From a Swarm to a Biological Computer

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Abstract: According to behaviourism, any swarm behaviour can be managed by outer stimuli: attractants (motivational reinforcement) and repellents (motivational punishment). In the meanwhile, there are the following two main stages in reactions to stimuli: (i) sensing (perceiving signals) and (ii) motoring (appropriate direct reactions to signals). Hence, by placing attractants and repellents at different sites we can manage and program the swarm behaviour. This opportunity allows us to design a biological computer – an abstract machine (i) with inputs presented by stimuli coming from attractants and repellents and (ii) with outputs presented by the swarm reactions to appropriate stimuli. This computer can be realized on different swarms differently. The point is that different matters are attractants and repellents for different animals. They differ a lot even for microorganisms. Nevertheless, their logic and mathematics are the same. Behaviourism means that (i) the complex of swarm behavioural patterns can be reduced to a composition of some elementary swarm patterns, (ii) if we know an appropriate attractant or repellent for each elementary pattern, then from a complex of attractants and repellents we can deduce a complex of patterns. Nevertheless, it can be shown that both assumptions are false. The point is that swarms are populations which behave as a distributed network, capable of responding to a wide range of spatially represented stimuli so that in their behaviours we can observe effects of neural networks with lateral activation and lateral inhibition mechanisms. As a result, behavioral patterns cannot be additive. In the paper it is discussed what we can do with this feature of swarm behaviour to program swarms.

1 INTRODUCTION

Within *swarm intelligence* (Bonabeau et al., 1999), (Kennedy and Eberhart, 2001), (Zelinka and Chen, 2017) we can formalize different swarm patterns to implement them in robotics. There are many well-studied forms of swarm intelligence: ant colonies (Dorigo and Stutzle, 2004), (John et al., 2008), bee colonies (Karaboga, 2005), (Karaboga and Akay, 2009), (Michener, 1969), fish schooling (Abrahams and Colgan, 1985), (Viscido et al., 2004), bird flocking and horse herding (Reynolds, 1987), bacterial colonies with a kind of social behaviour (Ben-Jacob, 2008), (Ingham and Ben-Jacob, 2008), (Ingham et al., 2011), (Ivanitsky et al., 1984), (Margenstern, 2011), (Passino, 2002), multinucleated giant amoebae *Physarum polycephalum* (Tsuda et al., 2004), etc. The main feature of all these systems is that their individual agents behave locally without any centralized control, but their interactions lead to the emergence of global behaviour of the whole group that cannot be reduced to subsystems additively.

Each swarm such as ants, bees, some social bacteria, *Physarum polycephalum*, etc. can solve logistic and transport problems very effectively (Kassabalidis et al., 2001). For instance, there is a collective navigation of bacterial swarms (Ariel et al., 2013), (Shklarsh et al., 2012) and there is an effective path finding by amoebae and a possibility of traffic optimization by them (Nakagaki et al., 2007), (Nakagaki et al., 2001), (Shirakawa et al., 2012), (Watanabe et al., 2011), (Whiting et al., 2015). Swarms can easily solve some complex (NP-hard) logistic problems: (i) the travelling salesman problem can be solved by ants (Dorigo and Gambardella, 1997) and by amoebae (Zhu et al., 2013); (ii) the Steiner tree problem can be solved by amoebae (Tero et al., 2010); (iii) the generalized assignment problem can be solved by bees (Ozbakir et al., 2010); (iv) mazes can be solved by amoebae (Nakagaki et al., 2000), (Ntinas et al., 2017), etc. As we see, even unicellular organisms can solve logistic problems effectively. Also, they can be involved in constructing algorithms for simulating the crowd evacuation (Kalogeiton et al., 2015)

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and for simulating transport systems such as the route systems in China (Adamatzky et al., 2013) and in the United States of America (Adamatzky and Ilachinski, 2012).

The main characteristics of any swarm consists in a possibility to optimising the own traffic in reactions to attractants and repellents. *Attractants* are things or sites in the environment, such as food pieces and sex pheromones, which attract individuals of swarm. *Repellents* are things or sites in the environment, such as predators, which repel individuals of swarm. The same swarm behaviour is observed even among human beings in the following two cases: (i) an addictive behaviour such as the behaviour of alcoholics (Schumann and Fris, 2017) or gamers – in this case the role of human attractants causing addiction increases strongly; (ii) an escape panic (Helbing et al., 2000) – in this case the role of human repellents like a terrorist act increases strongly, also.

