# Trajectory Planning for Automated Merging on Highways

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Abstract: This article introduces a new approach for trajectory planning for merging on highways. The aim of the algorithm, is to find a comfortable driving strategy to merge in a gap on the target lane. Therfore, the proposed algorithm determines a bunch of trajectories to reach surrounding gaps. The trajectory with the lowest costs for each gap is chosen. To obtain the longitudinal component of the trajectory, a five-part section-wise defined polynomial in Frenet space is used to generate comfortable driving behaviour, with as few changes in the acceleration profile as possible. Based on the prediction of surrounding traffic, different variations of deceleration and acceleration are combined. For each longitudinal part, a lateral component to perform a lane change into the target gap is evaluated. The concept allows to evaluate the influence of the longitudinal driving strategy on the dynamics required to change lanes. The algorithm is evaluated in a MATLAB simulation including a runtime estimation.

## 1 INTRODUCTION

To integrate automated vehicles into disordered manual traffic, they have to solve complex driving tasks. For example, when driving on highways, automated vehicles have to change lanes in high density traffic. The wrong assessment of distances in the longitudinal direction or wrong expectations of other traffic participants' behaviour are the main reasons for traffic accidents (Gründl, 2005). Therefore, the automated vehicle needs a distinct interpretable strategy to interact with manual traffic, respecting safety distances as well as driving comfort. One popular method of motion planning in structured environments is to generate a set of alternative trajectories to different target positions and select the best trajectory based on a cost function. For instance, Werling et al. (Werling et al., 2010) use quintic polynomials for the lateral and the longitudinal components. Based on a set of trajectories they formulate an optimization problem in order to obtain a trajectory that satisfies the comfort criteria of the car occupants, is collision free and minimizes the deviation from to the reference path. McNaughton et al. (McNaughton et al., 2011) introduce a lattice structure to obtain various lateral and longitudinal offsets to the lane's centre. They connect lateral poses with different longitudinal positions using cubic curvature polynomials, determining the optimal trajectory based on a cost function. Wei et al. (Wei et al., 2014) also use a lattice structure to generate feasible candidate strategies. Thereafter, they are taking static and dynamic obstacles into account, due to an interaction aware predicting of surrounding traffic. The optimal strategy is found based on a cost function. In contrast, Schwesinger et al. (Schwesinger et al., 2013) use a closed-loop vehicle model to obtain a set of feasible sample trajectories to certain goals considering environmental constraints. To plan the lateral trajectory Lee and Litkouhi (Lee and Litkouhi, 2012) propose quintic polynomials. The algorithm adapts the lateral trajectory in order to not exceed a defined maximum lateral acceleration. Heil et al. (Heil et al., 2016) use an asymmetrical lateral trajectory to achieve a human-like lane change behaviour according to Sporrer et al. (Sporrer et al., 1998), where a human driver uses a higher lateral acceleration for steering out of the initial lane than for steering back into the target lane. Ulbrich and Maurer (Ulbrich and Maurer, 2015) propose a decision algorithm for tactical lane changes based on Bayesian networks to decide if it is possible or beneficial to execute a lane change. Hansen et al. (Hansen et al., 2016) divide driving on highways in a lane keep and a merging maneuver. The lane keep maneuver is based



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on a common ACC vehicle controller. If the decision is made that a lane change is necessary or beneficial to the automated vehicle, they use two quintic polynomials to synchronize to a gap. They define the optimum intersection of the automated vehicles with the border of the safety area of the first vehicle building the gap and the entrance in the safety area of the target vehicle on the current lane by solving an optimisation problem minimizing the average velocity during both parts, as well as the velocity of the automated vehicles at the end of the trajectory. Based on those intersections the longitudinal and lateral component of the trajectory is defined.

The aim of our planning algorithm is to achieve comfortable driving behaviour with as few jerk minimal changes in the longitudinal acceleration profile as possible respecting the lateral dynamics and safety distances to interacting traffic. Therefore, we propose a planning algorithm that generates a set of trajectories for reasonable gaps and finds the trajectory with the lowest cost per gap. The longitudinal component consists of a new planning approach presenting an analytical solution using a piece-wise defined function. The approach allows to respect the maximum longitudinal and lateral dynamics of the trajectory as well as to check for possible collision analytically.

The structure of this article is as follows. Section 2 proposes a way to integrate the algorithm into a system architecture. In section 3 the planning algorithm is presented, where the sector to change lanes, the longitudinal and lateral components of the trajectory, cost functions and the algorithm are introduced. The concept is evaluated using a MATLAB simulation in section 4. A conclusion is given in section 5.

