Computer Vision and Deep Learning Tools for the Automatic Processing of Wasan Documents

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Abstract: ”Wasan” is a type of mathematical texts unique from Japan developed during the Edo period (1603-1867). These ancient documents present a wealth of knowledge and are of great cultural and historical importance. In this paper we present a fully automatic algorithm to locate a landmark element within Wasan documents. Specifically, we use classical computer vision techniques as well as deep learning tools in order to locate one particular kanji character called the ”ima” kanji. Even though the problem is challenging due to the low image quality of manually scanned ancient documents and the complexity of handwritten kanji detection and recognition, our pipeline including noise reduction, orientation correction, candidate kanji region detection and kanji classification achieves a 93% success rate. Experiments run on a dataset with 373 images are presented.

1 INTRODUCTION

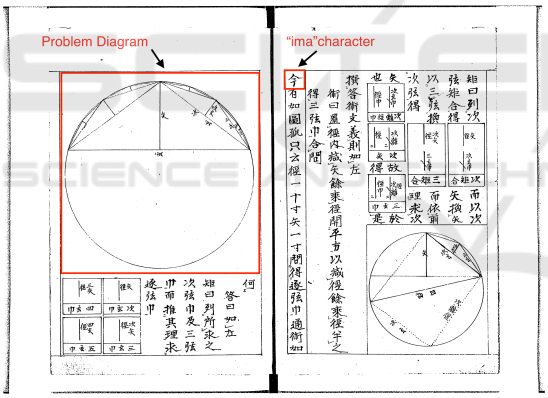

Figure 1: Wasan document example. The ”ima” kanji character marks the starting point in the description of a new problem and is typically placed beside the graphical description of the geometric problem.

”Wasan” is a type of mathematical texts unique to Japan developed during the Edo period (1603-1867). Japanese citizens have used Wasan documents to learn mathematics or as a type or mental training hobby ever since(Matsuoka, 1996; Martzloff, 1990).

This paper belongs to a line of research aiming to construct a document database of Wasan. This database will enable users to search and retrieve problems based on their geometric properties. This effort aims at dealing with the problem of the aging of researchers within the Wasan research area as it has been estimated that the whole area is in danger of disappearing within 15 years. By building an online-available database, searchable with state-of-the-art tools and built using modern computer vision and deep learning techniques, we aim at collecting the knowledge of traditional Wasan scholars and making it available for young researchers and educators. Specifically, we believe that the possibility of automatically describing Wasan problems in terms of their geometrical components will represent a powerful new tool of interpretation and description of these documents. It is our hope that this future electronic Wasan document data base will help find new insight on the historical and cultural importance of traditional Japanese mathematics.

In the current paper, we present the first of a set of tools that use computer vision and deep learning techniques for the automatic analysis of Wasan documents. Wasan problems are mostly geometric in nature. Their description is composed of a diagram accompanied by a textual description using handwritten Japanese characters known as *kanji*. Figure 1 presents an example of a Wasan document. The start of the description of a problem is marked by one particular kanji called ”ima” and representing the initial part of the sentence: ”Now, as shown,...”. This ”ima” (now) kanji is typically placed beside or underneath the graphical description of the geometrical problem and, thus, represents, a necessary starting point for any algorithm aiming at automatically processing these documents.

Therefore, in this paper our goal is to automatically determine the position of the ”ima” kanji in

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Wasan documents. In order to do this, we use several classical computer vision techniques (Agrawal and Doermann, 2013; Marr and Hildreth, 1980; Fernandes and Oliveira, 2008) as well as some of the most recent developments in the Deep Learning branch of Artificial Intelligence (Krizhevsky et al., 2012). Specifically we use the following steps:

First we preprocess the documents to alleviate some of the problems that they present in terms of image quality: On the one hand, we use a Hough line detector to correct possible tilts in orientation introduced during the process of digitizing the original documents. On the other hand, we use a noise removal step to clear small spurious regions of the image and improve the performance of the subsequent steps of the pipeline. In the second step of the pipeline the images resulting from the preprocessing steps are then used in blob detector algorithms in order to determine regions that are candidates to containing kanji characters. Each candidate region is transformed into an image and passed on to the final step. Finally, we run a Convolutional Neural Network (CNN)-based classifier in order to classify the single kanji contained in each of the images resulting from the previous step. The position of the images classified as containing the "ima" kanji constitute the final result of our algorithm.

