L is greater than the corresponding L^*. However, L^* cannot be less than one or arbitrarily high. An L value less than one violates the basis on which Eq. (3) was derived. An arbitrarily high L implies a false-alarm probability that is unachievably low. For example, a difficult-to-achieve 0.001 false-alarm probability for Synthetic Aperture Radar (Li, 1994) yields $log(L) = log(1/0.001) = 3$. In Table 4, this and similar values are coloured yellow indicating that it may be unachievably low. Red cells indicate invalid or values that are too low. Green cells indicate nominal values.

Table 4: $Log(L^*)$ as a function of $Log(C_A)$ and $Log(C_{\bar{A}})$, where red cells are invalid or too low, green contain normal and useful values, and yellow indicates that the required false-alarm probabilities may be unrealistic.

		Log($C_{\bar{A}}$)							
Log(s)	Log(C_A)	2	3	4	5	6	7	8	9
2	-0.49	2.38	3.38	4.38	5.38	6.38	7.38	8.38	9.38
3	1.37	0.52	1.52	2.52	3.52	4.52	5.52	6.52	7.52
4	2.47	-0.59	0.41	1.41	2.41	3.41	4.41	5.41	6.41
5	3.21	-1.32	-0.32	0.68	1.68	2.68	3.68	4.68	5.68
6	3.72	-1.84	-0.84	0.16	1.16	2.16	3.16	4.16	5.16
7	4.09	-2.20	-1.20	-0.20	0.80	1.80	2.80	3.80	4.80
8	4.35	-2.47	-1.47	-0.47	0.53	1.53	2.53	3.53	4.53
9	4.54	-2.65	-1.65	-0.65	0.35	1.35	2.35	3.35	4.35

The consequences of acting on a false-alarm are potentially far more serious than the consequences of inaction. For example, a technical glitch could result in a full-scale nuclear war. Thus, deterrence is as much about detection and false-alarm as it is about the quantity and destructiveness of weapons. Consider again the last row in Table 3 for which there is COW data, marked "1M < Dead ≤ 10 M", where the data indicates a severity probability of 0.022 and an annual risk to NATO countries equal to 17K deaths. For this case, it would be rational for NATO to take an action intended to negate war only if that action resulted in annual risk less than 17K. One example of such an action is to put strategic bombers on alert so they can be launched before being destroyed in a surprise attack. While this action may have economic impact, it risks few NATO combatant lives directly. So long as this action increases the riskto the enemy for attacking, the enemy cannot rationally choose to attack because the risk table applies equally to them. Thus, the U.S. Administration's recent decision to put bombers back on alert makes sense provided that the enemy is confident their attack would be detected and that the bombers could put at risk more of the enemy's lives than would be saved in a pre-emptive attack.

In all cases, robust and hardened detection and alerting systems are paramount. These systems require hardware, software, and human operators that don't automatically reject the possibility of a surprise attack. Deterrence is also improved if detection information is shared with the enemy because it helps ensure they too will react correctly to alarms. If the log-gamma model results are to be believed, even for a just a few rows past the data, then the risk continues to increase and the maximum false-alarm probability rapidly decreases to unachievably small values. This trend, partially seen in Figure 6, eventually reverses, but well past the end of the table where human population is exceeded. This result holds despite all of the modelled uncertainties and is true even though we have replaced the power-law with a distribution that is probabilistically valid. The high risk behaviour of war remains and it leads to the following observation. A sufficiently low false-alarm probability to justify a launch-on-warning decision is not achievable if that decision results in an arbitrary scale of retaliation. However, the same is not true if the scale of the attack is known. For example, if satellites are able to confirm that only ten missiles are inbound and each can kill at most 1M, then the row "1M < Dead ≤ 10M" applies and launch-on-attack is rational for log-likelihood ratios equal to and greater than 0.80. Referring to Table 4, these are cells in the $Log(C_A) = 4.09$ row, to the right of the $Log(C_{\bar{A}}) =$ 4 column.

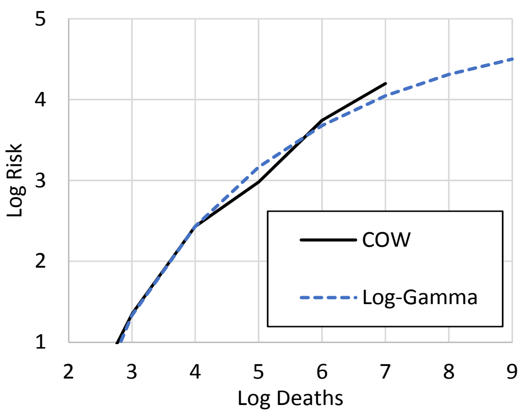

Figure 6: Log of attack risk vs. log deaths.

One can now make sense of power-law results by computing pseudo q-values based on the log of the slope of the upper end of exceedance probabilities for the log-normal and log-gamma fits. The only strategy with a pseudo q-value greater than one corresponds to the log-normally distributed defend-win case. See

Table 5 below. Thus, for high-magnitude war only defensive strategies generate bounded results.

Table 5: Pseudo q-values for log-gamma and log-normal exceedance probabilities based on indicated COW deaths.

	Attacker(s)	Defender(s)
Win	q=0.83, 0.8M total	q=1.2, 17M total
Lose	q=0.5, 5.4M total	q=0.78, 6.M total

5 CONCLUSION

Risk of military attack, in terms of combat fatalities, can be minimized using a Bayesian detection criteria based on prior probability distributions derived from COW Inter-State war data. Use of the power-law in this context is invalid and should be abandoned. A game-theoretic model with two Nash Equilibrium points help explain why combatant fatalities follow a log-gamma or log-normal probability distribution depending on if a state is offensive or defensive. De-correlating combat fatalities from alliance effects exposes the log-gamma structure of the defend-lose case and enables a calculation of attack risk per year for ranges of deaths in powers of ten. Further, the data indicate that war occurs with predictable temporal frequency where the likelihood of one or more wars in a year follows the Poisson distribution. After being initiated, a war escalates or deescalates proportional to the combat losses already incurred. The data also shows that the risk of nuclear war level fatalities increases despite decreasing in probability. Taking into account detection and false-alarm probabilities, an LRT advises that it is rational to escalate only when the consequence of inaction and action are about equal in magnitude, corresponding to nominal false-alarm maxima. A corollary is that act-on-warning is justified only if the detection system indicates an upper limit of impending fatalities. Lastly, only defensive strategies have a convergent mean for wars having fatalities greater 10^8.

In future work, a Bayesian detection criteria could be applied to automated detection of cyber-attack, informed by the correct prior and taking into account both positive and negative consequences.

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