## 2 SYSTEM ARCHITECTURE

We propose the system architecture illustrated in figure 1. To enable an automated vehicle (Ego) to merge on highways in dense traffic, we describe the free space in the form of gaps between other cars. A possible scenario is illustrated in figure 2. The first layer of the system architecture is divided into the parts perception and navigation. The second layer is the behaviour decision and the third layer is the vehicle controller. The perception extracts the environment information about other road users and the road. The navigation estimates the route and labels the lanes as regular lanes on-ramps or off-ramps. The behaviour decision builds the scene based on perception information and the route. The scene comprises all information relevant to the driving task (Ulbrich et al., 2015). Depending on the scene all reasonable gaps are extracted. For example, due to the desired route, gaps on the target lane are selected. The module needs to evaluate if it is feasible to reach the gap based on the front vehicle of the Ego, possible lane ends, the approximated gap velocity and predicted gap sizes. All selected gaps are given to the proposed planner. The displacement is predicted for every gap using a constant velocity model for the cars limiting the gap. Based on this prediction, it is possible to calculate the time span available to merge into the target gap. In a second step, a number of acceleration profiles are

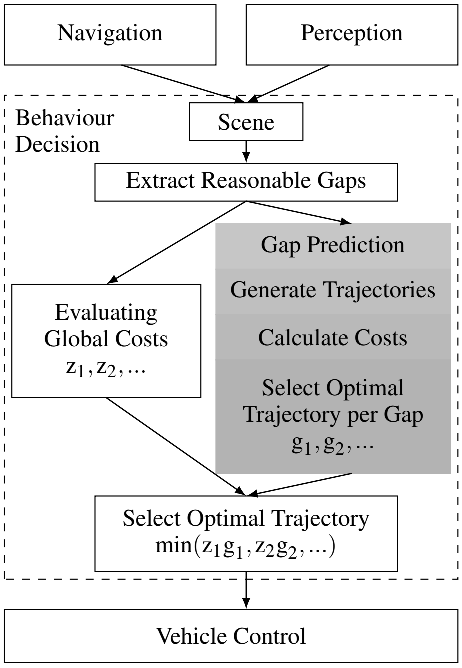


Figure 1: The figure shows a system architecture determining the best trajectory based on the surrounding gaps extracted from a scene representation. The parts of the proposed planning algorithm are the blocks in grey.

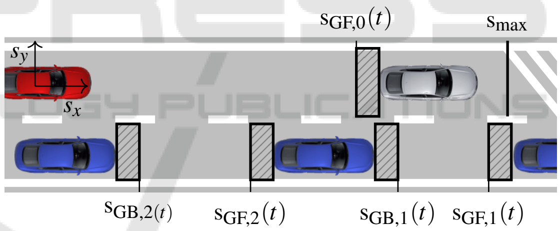


Figure 2: The reference coordinate system is placed in the middle of the Ego (red) on the left lane. The proposed planning algorithm determines the dynamics required to change lanes for example in gap 1, built by $s_{GF,1}(t)$ and $s_{GB,1}(t)$, or gap 2, built by $s_{GF,2}(t)$ and $s_{GB,2}(t)$, on the right lane. The algorithm also takes the front vehicle $s_{GF,0}(t)$ and possible lane ends $s_{max}$ into account. The black dashed area is an area that shall be avoided by the other vehicles, including the Ego.

evaluated to reach this space. For each longitudinal part a lateral planning to merge into the gap is made, considering safety margins in Frenet space (Werling et al., 2010). The optimal trajectory per gap according to comfort and safety margins is considered. Parallel global costs evaluate criteria like the velocity of each gap or strategic decision, for example going to the right lane as early as possible to increase the time reserve to reach an exit or the desired lane at an interchange.

The dynamic costs and the global strategic costs are evaluated together, the optimal trajectory is transformed from Frenet into global coordinates and is sent to the vehicle controller. The transformation is given in (Werling et al., 2010). The vehicle controller executes the desired plan.