Notice how the techniques used are general-purpose, so their adaptation to Wasan documents developed in the current work for the particular problem of locating the "ima" kanji will serve as basis for future applications. The rest of the paper is organised as follows: Section 2 presents details and references of the algorithms used throughout our image-processing pipeline. Section 3 discusses experimental results dealing with the kanji detection and classification steps. Finally, conclusions and future work are presented in section 4.

2 MATERIALS AND METHODS

Our Database was composed of images that had been previously manually scanned from original Edo period Wasan documents. Our documents were downloaded from the publicly accessible database in (YUWasanDB, 2018). This database includes "The Sakuma collection " consisting of the works of the famous Wasan mathematician "Aida Yasuaki" and one of his disciples know as "Sakuma". We had access to images corresponding to 431 books with each book containing between 15 and 40 pages with problem descriptions. For the purposes of this paper, we manually chose 373 images that contained the start of the description of mathematical problems and, thus, the "ima" character. Both the scanning procedure and the conservation state of some of the document resulted in some issues that make the automatic processing of the images challenging. In the following sections we will deal with two of these issues: Noise and image tilt.

2.1 Noise Correction

Noise can be defined as any unwanted information contained in a digital image. The digital image acquisition process is the primary source of noise. Noise can produce undesirable effects such as blurred objects, artifacts, unrealistic edges and unseen lines. It is almost impossible to totally remove noise without distorting an image, but it is essential to reduce it in order to perform image analysis.

Noise reduction in ancient documents is a particularly challenging issue (Arnia et al., 2015). One of the main problems that it entails is that it is often not possible to affect the binarization process of the documents. This makes it necessary to consider only operations on what typically are very low quality (in terms of resolution and information-to-noise ration) binary images (Agrawal and Doermann, 2013).

In order to reduce the noise present on the images, we used morphological transformations. Morphological transformations are a set of shape-based operations that process binary images. Morphological operations apply structuring elements or "kernels" to an input image and generate an output image. The use of these operations for noise removal in documents represents a very active research area (Barna et al., 2018; Goyal et al., 2018; Tekleyohannes et al., 2018).

The most basic morphological operations are erosion and dilation. They have a wide range of functionalities, such as removing noise, isolation of individual elements, joining disparate elements and finding bumps or holes in an image: The Dilation operation convolves an image A with a kernel (B). As the kernel B is scanned over the image, at each position we compute the maximal pixel value overlapped by B and replace one image pixel with it. This maximizing operation causes white regions within an image to grow. In a typical text document image, the white background dilates around the black regions of the text. On the other hand, Erosion computes local minima, causing the black areas to get bigger.

In this paper we were constrained by the large quantities of noise but also by the fact that kanji characters that we want to segment often present small parts that can be easily mistaken with noise. Consequently, we designed the following procedure using morphological operations:

First, an erosion operation is applied to the whole input image obtaining a new image with well-defined kanji. The goal was to obtain an image where all the elements belonging to each kanji are interconnected. Also we obtained much better defined lines. The erode operation is performed 8 times with a typical square kernel of 3x3 pixels. Subsequently, an algorithm to remove lines is used. This algorithm consists in convolving the image using a kernel with liner shape. When all kernel pixels are black, they are converted to white. Then the image obtained is used as a mask, applying a binary OR operation, to obtain a new, cleaner image. In the next step, blobs which have a small enough size, so that they are not confused with any element of a kanji, are removed. Then, the first step is applied again to this new image in order to interconnect the elements of each kanji. Similarly to the fourth step, the blobs which have a size smaller than an eroded kanji are removed. Finally, and similar to the third step, the image obtained in the sixth step is used as a mask to obtain the final resulting image.