Hence, by placing attractants and repellents at different sites we can manage and program the swarm behaviour. This opportunity allows us to design a *biological computer* – an abstract machine (i) with inputs presented by stimuli coming from attractants and repellents and (ii) with outputs presented by the swarm reactions to appropriate stimuli. This computer can be realized on different swarms differently. The point is that different matters are attractants and repellents for different animals. They differ a lot even for microorganisms. Nevertheless, their logic and mathematics are the same. In the biological computer we have two main stages: (i) *sensing* – when a swarm detects neighbour attractants and repellents; and (ii) *motoring* – when this swarm reacts to founded attractants and repellents, e.g. to exploit attractants and to avoid repellents.

In this paper, there are considered some basic expectations how we can design the biological computer (Section 2). These expectations correspond to behaviourism. Then it is shown that these expectations cannot be realizable at all in the behaviouristic way (Section 3) because we observe a kind of cognitive biases even at the level of unicellular organisms. Then there is proposed a context-based game theory which can be used for programming the biological computer (Section 4).

2 DESIGNING A BIOLOGICAL COMPUTER

Designing the biological computer is based on *behaviourism* as theoretical frameworks explaining the animal behaviour by reflexes produced by responses to external stimuli with a possibility to track individual's history of reinforcement and punishment implied by those stimuli. Each *reflex* is a direct response of animal to a stimulus, connecting the stimulus to a behaviour within two basic modes: either attracting (e.g. reinforcement) or repelling (e.g. punishment) coming from this stimulus. If we know what attracting and repelling matters are for the animal in fact, we can manage a control of its reactions by placing these matters. It means, simply put that *controlling stimuli causes controlling behaviour* according to behaviourism. So, we assume, the environment determines each animal behaviour.

In the theory of reflexes there are distinguished the following two types of responses/reactions/reflexes: (i) the *unconditioned response* to a stimulus (then an appropriate stimulus is called *unconditioned*); (ii) the *conditioning response* to a stimulus (then an appropriate stimulus is called *conditioning*). The unconditioned stimuli are represented by biologically active matters: either *attractants* (e.g. food or sex pheromones attracting an animal) or *repellents* (e.g. places evaluated automatically as dangerous by an animal). These stimuli are strongly connected to some direct responses from the very beginning, i.e. they are given *a priori*, just because of chemical processes of organisms. In the meanwhile, at the beginning, unconditioned stimuli are biologically neutral and assume no direct reactions of organism, i.e. they are rather ignored under standard conditions. Nevertheless, they can be associated with some unconditioned responses, too. Then they become non-neutral, but attractive (if they were associated with attractants) or negative (if they were associated with repellents).

At the time of Pavlov, one thought that a possibility of conditioning responses is one of the basic features of brain and nervous system. But it is known now through experiments that it is a fundamental feature of any adaptive behaviour including the adaptive behaviour of unicellular organisms, see (Shirakawa et al., 2011). Conditioning responses mean a memory, when an organism was taught that a neutral stimulus is a context of appearing an attractant or repellent. The knowledge of this context allows the organism to behave more adaptively in the feature. Thus, the possibility of conditioning responses is connected to a possibility of life to have a memory and then to be more adaptive to the environment.

Let us consider some examples of memory of unicellular organisms. From (Ball, 2008), we know that the amoeba *Physarum polycephalum* can learn the patterns of shocks at regular intervals, and then it changes its behaviour in anticipation of the next shock to come. In (Saigusa et al., 2008), this experiment

is performed in the following manner. Unfavorable conditions for *Physarum polycephalum* are presented as three consecutive pulses at constant intervals. Under these unfavorable conditions, the amoebae reduce their locomotive speed in response to each episode. When the amoebae move under the favorable conditions, they spontaneously reduce their locomotive speed at the time when the next unfavorable episode is expected to occur. This fact shows that we deal with the anticipation of impending environmental change. *For the amoeba the regular interval of shocks became a conditioning stimulus.*

Another experiment with the amoeba *Physarum polycephalum* was performed in (Shirakawa et al., 2011) to show that the temperature fluxes can become a conditioning stimulus for the amoebae. So, we know that the amoebae avoid cold temperatures and under these conditions they become slow. In the experiment there was shown that when the temperature changes had stopped, the amoebae slowed down, anticipating a cold flux. It is an evidence of *memory of temperature fluxes*.