# 3 PLANNING ALGORITHM

The proposed planning approach is based on Hansen et al. (Hansen et al., 2016). Our goal was to develop an algorithm to determine the most beneficial driving strategy by finding the optimal trajectory in a set of candidate trajectories, like in (Lee and Litkouhi, 2012) or (Werling et al., 2010), for several gaps on the target lane. The concept allows to start a lane change at a point in the future and to combine an accelerating and decelerating longitudinal driving strategy to reach the gap by taking the front vehicle or a possible lane end into account. To obtain a comfortable driving behaviour with as few changes in the longitudinal acceleration profile as possible, we use a section-wise defined longitudinal component with no more than five parts. For every longitudinal component, a lateral component is planned. Therefore, it is possible to model the coupling of longitudinal and lateral dynamics when changing lanes. The optimal trajectory is found based on a cost function. Thus, the longitudinal and lateral jerk, the deviation from the target position to the end position of the longitudinal trajectory and the number of parts of the section-wise defined function is taken into account. Based on the gap prediction built by surrounding vehicles, the planning is made.

The prediction model for the extracted reasonable gaps is introduced in section 3.1. Second, the longitudinal component in section 3.2 and thereafter the lateral component in section 3.3 of the trajectory is explained. Then, the cost function to determine the optimal trajectory per gap is introduced in section 3.4. The algorithm is given in section 3.5.

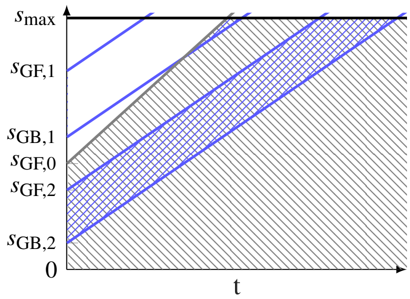


Figure 3: The figure illustrates as grey dashed area the displacement of the Ego gap and as blue dashed area, where a lane change to gap two is possible.

## 3.1 Gap Prediction

The prediction estimates the space where following the Ego lane or changing into a gap on the target lane is possible. An interaction-aware prediction and uncertainties in perception are neglected. The prediction is not able to anticipate the reactions of other traffic participants to the action of the Ego and therefore does not depict interactions between road users. Since the planning is constantly repeated, only the first time step is executed until a new plan is available, the planning reacts to changing behaviour of surrounding traffic participants. Surrounding vehicles are predicted with constant velocity as given in equation (1), for each gap n:

$$\begin{aligned} s_{\mathrm{GF,n}}(t) &= s_{\mathrm{GF,n}}(t=0) + v_{\mathrm{GF,n}}(t=0)\cdot t, \\ s_{\mathrm{GB,n}}(t) &= s_{\mathrm{GB,n}}(t=0) + v_{\mathrm{GB,n}}(t=0)\cdot t, \end{aligned} \tag{1}$$

where $s_{\mathrm{GF,0}}(t=0)$ and $s_{\mathrm{GB,n}}(t=0)$ are the starting position and $v_{\mathrm{GF,n}}(t=0)$ and $v_{\mathrm{GB,n}}(t=0)$ are the starting velocity of the gap's front and back vehicle. Moreover, the longitudinal distance to traffic participant should not fall below a minimum. In this case the maximum velocity of vehicle is the velocity of its vehicle in front. Figure 3 illustrates the perception model. The grey dashed area illustrates the displacement of the Ego's front vehicle $s_{\mathrm{GF,0}}(t)$. The dashed blue area shows the predicted areas, where a lane change to gap $n=2$, built by the gap's front $s_{\mathrm{GF,2}}(t)$ and back vehicle $s_{\mathrm{GB,2}}(t)$ is possible. In addition, the concept allows to restrict the maximum displacement $s_{\max}$ allowed for the maneuver due to a lane end. The prediction can be extended by interaction aware prediction (Kesting et al., 2010), (Treiber and Kesting, 2009) or by considering uncertainties in prediction and perception (Suh and Yi, 2017).

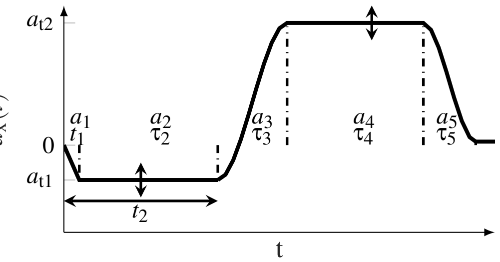


Figure 4: The parts one, three and five use cubic acceleration equations. The parts two and four use constant acceleration equations. After part five zero acceleration is assumed. The target accelerations $a_{t,1}$ and $a_{t,2}$ as well as the time span $t_2$ are varied in the algorithm.