2.2 Orientation Correction

The second issue that our data presented was that during the process of manually scanning the original documents, small tilt angles were introduced. This represented a problem as the structure of Wasan documents relies on some orientation-dependant characteristics. Specifically, the position of the diagram explaining the problem respect to the ”ima” character that we want to detect is either placed immediately over or just to the right of the character. Consequently, properly identifying the vertical and horizontal directions in the image is an important step towards further automatic processing of Wasan documents.

Wasan documents often present ”pattern lines”. These are vertical and/or horizontal lines that helped the writers maintain a coherent structure and orientation over each of the pages. In our case, these lines clearly show the tilt angle introduced during the scanning process as, instead of following the vertical direction on the scanned image their slope deviates slightly from it. As the noise removal process also removed these lines, we started this orientation correction process from the original images:

Taking into account the peculiarities of Wasan documents and the quality limitations in our images, we focused on finding the lines that used to indicate the vertical direction. Thus, our orientation correction procedure, can be divided into three main parts 1) Line detection 2) Line Filtering to remove lines that have slopes too far from the image vertical 3) Among the resulting lines, detection of the main lines in the image by grouping those lines that were close in the image.

Figure 2 presents an example of the process. In the first step we detected lines in the image using a Hough Line transform algorithm. The Hough transform is a well known feature extraction technique dating as far back as 1962 that is frequently used in many applications in Computer Vision and related areas . For this work we explored the two versions of the Hough transform implemented in the *OPENCV* library (Bradski, 2000): Probabilistic and deterministic. The visible ink density of the original documents resulted in discontinuous lines at pixel level. This caused the deterministic version of the Hough transform to yield poor results. The probabilistic version (Matas et al., 1998) , though, was able to find many lines in our images. These lines, however, did often not contain information that was useful in terms of orientation as they were often made up of kanji parts and were very short. Consequently we filtered these results to consider only lines over a certain length threshold. We also filtered out the lines that were too far away from the image vertical as our goal was to find the original vertical lines and the tilt angles in the images were found not to be very large. Once a small number of lines was left, we observed that many of the smaller lines were actually part of longer lines that had some part in the middle that was either faint or had been totally erased. We grouped those lines by proximity of their endpoints. All grouped lines were then converted to a single ”summary line” that aimed at following the path of the original vertical in the images.

