Maximization of Profit for a Problem of Location and Routing, with Price-sensitive Demands

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Abstract: This article seeks to study and solve a problem of profit maximization of a company by defining the location of an optimal number of facilities, allocation and routing of vehicles, and costs for home delivery to meet the demand of its customers. This study is based on an article in which, for the first time, a problem of location and routing and maximization of utilities with price-sensitive demands is integrated. This problem, unlike other studies that only minimize other metrics such as waiting times, route distances, and transportation costs, seeks a greater benefit by increasing profits by discriminating prices depending on the customer's location or adding an additional cost to retail sales. This paper presents a model for small instances based on the model proposed in the aforementioned article. Next, a two-phase heuristic is proposed that solves larger instances with a result close to that obtained in the previous article where a branch-and-price heuristic was used.

1 INTRODUCTION

In problems of transport and distribution, there are cases where decisions must be made that affect the supply chain in the long term, in the short term and daily; these are called strategic, tactical and operational decisions, respectively (Fazayeli et al., 2017). The location and routing problem (LRP) includes two types of problems fundamental to supply chain management: the problem of facility location of and the problem of vehicle routing. Because both problems are related, LRP problems have recently become an interesting area of study (Archetti et al., 2017).

The classic problems of location and routing, where a set of potential distribution centers, opening costs, identical vehicles and a set of known demands are defined, consist of selecting which distribution centers will be opened, assigning customers and determining the route of each available vehicle. The objective is to minimize the total cost, which includes the cost of opening each center, the fixed cost of the vehicles and the total cost of transportation (Prodhon and Prins, 2014). Panicker et al. (2018) solve a location-routing problem using an ant-colony optimization heuristic, for instances generated by the authors. Ferreira and Alves de Queiroz (2018) solve a LRP using heuristics based on simulated annealing with good results for instances of up to 200 customers.

There are several extensions to the Location-Routing Problem in the literature. Sarham et al. (2018) developed a column-generation approach to solve the LRP with time windows. Chen et al. (2018) investigate a LRP with full truckloads for designing a low-carbon supply chain. They developed a hybrid heuristic combining NSGA-II and Tabu Search. Guo et al. (2018) study a closed-loop supply chain where location, inventory, and routing decisions must be made. They propose the mathematical model and develop a hybrid heuristic that combines simulated annealing and genetic algorithms to solve several instances.

Ahmadi-Javid et al. (2018) studied a problem that, unlike the classic problems of location and routing, besides minimizing costs, seeks to maximize profits managing delivery costs considering the demand and location of the customers. However, due to the complexity of this situation, in addition to proposing a mixed integer linear model (MILP), the authors developed a branch-and-price heuristic to obtain a feasible solution for large instances.

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For this study, an alternate two-phase heuristic is presented to solve the problem proposed by Ahmadi-Javid et al. (2018). The heuristic consists of pre-grouping the customers, and assigning to the nearest centers to create small instances that can be solved with the MILP model. Despite its simplicity, this heuristic achieves results similar to those obtained with branch-and-price heuristics.

2 LITERATURE REVIEW

Ahmadi-Javid et al. (2018) made reference to Laporte (Albareda-Sambola et al., 2007), who has contributed to the study of this problem with different formulations, solution methods and computational results, as well as other authors (Nagy and Salhi, 2007; Borges Lopes et al., 2013). In addition, they mentioned recent investigations of variants of this model, such as a model for a stochastic supply chain system (Ahmadi-Javid and Azad, 2010) and a location and routing model with production and distribution with risks of interruption in a supply chain network (Ahmadi-Javid and Seddighi, 2013).

In most cases, the problems of location and routing establish that all customers must be visited and their demands must be met (Ahmadi-Javid et al., 2018). However, the problem seeks to maximize the total utility, minimizing the cost of transportation and the cost of establishing distribution centers without necessarily having to attend to all their potential customers. Likewise, there are other articles (Nagy and Salhi, 1998), where it is allowed to visit the customer more than once, others where some do not need to be visited (Averbakh and Berman, 1994; 1995), and others where some randomly selected customers are not visited (Albareda-Sambola et al., 2007). This is because sometimes the cost exceeds the income generated by serving them.

Although the model of Ahmadi-Javid et al. (2018) is one of the few investigations on problems of location and routing with multi-objectives, there are other similar models such as the Traveling Salesman Problem (TSP) or Vehicle Routing Problem (VRP), which are among the most studied combinatorial optimization problems. In addition, there are extensions of these that make decisions based on the profits generated by visiting only certain customers, such as in the case of the traveler with profit (TSPPs), or the problem of vehicle routing with profits (VRPPs).

Of the previously mentioned models, the one that most resembles maximization of profit for a problem of location and routing (Ahmadi-Javid et al., 2018) is the problem of vehicle routing with profits. Unlike the classic problems, the customers that will be attended must be selected, since the set is not defined, and the route in which these customers will be served, taking into account how attractive the customer is for the profit that can generate (Archetti and Speranza, 2014).

However, in this case of routing with profits, only one distribution center is available. The problem of routing vehicles with multiple deposits is a variation of VRP, which has the same objectives. However, it has several vehicles and potential distribution centers (Archetti et al., 2014). Aras et al. (2011) presented a selective model of vehicle routing with multiple deposits and prices, where only those customers that are profitable are served.