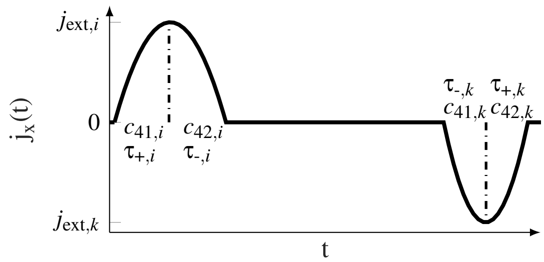


Figure 5: In case of a jerk maximum $j_{\text{ext},i}$, the time span $\tau_{+,i}$ before the extreme and the time span $\tau_{-,i}$ after the extreme is used. In case of a jerk minimum $j_{\text{ext},k}$ it is vice versa. For both cases we use the constants $c_{41,i}$ before and $c_{42,i}$ after the extreme point.

## 3.2 Longitudinal Component

The longitudinal component of the trajectory evaluates different variations of acceleration and deceleration to reach the target gap. To ensure collision free driving, all acceleration profiles that do not reach the blue or are outside the grey dashed area are discarded, as illustrated in figure 3. To generate a comfortable driving behaviour with as few changes in the acceleration profile as possible, a five-part section-wise defined function is used, as given in equation (2):

$$S_x(t) = \begin{cases} S_{x,1}(t), & t \le t_1 \\ S_{x,2}(t-t_1), & t_1 < t \le t_2 = t_1 + \tau_2 \\ S_{x,3}(t-t_2), & t_2 < t \le t_3 = t_2 + \tau_3 \\ S_{x,4}(t-t_3), & t_3 < t \le t_4 = t_3 + \tau_4 \\ S_{x,5}(t-t_4), & t_4 < t \le t_5 = t_4 + \tau_5 \end{cases} \tag{2}$$

The time span for each polynomial is described by $\tau_i$. The section-wise defined acceleration profile is illustrated in figure 4. The algorithm varies the first $a_{t1}$ and second $a_{t2}$ target acceleration, as well as the time $t_2$. The time $t_1$ is maximum $\tau_1$. To determine the polynomials of all five parts, the matrix $A(t)$ is used, as given in equation (3):

$$A(t) = \begin{pmatrix} 1 & t & t^2 & t^3 & t^4 & t^5 \\ 0 & 1 & 2t & 3t^2 & 4t^3 & 5t^4 \\ 0 & 0 & 2 & 6t & 12t^2 & 20t^3 \\ 0 & 0 & 0 & 6 & 24t & 60t^2 \\ 0 & 0 & 0 & 0 & 24 & 120t \end{pmatrix}. \tag{3}$$

Cubic acceleration leads to quintic position polynomials. Those can be described by the following equation (4), where i stands for the part of the section-wise defined function:

$$\begin{aligned} c_{x,i} &= \left(c_{0,i}, c_{1,i}, c_{2,i}, c_{3,i}, c_{4,i}, c_{5,i}\right)^{\mathrm{T}}, \\ S_{x,i}(t) &= \left(s_{x,i}(t), v_{x,i}(t), a_{x,i}(t), j_{x,i}(t), \dot{j}_{x,i}(t)\right)^{\mathrm{T}} \\ &= A(t) c_{x,i}. \end{aligned} \tag{4}$$

At first the time for each part of the section-wise defined function $\tau_i$ and the constants $c_{2,i}$, $c_{3,i}$, $c_{4,i}$ and $c_{5,i}$ are determined. Therefore, it is differentiated between cubic and constant acceleration equations. The first, third and fifth part of the five-part section-wise defined solution are cubic acceleration parts. The second and fourth part use constant acceleration equations. Thus, the time span to reach a certain target velocity is determined for the fourth part. Thereafter, the constants $c_{0,i}$, $c_{1,i}$ for each part are determined by solving the trivial solutions by using the starting position $s_{s,i}$ and velocity $v_{s,i}$ of each part.

### 3.2.1 Cubic Acceleration

Solving the trivial solutions leads to $c_{2,i}$ and $c_{3,i}$ by using the starting acceleration $a_{s,i}$ and jerk $j_{s,i}$ of each part. The time to reach the target acceleration $\tau_{+/-,i}$ and the constants $c_{4,i}$ and $c_{5,i}$ are extinguished by solving the following boundary conditions: $a_{x,i}(\tau_i) = \mathrm{a}_{t,i}$, $j_{x,i}(\tau_i) = 0$, $\dot{j}_{x,i}(\tau_{\text{jext},i}) = 0$ and $j_{x,i}(\tau_{\text{jext},i}) = j_{\text{ext},i}$. At the time $\tau_{\text{jext},i}$ the jerk extreme point $j_{\text{ext},i}$ is reached. The time to reach the target acceleration $a_{t,i}$ is given in equation (5):

$$\tau_{+/-,i} = \frac{3(a_{t,i} - a_{s,i})}{j_{\text{ext},i} + j_{s,i} \pm \sqrt{j_{\text{ext},i}^2 - j_{\text{ext},i} j_{s,i}}}. \tag{5}$$

The time $\tau_i$ differentiates between a positive case $\tau_{+,i}$ and a negative one $\tau_{-,i}$ as illustrated in figure 5. In case of a positive jerk extreme the positive case is used, after the extreme point we use the negative one. In case of a negative extreme point it is vice versa.