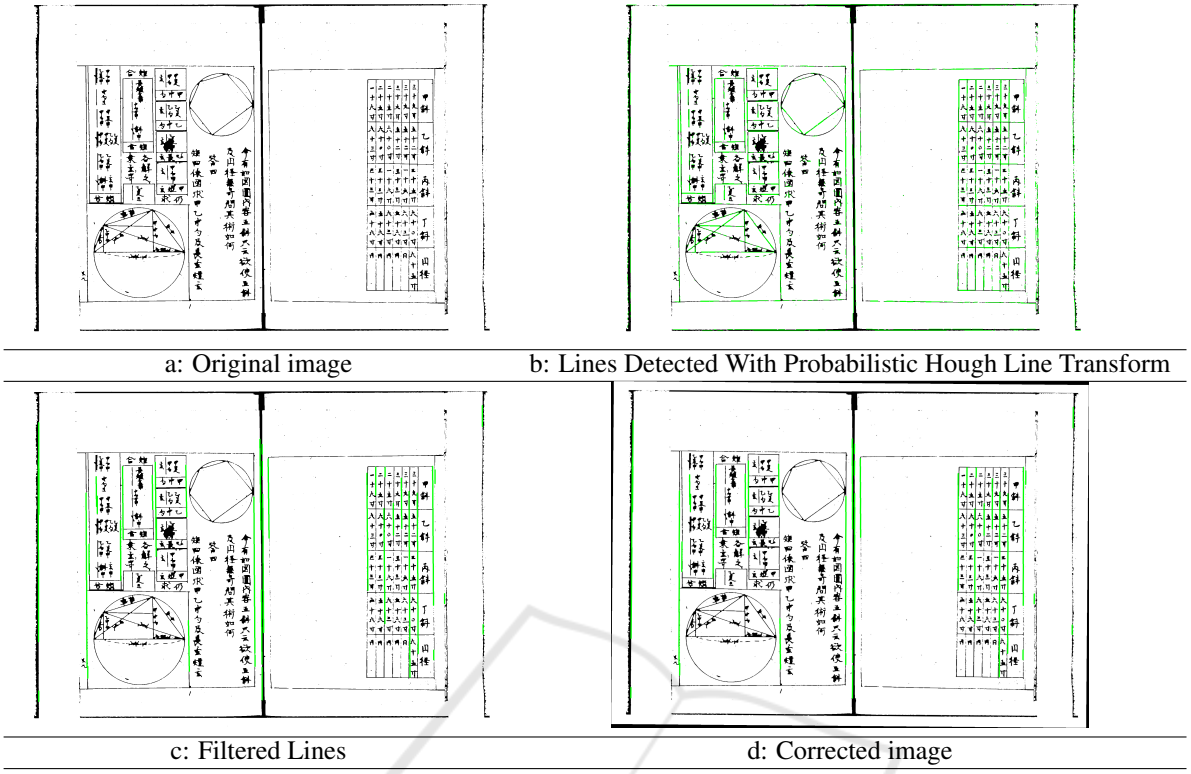
2.3 Individual Kanji Character Segmentation Kanji

In this section we describe the part of our pipeline that segments individual kanji characters.

kanji Characters are made up of a number of strokes ranging from one to around 20 strokes for the most common kanji. From the point of view of image processing, they are small regions made up of even smaller and disconnected regions. Furthermore handwritten kanji (such as the ones in Wasan documents) often present irregular or incomplete shapes.

In order to segment regions that are candidates to containing kanji (and, specifically, the ”ima” kanji that we are aiming at segmenting) we will use a computer vision technique known as *blob detection*. We tested four different blob detectors, Figure 3 presents an example of their results.

The first detector used was implemented using the *OPENCV* library (Bradski, 2000) while the other 3



a: Original image b: Lines Detected With Probabilistic Hough Line Transform

c: Filtered Lines d: Corrected image

Figure 2: Orientation Correction.

where implemented using the *SciKit* library (van der Walt et al., 2014).

Simple Blob Detector

This algorithm aims at exploiting differences in pixel intensity by thresholding the source image at different levels of gray and produce a a number of binary images. Then connected components are extracted from every binary image and their centers of masses are considered. These centers are then grouped among several binary with the aim of detecting blobs by proximity of the coordinates of the centers. Finally, blob connected components are merged into single blobs represented by a center and a radius.

Laplacian of Gausian LoG

The input image is convolved by a Gaussian kernel with t representing a scale parameter. The Laplacian of the space of the scales is considered:

$$\nabla^2 L = L_{xx} + L_{yy}$$

The goal is to highlight dark blobs (indicated by large positive values) or bright blobs (large negative values) of radius $r = \sqrt{2t}$ (Lindeberg, 2015).

Difference of Gaussians DoG

The Laplacian operator of the scale space can be approximated by finite differences:

$$\nabla^2_{norm} L(x,y;t) \approx \frac{t}{\Delta t}\left(L(x,y;t+\Delta t) - L(x,y;t)\right)$$

This approach is known as the difference of Gaussians (DoG) approach (Marr and Hildreth, 1980).