All these experiments demonstrating a kind of memory of unicellular organisms can be explained by *chemotaxis*, the unconditioned response to some chemical cues (such as pheromones), that is different for static gradients and dynamic cues, see (Skoge et al., 2014), where this difference is shown for the migration of *Dictyostelium* cells in response to the source of traveling waves of chemoattractant during aggregation.

Hence, some primitive forms of conditioning reflexes (that is a primitive memory) can be detected even at the level of cellular proteins (including actin filaments) responding to the changes of the environment, such as shock intervals or cold fluxes. Not only neurons can cumulate conditioning responses, as Pavlov thought first, but also cellular proteins. This basic feature of chemotaxis to have a short memory is involved into the particle swarm optimization (Wang et al., 2011), (Zelinka and Chen, 2017) and in designing *organic memristors* – organic devices with a memory. For example, in (Traversa et al.,) for designing circuit models there was applied an ability of slime mould of *Physarum polycephalum* to both memorize the period of temperature and humidity variations and anticipate the next variations to come. Some other organic memristors are proposed in (Dimonte et al., 2014), (Erokhin, 2013), (Erokhin et al., 2012), (Pershin et al.,), (Pershin et al., 2016).

The idea of biological computer satisfies the presupposition of *radical behaviourism*: attractants, repellents and conditioning stimuli (connected to attractants or repellents) as environmental variables can completely control observable behaviours of animals (Baum, 2005), (Cheney and Ferster, 1997), (Skinner, 1976). This presupposition is used also in functional analysis of behavioural psychology to define relationships between stimuli and responses on the basis of explicating the following four elements: motivating operations, trigger of behaviour, behaviour itself, consequence of behaviour (Bandura, 1982). Thus, *the biological computer is a logic for radical behaviourism*. In other words, it defines how different Boolean combinations of attractants and repellents determine the patterns of animal behaviour, first of all their swarm behaviour.

3 BOUNDED RATIONALITY AND COGNITIVE BIASES

Radical behaviourism means that (i) the complex of swarm behavioural patterns can be reduced to a composition of some elementary swarm patterns, (ii) if we know an appropriate attractant or repellent for each elementary pattern, then from a complex of attractants and repellents we can deduce a complex of patterns. Hence, following this point of view we can suppose that the animal behaviour is *rational* – it is the same under the same conditions with the same complex of attractants and repellents. Each attractant is more or less preferable and each repellent is more or less avoidable. Then this rationality would mean that at each step if a swarm faces several attractants, then the most preferable attractant is always contained among chosen attractants; and if it faces several repellents, the most avoidable repellent is always contained among rejected repellents. In other words, the rational animal tries to maximize its satisfaction and, at the same time, to minimize its frustration. If preferences affect decisions indeed, then a Boolean complex of stimuli based on these preferences determine a complex of behavioural patterns. Is it so? Can we design the biological computer as a complex of Boolean compositions?

The assumption of rationality of agents is fundamental for game theory and decision theory. According to these theories, each rational agent always follows his or her preferences in his or her choices of items or strategies. So, the rationality of human beings in microeconomics is understood in the same way of utility-maximizing consumers and profit-maximizing firms. Each rational agent of the market follows the same decision model to maximize their own profits: I choose items to maximize my satisfaction as a consumer and I offer something to maximize my profits as a firm. We face a zero re-

flexion of rational humans there. Everyone is transparent for others, e.g. satisfies their expectations, if all them are rational. This transparency in decisions means that rational agents appeal only to facts which are the same for everyone, therefore these agents always can agree (see the Aumann's agreement theorem (Aumann, 1976), (Aumann, 1989)). Preferences are instances of facts that should be seen before meeting. Furthermore, due to rationality and zero reflexion of individuals we can always agree about long-term joint actions.