For $c_{4,i}$ between two constants is distinguished. The constants are given in equation (6):

$$\begin{aligned} c_{41,i} &= -\left(\sqrt{\frac{-j_{\text{ext},i}}{j_{s,i} - j_{\text{ext},i}}} + 1\right) \frac{j_{s,i} - j_{\text{ext},i}}{12\tau_i} \\ c_{42,i} &= \left(\sqrt{\frac{-j_{\text{ext},i}}{j_{s,i} - j_{\text{ext},i}}} - 1\right) \frac{j_{s,i} - j_{\text{ext},i}}{12\tau_i}. \end{aligned} \tag{6}$$

The constant $c_{41,i}$ is used before and the constant $c_{42,i}$ after the jerk extreme. The solution for $c_{5,i}$ is given in the following equation (7):

$$c_{5,i} = \frac{12c_{4,i}^2}{5(j_{s,i} - j_{ext,i})}. \quad (7)$$

If $j_{s,i}$ is equal to $j_{ext,i}$, the constant $c_{4,i}$ is equal to zero and the constant $c_{5,i} = -j_{s,i}/(60\tau_i)$.
We determine if the jerk profile is before or after its extreme point, based on the last plan $S_{x,t-1}$, the current acceleration $a_e$ and the current jerk $j_e$. Based on the last plan $S_{x,t-1}$ it is solved for the times where the jerk component is equal to the current jerk $j_e$. At the smaller time $t_1$ and greater time $t_2$ the accelerations $a_{e,t-1,1}$ and $a_{e,t-1,2}$ are evaluated based on the last plan, respectively. Those accelerations are compared with the current acceleration $a_e$.
If the current acceleration $a_e$ is equal to the acceleration $a_{e,t-1,1}$, the current jerk is before the extreme point. If the current acceleration $a_e$ is equal to the acceleration $a_{e,t-1,2}$, the current jerk is after the extreme point. If the current acceleration $a_e$ is not equal to both accelerations $a_{e,t-1,1}$ and $a_{e,t-1,2}$ the Ego stayed not on the last plan. In such cases the jerk is always before the extreme point.

#### 3.2.2 Constant Acceleration

Solving the trivial solutions leads to $c_{2,i}$ and $c_{3,i}$ by using the starting acceleration $a_{s,i}$ and jerk $j_{s,i}$. While in part two the algorithm varies the time length to hold the target acceleration $a_{t,1}$ in part four also the time span to reach the target velocity $v_{GF,n}$ with a certain target acceleration $a_{t,2}$ is determined. The solution is given in equation (8):

$$\tau_4 = \frac{v_{GF,n} - (v_3 + \Delta v_5)}{a_{t,2}}. \quad (8)$$

The velocity $v_3$ is the Ego velocity at the end of the third part of the section-wise defined function and the velocity $\Delta v_5$ is caused by the acceleration of the fifth part of the section-wise defined function.

### 3.3 Lateral Component

It is possible to start the lane change if the Ego has reached the height of the target gap. Therefore, the intersection point of the back of the Ego with the safety area of the back vehicle $t_{y,s1}$ of the target gap is determined. Moreover, the intersection point of the front of the Ego with the safety area of the front vehicle $t_{y,s2}$ of the target gap is evaluated. The earliest intersection point is considered as starting time given in equation (9):

$$t_{y,s} = \min(t_{y,s1}, t_{y,s2}) \quad (9)$$

Moreover, the lateral planning needs to be executed if the distance to vehicle in front $t_{y,e1}$ of the Ego or to a possible lane end $t_{y,e2}$ falls below the safety distance or the lateral planning time does exceed a maximum time $t_{y,e3}$ to change lane, defined by Lange et al. (Lange et al., 2013) to reach a comfortable driving behaviour, given in equation (10):

$$t_{y,e} = \min(t_{y,e1}, t_{y,e2}, t_{y,e3}) \quad (10)$$

To determine the intersection, it is evaluated if each part of the section-wise defined longitudinal component, considering the length of the Ego, is in front or in back of the safety area of the considered vehicle. If the intersection is in the constant acceleration part of the component, it is possible to determine the time analytically. Otherwise, to avoid a numerical solution, the end time of a part is considered for the times $t_{y,s1}$ and $t_{y,s2}$. Moreover, the start time of a part is considered for the times $t_{y,e1}$ and $t_{y,e2}$. The time to change lane $t_{TTLC}$ is given in equation (11):

$$t_{TTLC} = t_{y,e} - t_{y,s}. \quad (11)$$

To calculate the lateral component quintic polynomials, introduced in (Werling et al., 2010), are used.