Determinant of the Hessian DoH

This operator is defined by considering the Hessian matrix of the scale-space representation L. The determinant of this operator

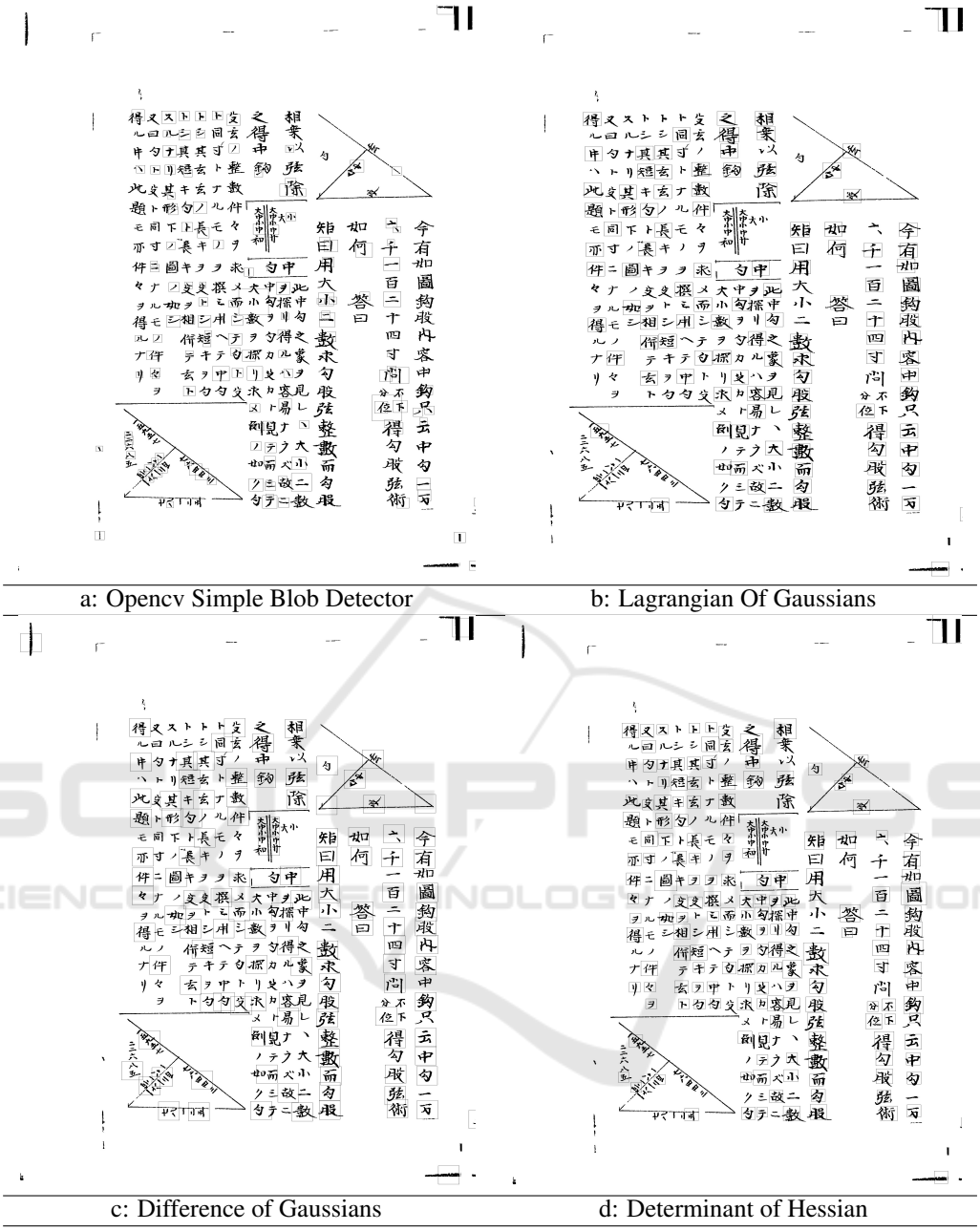
$$\det H_{norm} L = t^2 (L_{xx} L_{yy} - L^2_{xy})$$

is then used to detect maxima of this operator. The blobs in the image are defined from this scale-space maxima. Blobs defined using this technique are reported to present better properties under non-Euclidean affine transformations than the LoG and Dog operators ((Lindeberg, 2015)).