Nevertheless, there are many empirical evidences which contradict the assumption that, on the one hand, consumer behaviour is reasonably characterized as the maximization of expected lifetime utility subject to a budget constraint and conditional on the available information and, on the other hand, firm behaviour is characterized as the maximization of expected profit subject to investments and conditional on the correct programming of consumer behaviour. For example, Thaler ((Thaler, 1990), (Thaler, 1994)) shows the *bounded rationality* and impatience of consumers and, as a result, he proposes the behavioural life-cycle theory emphasizing self-control, mental accounting, and framing. Psychologists Amos Tversky and Daniel Kahneman established a research programme of studying *cognitive heuristics and biases*, i.e. the programme of studying contexts and patterns which influence on human decisions beyond rationality (Tversky and Kahneman, 1974) (i.e. beyond logical positivism and classical game theory and decision theory).

As we see, in behavioural economics and experimental psychology it was made evident through experiments that classical game theory and decision theory assuming the rationality of agents are too abstract, because real agents are too far from rational. The problem is that preference relations are not so strongly linked to decisions. As a consequence, a Boolean composition of preferences does not give a complex preference for a complex decision. Mathematically, it means that complexes of preferences as well as complexes of behavioural patterns are not additive.

Let us return to our task of designing the biological computer. From these examples taken from behavioural economics, we can assume that logical positivism is unapplied in the logical modelling of the swarm behaviour, as well – because we deal with a non-additivity of all natural behaviours. As a result, a Boolean composition of attractants and repellents cannot give an expected complex of behavioural patterns in the standard way of constructing inductive sets.

Hence, due to recent results in behavioural economics we know that even in human cognitions (which evaluated as the most rational among all animals) we cannot avoid cognitive biases and cognitive heuristics which are a substantial part in our decision making not only in everyday situations, but also in firms. Therefore, cognitive reflexion and cognitive biases are evaluated now as a natural mechanism of cognitions (Frederick, 2005), (Gigerenzer and Brighton, 2009).

Some primitive forms of cognitive biases and cognitive heuristics of unicellular organisms were found out by us within the project *Physarum Chip: Growing Computers from Slime Mould* (Adamatzky et al., 2012). Even unicellular organisms behave differently in stress or under favourable conditions. For instance, under the favourable condition (meeting attractants) bacteria behave more predictable in choosing a trajectory of motion – so, if a concentration of attractant increases, bacteria tumble less frequently. In the stress (meeting repellents) bacteria try to change a trajectory stochastically – so, if a concentration of repellent increases, bacteria tumble more frequently. As we see, we face a duality in basic reactions of bacteria to their environment. It is a kind of the most primitive "cognitive heuristics" detected at the bacterium level – to be more or less predictable in a favourable or stress situation, respectively.

Amoebae of *Physarum polycephalum* can behave both individually and collectively. Therefore, they can be considered a swarm, although they are unicellular organisms in fact, but with myriad nuclei. We have discovered that the main feature in perceiving external signals even by one cell due to actin filament networks is that the same signal can be perceived differently – it depends on the cell shape and many other internal circumstances. As a result, one unicellular organism bears more outputs than inputs in possible reactions to external signals. So, even one cell can have "cognitive biases" and behave quite unpredictably and irrationally under the same conditions. It means that amoebae of *Physarum polycephalum* and *Amoeba proteus* can modify their own elementary actions – they possess a kind of "free will".

4 CONTEXT-BASED GAMES

Let us start with considering preference relations on the level of neurons. It is known from neurobiology that there are two different synaptic effects: (i) excitatory effect (depolarization) that increases the membrane potential to make neuron more negative and to decrease the likelihood of an action potential

and (ii) inhibitory effect (hyperpolarization) that decreases the membrane potential to make neuron more positive and to increase the likelihood of an action potential. So, *lateral activation* is the structuring of a neural network so that neurons activate their neighbours to decrease their own responses and *lateral inhibition* is the structuring of a neural network so that neurons inhibit their neighbours in proportion to their own excitation. In other words, the more neighbouring neurons stimulated, the less strongly a neuron responds and the fewer neighbouring neurons stimulated, the more strongly a neuron responds. For example, the lateral activation decreases the contrast and sharpness in visual response to describe more explicitly all the edges and regions in the image. In this case we deal with the so-called *low-level vision*. The lateral inhibition increases the contrast and sharpness in visual response to perform an overall action-oriented interpretation of the scene. It is the so-called *high-level vision*.