### 3.4 Cost Function

The cost function evaluates the driving comfort caused by maximum longitudinal and lateral jerk, the parts of the section-wise defined longitudinal component and the distance to the target point at the end of the fifth part of the longitudinal component. The trajectory with the lowest costs per gap is chosen.
The cost of the longitudinal jerk $j_x(t)$ is given in equation (12), where $a_{tmax,1}$ and $a_{tmax,2}$ are the maximum acceleration of the first $a_{t,1}$ and second $a_{t,2}$ target acceleration:

$$g_1 = \left( \frac{\int_0^{t_5} |j_x(t)| dt}{|a_{tmax,1}| + |a_{tmax,2}|} \right)^2. \quad (12)$$

The cost of the lateral jerk $j_y(t)$ is given in equation (13), where $a_{ymax}$ is the maximum allowed lateral acceleration:

$$g_2 = \left( \frac{\int_0^{t_5} |j_y(t)| dt}{a_{ymax}} \right)^2. \quad (13)$$

To calculate the jerk $j_y(t)$ to evaluate the cost function the time to change lane is not constraint by $t_{y,e3}$, to reward solutions that already changed lane with a greater time distances to the lane end or possible vehicles in front of the Ego.

In the first planing step of the longitudinal component at most five section-wise defined parts are possible to reach the target velocity. With cyclical replanning, in every time step at most five section-wise defined parts can be added to the initial plan and therefore the possible number of parts of the section-wise defined longitudinal component with respect to the initial plan increases. Moreover, to accelerate or decelerate to a target acceleration in several steps implies the same total jerk as accelerating at once to the target acceleration. To keep the optimal trajectory at the initial plan and to gain as few changes in the acceleration profile as possible, the number of parts $p$ of the section-wise defined function, are evaluated as well, as given in equation (14):

$$g_3 = \left(\frac{p}{5}\right)^2. \tag{14}$$

Furthermore, the distance at the end of the longitudinal part of the trajectory $s_x(t_5)$ to the position to the gap's front vehicle or a possible lane end $s_t = \max(s_{GF}(t_5), s_{\max})$ is taken into account. The cost due to the target point are given in equation (15), where $t_s$ is the safety time for car following:

$$g_4 = \left(1 - \frac{s_t - s_x(t_5)}{v_{GF,n} \cdot t_s}\right)^2. \tag{15}$$

The overall cost per trajectory is the weighted sum of all introduced partial costs.

### 3.5 Algorithm

The algorithm samples the first and the second target acceleration $a_{t1}$ and $a_{t2}$, as well as the time span $t_2$ to generate the set of longitudinal components. Moreover the algorithm allows combinations of non-negative values for $a_{t1}$ and non-positive values for $a_{t2}$ as target acceleration for target gaps with a smaller target velocity than the Ego. If the target gap has a higher velocity it is vice-versa.

At first the algorithm samples $a_{t1}$. Thereafter, we determine if the Ego acceleration $a_e$ is equal to the target acceleration $a_{t1}$ and if the Ego jerk $j_e$ is equal to zero. In this case also $\tau_1$ is equal to zero and the starting states of the second part are equal to the Ego states. Otherwise we determine $\tau_1$ and $c_{x1}$ using the cubic acceleration equations. Second, the time span $t_2$ is sampled by the algorithm. Since we want to obtain one to five part-section wise defined trajectories, the time span $t_2$ starts from zero. The resulting times $t_1$ and $\tau_2$ are calculated according to equation (16):

$$t_1 = \begin{cases} 0, & t_2 = 0 \\ \tau_1, & t_2 > 0 \end{cases} \qquad \tau_2 = \max(0, t_2 - t_1). \tag{16}$$

The second part is a constant acceleration part. Therefore, $c_{x2}$ is determined by constant acceleration equations and $\tau_2$ by sampling the time $t_2$. Third, the second target acceleration $a_{t2}$ is sampled. The second target acceleration is used to accelerate the Ego to the target velocity of each gap. Here, the target vehicle is the front vehicle of each gap. If $t_2$ is equal to zero we try if the acceleration of the fifth part leads directly to the target velocity considering the end states of the second part as the starting states. If this does not occur the third part is similar determined as the first part. Next, the acceleration profile for the fifth part is extinguished. Therefore, the time span $\tau_5$ and the constants $c_{25}$, $c_{35}$, $c_{45}$ and $c_{55}$ are determined using cubic acceleration equations. Thereafter, it is possible to determine the fourth part $\tau_4$ and $c_{x4}$ using constant acceleration equations. By then, it is possible to determine the missing constants $c_{05}$ and $c_{15}$ of the fifth part.