We also took advantage of the orientation correction and used the relative position of the blobs detected to merge small blobs that were very close and could be fit into a bounding box of a maximum given size along the y axis. This step slightly improved the results of the blob detector based on pixel intensities and had a very small effect on the results of the DoG, LoG and DoH blob detectors.

2.4 Classification of Kanji Images

In the final step of our algorithm we focused on the classification of the obtained kanji images to automat-



a: Opencv Simple Blob Detector

b: Lagrangian Of Gaussians

c: Difference of Gaussians

d: Determinant of Hessian

Figure 3: Kanji detector Methods.

ically locate the occurrences of the ”ima” kanji. Image classification has been for some time one of the areas where artificial intelligence approaches (more recently in their Deep Learning variants) have obtained best results.

Specifically, classification of kanji characters (both Chinese and Japanese) is a well established research area producing a large number of commercial applications as well as research papers. Among the first, we find, on the one hand the OCR applications aimed mainly at automatic text recognition (generally working with printed characters, see, for example, (OCRconvert, 2018)) and on the other software solutions aimed at students of the Japanese and Chinese languages (using both printed and handwritten characters, (see, for example (Yomiwa, 2018))).

Concerning the research papers, we will only mention the methods that mostly resemble the approach that we have used in this paper. Basically, we have followed closely the Deep Learning, CNN

approach used in (Tsai, 2016; Simonyan and Zisserman, 2014) and also broadly followed in (Grębowiec and Protasiewicz, 2018) in terms of the architecture. However, we have also considered data augmentation following (Cireşan et al., 2010) and used the publicly available ETL Database (ETL, 2018) to obtain kanji image to retrain our deep learning architecture.

Convolutional neural networks (CNN) consists of layers that transform an input image into a value that is then used to classify it in one of the existing classes. In our case, each kanji candidate image is assigned to one of the existing kanji classes. These layers contain element-wise operations to introduce non-linearities and hence increase the capacity of the network to represent concepts as complex as the shape of a handwritten character. See (Tsai, 2016) for a description of the different types of layers and their function. Regarding the shape (architecture) of our network, we followed the ideas described in (Tsai, 2016; Simonyan and Zisserman, 2014). Convolutional layers are followed by activation and max-pooling layers. This is repeated with convolutional layers of increasing size. Finally, a fully-connected layer is used before computing the final class scores with a softmax function. Non-linearity is ensured by placing ReLU layers both after every convolutional layer.

In order to train our network we used images of handwritten Japanese characters obtained from the ETL Character Database. Specifically we used the images corresponding to 876 different kanji characters contained in file ETL-8. We also explored the possibility of using Elastic distortions for data augmentation as presented in (Cireşan et al., 2010). However, we were not able to improve our classification results. The most likely explanation is that as our training dataset already has a large enough number of images per class (around 80).

3 RESULTS AND DISCUSSION

In this section we will analyze the performance of the algorithms presented throughout the paper. All tests were run on a 2.6GHz computer under a linux Ubuntu environment with an NVIDIA GTX 1080 graphics board. The computer vision techniques were implemented using the SciKit (van der Walt et al., 2014) and OpenCV (Bradski, 2000) libraries. The CNN presented where implemented using Keras and Tensorflow.

Regarding the images used in the experiments, all 373 images containing the "ima" character were used. In the experiments that required manual checking, a subset of images was chosen due to the long time needed for manual checks. Specifically, this involved 25 images per method in for the counting of correctly detected kanji and false positive kanji in section 3.2 and 100 images per method for the rest of checks in section 3.2 (detection of "ima") and all the checks in section 3.3. Some of these images contained more than one "ima" kanji.