Swarms are populations which behave as a distributed network, capable of responding to a wide range of spatially represented stimuli, for example, colonies of ants or fungi have such a behaviour. In their behaviours we can observe effects of neural networks with lateral activation and lateral inhibition mechanisms. It was shown experimentally by us in our project *Physarum Chip: Growing Computers from Slime Mould* that effects of lateral activation and lateral inhibition are detected in the plasmodium of *Physarum polycephalum*, the supergroup *Amoebozoa*, phylum *Mycetozoa*, class *Myxogastria*. This means that we can perform experiments how the plasmodium prefers items in the two different modes: within lateral inhibition and lateral activation. Let us recall that the plasmodium is an active feeding stage of *Physarum polycephalum* that moves by protoplasmic streaming and can switch its direction or even multiply in accordance with appropriate attractants (chemical signals which attract the organism) and repellents (chemical signals which repel it). This behaviour is intelligent and can be controlled by different locations of chemical signals attracting and repelling the plasmodium.

The true slime mould (plasmodium) of *Physarum polycephalum* has the following two distinct stages in responding to signals: first, the sensory stage (perceiving signals) and, second, the motor stage (action as responding). The effect of lateral activation in the plasmodium is to decrease contrast between spatial environmental stimuli at the sensory stage and to split protoplasmic tubes towards two or more attractants at the motor stage. The effect of lateral inhibition is to increase contrast between spatial environmental stim-

uli at the sensory stage and to fuse protoplasmic tubes towards one attractant at the motor stage. Hence, in the lateral activation the plasmodium prefers items by splitting tubes and in lateral inhibition the plasmodium prefers items by fusing the same tubes.

To program the biological computer we need a context-based game theory we are working on. Its main presuppositions:

1. Each *game* can be assumed *infinite*, because its rules can change depending on context – to be laterally activated or laterally inhibited.

2. Players can have *different levels of mood or reflexion*: at the level of lateral activation they are more unintentional (zero reflexion) and at the level of lateral inhibition they are much more intentional and a degree of intentionality depends on a degree of inhibition.

3. Some utilities can have *proto-symbolic meanings* or *symbolic meanings*. These meanings are results of accepting (proto-)symbolic values by some players, e.g. swarms can consider the same item being laterally inhibited or laterally activated. The higher symbolism of payoffs, the higher level of reflexion of appropriate players. On the zero level of reflexion, the payoffs do not have symbolic meanings at all. For consensus the players are looking for joint symbolic meanings.

4. *Resistance points* for players are reduced to the payoffs of the *zero level of reflexion*.

5. The *joint* (proto-)*symbolic meanings can change* through the game if a player increases his/her level of reflexion.

6. For any game there is *performative efficiency*, when all (proto-)symbolic meanings of one player are shared by other players.

In the case of these new game-theoretic assumptions we can calculate some aspects of (proto-)symbolic interactions by probabilistic tools in non-Archimedean numbers (Schumann, 2014). These new assumptions correspond to bio-inspired game theory as well as to theory of reflexive games. So, on the one hand, in bio-inspired game theory all the game moves are performed under the lateral inhibition or lateral activation conditions, therefore the swarm behaviour is not forecasted by additive measures. On the other hand, in reflexive games players can lie to each other, therefore their behaviours are not predictable by additive measures, too.

The context-based game is defined as follows. It is a tuple

$$\mathcal{G} = \langle (States_t)_{t \in \mathbf{N}}, Agt, (Act_{t,n})_{t,n \in \mathbf{N}}, (Mov_t^n)_{t,n \in \mathbf{N}}, (Tab_t^n)_{t,n \in \mathbf{N}} \rangle, \text{ where}$$

- $States_t$ is a (finite) set of *states* at time $t = 0, 1, 2, \ldots$;

- $Agt = \{1, \ldots, k\}$ is a finite set of *players*;

- $Act_{t,n}$ is a non-empty set of *concurrent actions* with radius n at $t = 0, 1, 2, \ldots$, an element of $Act_{t,n}^{Agt}$ is called a *move* at time $t = 0, 1, 2, \ldots$;

- $Mov_t^n : States_t^n \times Act_{t,n}^{Agt} \to 2^{Act} \setminus \{\emptyset\}$ is a mapping indicating the *available* sets of actions to a given player in a given set of states, $n > 0$ is said to be a radius of hybrid actions, a move $m_{Agt}^n = (m_A^n)_{A \in Agt}$ is legal at $\langle s_1, \ldots, s_n \rangle$ if $m_A^n \in Mov^n(\mathbf{s}, A)$ for all $A \in Agt$, where $\mathbf{s} = \langle s_1, \ldots, s_n \rangle$;

- $Tab_t^n : States_t^n \times Act_{t,n}^{Agt} \to States_{t+1}^n$ is the transition table which associates, with a given set of states at t and a given move of the players at t, the set of states at $t + 1$ resulting from that move.