All solutions that do not reach the blue-dashed area or are outside the grey-dashed area are neglected. Thereafter, for every longitudinal planning a lateral planing to change lane is made. All lateral plans where the acceleration exceeds a defined maximum are also neglected. Then, the costs for every trajectory are calculated. The algorithm returns the trajectory with the lowest costs per gap.

## 4 EVALUATION

The proposed planning algorithm is evaluated in a MATLAB simulation. Therefore, surrounding traffic is simulated with constant velocity and the Ego is assumed to follow the planned trajectory. Figure 6 shows the generated set of trajectories for the initial planning, for a lane change into a gap with a smaller velocity. Due to the vehicle in front of the Ego it is not possible to reach gap 1. Therefore, the algorithm only determines trajectories reaching gap 2. The thick black trajectory is the optimal one. To solve the situation, the car first accelerates to reach the gap and to gain enough time to change lane with small lateral dynamics. Second the car decelerates to target speed. Since the number of parts of the section-wise defined longitudinal component are considered, the Ego trajectory has as few changes in the acceleration profile as possible. Without the vehicle in front of the Ego or a lane end in a greater distance the Ego would only decelerate. To evaluate the cost function it is evaluated if the Ego stays on the initial plan over the whole maneuver. Therefore, the Ego is located cyclically on the next time step $t_{\text{step}} = 0.2\,\text{s}$ of the plan with the lowest costs, as illustrated in figure 7. At every time step

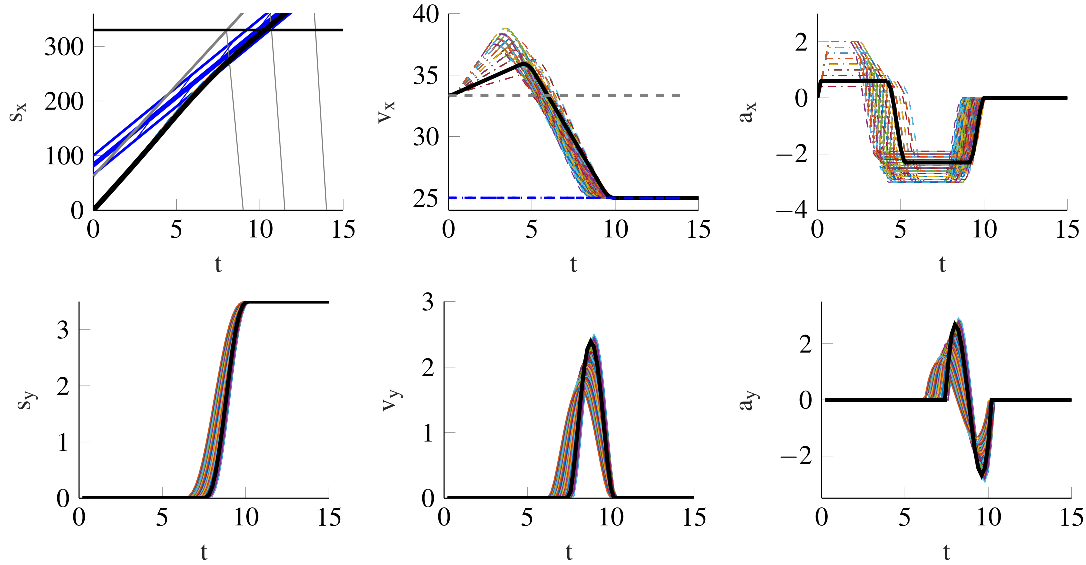


Figure 6: Longitudinal and lateral component of the set of trajectories in Frenet coordinates: one planning step is executed with $a_{t1min} = -2\,\mathrm{m/s^2}$, $a_{t1max} = 2\,\mathrm{m/s^2}$, $a_{t2min} = -3\,\mathrm{m/s^2}$ and $a_{t2max} = 3\,\mathrm{m/s^2}$. The discretization of the target acceleration is $a_{t1step} = 0.2\,\mathrm{m/s^2}$ and $a_{t2step} = 0.1\,\mathrm{m/s^2}$. The maximum time is $t_{2max} = 10\,\mathrm{s}$ and is sampled with $t_{2step} = 0.2\,\mathrm{s}$.