3.1 Running Time Considerations

Our pipeline consisted of four steps, the two initial steps were common and took, on average: 3.72s (noise removal) and 7.73s (orientation correction). Figure 4 presents these two runtimes as well as the subsequent kanji detection (for the four methods studied) and the classification of the resulting candidate kanji regions with the used CNN. All times are presented in seconds. Apart from these times, training the CNN took approximately 190 minutes. This process only needs to be performed once and, thus, we have left it out of the runtime discussion.

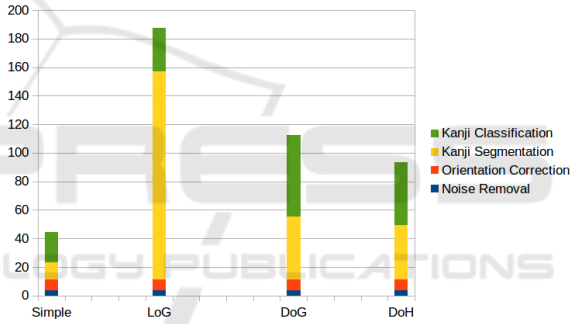


Figure 4: Runtimes for the 4 algorithm variants tested.

The simple blob detector was fastest overall, although, as we will see, it did not produce satisfactory results. For the rest of methods, the kanji detection step (using blob detectors) determined the final time, with methods producing more detection (including false positive ones) taking longer to run both the detection and classification steps. Overall, even the slowest pipeline took about 3 minutes to run in a single image (LoG, detection 145.49s, classification 30.70s). This shows that the automatic processing of large quantities of Wasan documents is feasible although code optimization and multi-process execution will need to be considered.

3.2 Kanji Detection Experiment

In this first experiment we aimed at determining which of the blob detectors produced best results for our problem. We automatically counted the number of blobs detected and then proceeded to manually

Table 1: Blob detector methods for kanji Segmentation. Summary Table.

Method Name 1	Detected kanji %	False Positive kanji %	”ima” kanji detection %
Opencv Simple Blob Detector	63.30	4.43	69.35
LoG	71.57	2.42	96.71
DoG	74.21	3.06	93.54
DoH	86.91	1.50	83.87

count the number of kanji that had not been detected. We note these ”False negative” (FN), detections. We also manually counted what of the detected blobs did not contain any kanji and noted them ”False positive” (FP), detections. Using these values we were able to determine the actual number of kanji present in each page. Then we were able to compute the percentage of kanji detected in every page (over the total number of kanij). The average over the test set for this correct detection percentage is presented in Table 1 second column. The ratio of FN detections over the total number of kanji is presented in column three. Finally, the average percentage of occurrence of the ”ima” kanji correctly detected is presented in column four. A typical page had around 200 kanij characters with only one occurrence of the ima kanji although in a small number of cases the ”ima” kanji appeared twice in the same page.

The table shows how the best performance in terms of the detection of the ”ima” kanji was obtained by the LoG blob detector. It’s failure rate of 3.29% ranked higher above all method and illustrates the adequacy of scale space blob detectors for our very specific problem. As this was the main goal of the current paper, this is the method that we will use in our full-pipeline experiment in section 3.3.

To complement the analysis, we observe that in terms of the detection of all the kanji in a page, the DoG and, specially, the DoH methods performed better than the LoG blob detector. however, specially for the DoH method, this higher capacity to detect kanji regions (expressed both in higher percentage of detected kanji and lower FP kanji detection) did not result in a better rate of location of the ”ima” kanji. This was slightly surprising to us as in the data analyzed the DoH method seemed to perform best over all but had a tendency to ”miss” the one kanji that mattered in our application. This is likely explained by the shape of the ”ima” kanji, which is small and made up of four or five (depending on the writer) small connected regions. Visual inspection of the results seemed to indicated that the DoH method was very good at detecting kanji regions but had some trouble with smaller kanji, specially if they were not totally surrounded by other kanji as is often the case with ”ima”.