Let us consider an example of context-based game for *Physarum polycephalum*. Let $States_0$ consist of attractants $a_1, a_2, \ldots, a_{m_0}$ and zero repellents, $States_1$ consist of attractants $a_1, a_2, \ldots, a_{m_1}$ and repellents $r_1, r_2, \ldots, r_{k_1}$, etc. Suppose, $Agt = \{1, 2\}$, i.e. we have just two players. At $t = 0$ there are no repellents. It means that both players are laterally activated and try to occupy all visible attractants at once. In our case, there are supposed m_0 visible attractants. So, at $t = 0$ we deal with concurrent actions with radius m_0. Let l attractants be neighbours for player 1 and $m_0 - l$ attractants be neighbours for player 2. Then we have l available actions for 1 at $t = 0$ and $m_0 - l$ available actions for 2 at the same time. At the end of step $t = 0$, l attractants are occupied by player 1 and $m_0 - l$ attractants are occupied by player 2.

At time $t = 1$, there are attractants $a_1, a_2, \ldots, a_{m_1}$ and repellents $r_1, r_2, \ldots, r_{k_1}$. It means that both players are laterally inhibited. Let l repellents be neighbours for player 1 and $k_1 - l$ repellents be neighbours for player 2. Then we have l available actions for 1 to avoid l places at once and $k_1 - l$ available actions for 2 to avoid $k_1 - l$ places at once, too. After avoiding all the dangerous places, both player 1 and player 2 decide whether there are some attractants from $a_1, a_2, \ldots, a_{m_1}$ at free places. It can be a situation that there are no attractants which can be occupied (they are too close to repellents). In this case, player 1 moves to one free place and player 2 moves to another fee place.

Hence, at time $t = 0, 1$ the rules of game changed. At $t = 0$ it was a lateral activation and $t = 1$ it was a lateral inhibition. Therefore, at $t = 0$, we have a zero reflexion of both players, but at $t = 1$ the level of reflexion increased. It means, while at $t = 0$ there were no (proto-)symbolic meanings for both players,

at $t = 1$ some (proto-)symbolic meanings appeared.

5 CONCLUSIONS

In accordance with behaviourism, any animal behaviour based on unconditioned and conditioning reflexes can be controlled or even managed by stimuli in the environment: attractants (motivational reinforcement) and repellents (motivational punishment). In the meanwhile, there are the following two main stages in reactions to stimuli: *sensing* (perceiving signals) and *motoring* (appropriate direct reactions to signals). In our research, the strict limits of behaviourism have been studied from the point of view of symbolic logic and algebraic mathematics: How far can animal behaviours be controlled by the topology of stimuli? In other words, how far can we design unconventional computers on the basis of animal reactions to stimuli?

On the one hand, we can try to design reversible logic gates in which the number of inputs is the same as the number of outputs. In our case, the behaviouristic stimuli for swarms are inputs and appropriate reactions of swarms at the motoring stage are outputs. It means that behaviourism can hold true, indeed. Nevertheless, on the other hand, at the sensing stage the same signal can be perceived so differently. The problem is that at the sensing stage even each unicellular organism can be regarded as a logic gate in which the number of outputs (means of perceiving signals) greatly exceeds the number of inputs (signals). It is connected to actin filament networks and other subcellular protein mechanisms perceiving the stimuli to react to them in various ways in accordance with the needs and tasks of the cell at the current moment. As a consequence, even one cell can resist outside influences. Hence, we face some strict biological limits in applying behaviourism. From the standpoint of symbolic logic and algebraic mathematics, this means that we cannot examine animal behaviours as conventional spatial algorithms, such as Kolmogorov-Uspensky machines. The mathematics of animal behaviours is much more complicated. The matter is that we should know how logical-mathematically we can design logic gates in which the number of inputs is far exceeded by the number of outputs. The universe of such incorrect mathematical "functions" is known for mathematicians well and called by them *non-well-founded*. In our research, some new mathematical tools for studying the non-well-founded universe of animal behaviours were proposed: context-based games and reflexive games.

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