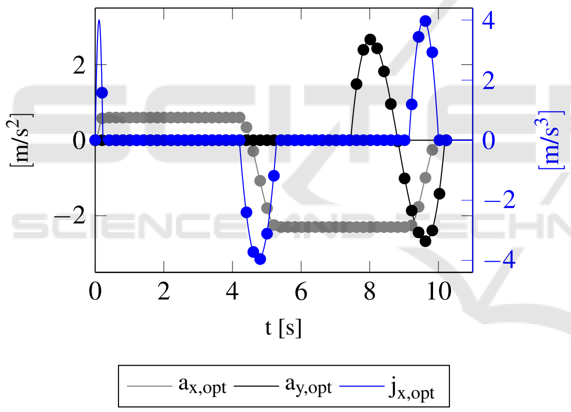


Figure 7: The figure illustrates the devolution of the Ego states $a_{x,opt}$, $a_{y,opt}$ and $j_{x,opt}$ by cyclic planning of the proposed algorithm. The dots visualize the Ego states at the different planning steps. The solid lines represent the initial planning.

the proposed algorithm is executed. The Ego stays on the initial plan over the whole maneuver.

In addition, to evaluate the computational costs, we evaluated different scenarios. Since, the calculation time depends not only on the discretization of the target velocities $a_{t1}$, $a_{t2}$ and the time to hold the first target acceleration $t_2$, but as well on the grey and blue dashed area. The simulation was running on an HP Elite Book with an Intel Core i-7 v Pro CPU at 2.60 GHz with 16.0 GB RAM. The time measurement is made over one scenario until the lane change was performed and the Ego is at target speed. In all evaluated

Table 1: The sampling is executed similar as illustrated in figure 6. The planning is performed due to one gap only.

| $s_{max}$ [m] | $t_G$ [s] | max [s] | mean [s] | std [s] |
|---|---|---|---|---|
| 400 | 1.8 | 0.2215 | 0.0805 | 0.0306 |
| 400 | 1.2 | 0.2143 | 0.0718 | 0.0364 |
| 400 | 0.6 | 0,1944 | 0.0626 | 0.0412 |
| 600 | 1.8 | 0.6996 | 0.2026 | 0.1783 |
| 600 | 1.2 | 0.3787 | 0.1071 | 0.0937 |
| 600 | 0.6 | 0.2463 | 0.0684 | 0.0566 |

scenarios, the initial distance is $s_{GF} = 100\,\mathrm{m}$ and the initial velocity of the front vehicle of the target gap is $v_{GF} = 25\,\mathrm{m\,s^{-1}}$ . The front and back vehicle of the target gap drives with constant speed. The initial Ego speed is $v_e = 33.3\,\mathrm{m\,s^{-1}}$. The size of the target gap, as well as the possible displacement on the starting lane influences the calculation time. Thus, the time distance from the front and the back vehicle of the target gap $t_G$ and the distance to the lane end $s_{max}$ is varied. A vehicle in front of the Ego is not taken into account. The mean, the maximum (max) and standard deviation (std) of the computation time is listed in table 1.

## 5 CONCLUSION

This article introduces a new approach for trajectory planning of automated vehicles for merging maneuvers on highways. To determine the dynamics necessary to reach several gaps a possible system architecture is proposed. To evaluate those dynamics, a set of trajectories is evaluated based on a cost function in each time step. The longitudinal component is composed of a five-part section-wise defined function, to accelerate to the target velocity of each gap. All solutions outside the drivable area are discarded. The lateral planning for potentially changing lane is executed, dependent on the situation, at a point in the future, considering safety distances. The cost function evaluates the longitudinal and lateral jerk, the number of parts and the distance to the target point at the end of the longitudinal component of the trajectory. Evaluations in MATLAB simulations show that the Ego stays on the same plan over the whole maneuver, executing the same planning algorithm in every time step, assuming constant behaviour of surrounding traffic. Moreover, a first evaluation of the computational costs of the algorithm is made.

To reach a higher discretization of one planning step, the proposed algorithm can easily be parallelised. Also, the gap prediction model can be extended to consider interactions between road users. Moreover, for use in real traffic, the model could be extended to deal with uncertainties in prediction or perception, to reward driving in less uncertain spaces.

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