This differences in behaviour of the different blob detector method indicate the possibility that combining their outputs might result in future work in improved results for ”ima” recognition as well as the development of an automatic tool for the automatic extraction of the full text of the description of Wasan problems.

3.3 Classification of the ”ima” Kanji

After considering the LoG blob detector as the best suited to locate the ”ima” kanji, in this section we analyze the results of running its output through the last step of our pipeline.

Specifically, the CNN described in section 2.4 was asked to classify every candidate kanji region in one of the 876 kanji classes that we had trained it with. During the trainig of the CNN the system achieved a 99.12% accuracy that is in line with previous kanji recognition applications (Tsai, 2016; Cireşan et al., 2010; Grębowiec and Protasiewicz, 2018).

Figure 5 shows the percentage of: 1) Success, defined as the percentage of correct ”ima” kanji detection without wrongly detection any other kanji as ”ima” at the same time. 2) Success with FP, the percentage of detections that happened along with the misclassified of some other kanji as ”ima”. 3) Detection Failure: The percentage of failed location of the ”ima” kanji because it was not inside a kanji candidate region and 4) Classification Failure: The percentage of cases in which a properly detected occurrence of the ”ima” kanji was classified as some other kanij class.

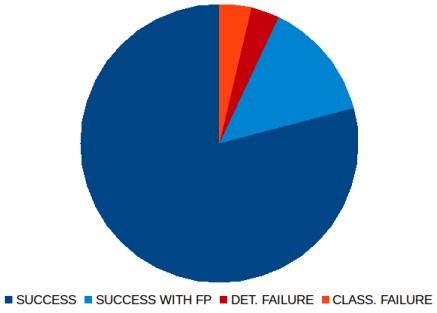


Figure 5: Summary of results.

The results show how the high success of the detection of the ”ima” kanji (93.02% total success rate

including success with FP) makes our method suitable to achieve our goals in this paper. There is, however, still room for improvement. Specifically, the 13.95% FP rate might result in extended human review time over the results to filter the wrong classification. The most likely explanation for these classification errors is that the CNN that we used had not been trained to include all possible kanji variants. We checked that the kanji most often confused with "ima" did not have its own class properly defined. Consequently, extending our kanji image library will likely result in a reduced false positive misclassification rate. The failure rate of 6.97% is acceptable in the context of our application and can be attributed in almost equal parts to failure in classification and detection. Regarding the failure due to the misclassificatio of a correctly detected "ima" kanji, a possible issue is that the ground truth present in the ETL database (ETL, 2018) corresponds to modern writing of the kanji which differ slightly from some of the ways in which they were written in the Edo period.

4 CONCLUSIONS AND FUTURE WORK

We have presented what is, to the best of our knowledge the first work using computer vision tools and deep learning for the analysis of Wasan documents. We have presented a tool to select an important structural element, the "ima" character that indicates the position of the start of the textual problem and lies beside or underneath the graphical description of the problem. The application presented in this paper represents, thus, the first step in the construction of a data base of Wasan documents that can be searchable in terms of the geometric properties of each problem. The results presented in this work illustrate how our algorithms are able to overcome the image quality problems present in the images (mainly noise in data, low resolution and orientation tilt) and locate the occurrences of the "ima" character with a high success rate of 93.02%. Among the issues that remain is the presence of a 13.95% of false positive detections and the 6.97% of cases in which the detection failed.

Both issues will be addressed in future work. In order to reduce False positives, we will widen our Kanji database so that it includes more than the present 876 kanji classes. Regarding the cases in what the algorithm failed, roughly half where due to detection failure and the other half to classification failure. The first issue will be addressed by combining the results of the three blob detectors obtaining best results in Experiment1 (LoG,DoG,DoH). while the second can be addressed by tailoring the ground truth for the "ima" kanji so it is closer to the writing peculiarities of the EDO period. The experiments included in this paper also indicate that modifying the code used to produce the experiments presented to obtain the full text of the description of each Wasan problems is feasible with minor modifications of the current code.

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