Another particularity of the model proposed by Ahmadi-Javid et al. (2018) is that the known demand of each customer changes according to the assigned price. They mention that price sensitive demand has been integrated into different models. However, it is the first time that sensitive demands are taken into account for a location and routing problem.

In addition, they mention that the model most similar to theirs is that of Archetti et al. (2014) who solve a VRP with profits, which consists of maximizing the difference of the obtained profits and the cost of transport, using a single distribution center and a fleet of identical vehicles. Unlike the study by Archetti et al. (2014), they model a problem with the same objectives, but with several potential distribution centers and with price sensitive demands, making their problem more complex since each center can offer different prices to each customer.

3 PROBLEM DESCRIPTION AND MATHEMATICAL MODEL

In this Profit-Maximization Location-Routing Problem (PM-LRP), we have a set of possible locations for distribution centers, with equal capacity and a set of locations for potential customers with their respective initial demands. Also, a number of available vehicles with equal capacity is defined. The objective is to maximize profits, minimizing the total cost of opening centers and transport. To achieve this, it is necessary to determine which centers to open, which customers to assign to each center with the possibility of not attending to all, the prices assigned to each customer taking into account the variation in demand based on the price assigned, the vehicles to each distribution center, and the route of each vehicle by visiting the selected customers only once.

To decide the delivery prices for each customer, Ahmadi-Javid et al. (2018) use a space price policy, which consists of assigning an equal retail price for all customers adding an additional cost depending on their location. This added cost per delivery is an additional percentage of the retail price, which is called markup. For this model 6 or 11 levels of markup are used depending on the instance ranging from the percentages p_l 0.1 to 0.2, in intervals of 0.1 for the instances of 6 markups, or in intervals of 0.05 for instances of 11 Markups. Therefore, the pricing decision is to define the level of markup l to add to overall price considering that initial claims assigned vary depending on the final price. This final price P_l is called the delivery price. This is a type of price discrimination that can only be applied if the exact location of the potential customers is known, and is given by $P_l = w(1 + p_l)$, where w is the retail price, and p_l the extra percentage at the level of markup l. For the demands to be modified depending on the final price, the following negative slope function was used (Greenhut et al., 1975):

$d_{il} = f(P_l) = d_i(a - bP_l)^{1/v}, \quad P_l < a / b$

Where d_{il} is the final demand of customer i with the level of markup l, P_l the end price of the product with the markup level l, and a, b and v are positive parameters which for this model were established as 10.1, 1.5, and 0.25, respectively.

Mathematical model proposed by Ahamid-Javid et al. (2018)

Sets

I Set of potential customers

H Set of potential distribution centers

L Set of markup levels

K_0 Set of Available Vehicles

Auxiliary Sets

M Set of all possible nodes (Distribution centers and customers), i.e., $M = I \cup H$

K_h Set of $|K|$ virtual vehicles assigned to the distribution center h

$K_h = \{v_1^h, \dots, v_{|K_0|}^h\},\ h \in H$

K Union of $|H|$ vehicle sets K_h, i.e.

$$K = \bigcup_{h \in H} K_h$$

A Set of triple arcs (i, j, k), where vehicle $k \in K$ can travel from node $j \in M$

Parameters

c_{ij} Distance from node i to node j, $i, j, \in M$

s Fixed cost per unit of distance

Cap^v *Vehicle capacity, same for all vehicles*

Cap_h^{DC} Capacity of Distribution Center h, $h \in H$

w Base price of the product

p_l Percentage associated with the level of markup l, $l \in L$

P_l Delivery price per unit of product associated with the markup level l, which is obtained by $P_l = w(1 + p_l), l \in L$

d_{il} The demand of the customer i to which the extra percentage of the level is charged markup l, $i \in I$, $l \in L$

F_h Fixed cost of establishing a distribution center $h, h \in H$

h_k The distribution center to which the vehicle k is assigned, i.e., $k \in K_{h_k}, k \in K$.

Decision Variables

x_{ijk} Binary variable that becomes *1* if node j is visited just after node *i* by vehicle *k*, or *0* otherwise, $(i, j, k) \in A$

y_{ikl} Binary variable that becomes *1* if node i is visited by vehicle *k* with the markup level *l*

t_h Binary variable that is used, *1* if distribution center *h* is selected to be set or *0* otherwise. , $h \in H$

u_{ik} Non-negative auxiliary for customer *i* used in MTZ sub-tour elimination constraint of the virtual path of $k, i \in I, k \in K$

Objective Function

Maximize:

$$w \sum_{k\in K} \sum_{i\in I} \sum_{l\in L} p_l d_{il} y_{ikl} - \sum_{h\in H} F_h t_h - s \sum_{(i,j,k)\in A} \left(c_{ij} x_{ijk}\right) \quad (1)$$

Subject to

$$\sum_{k\epsilon K} \sum_{l\in L} y_{ikl} \le 1\ ;\ i \in I \quad (2)$$

$$\sum_{i\in I} x_{hij} - \sum_{i\in I} X_{ihk} = 0;\ h \in H, k \in K \quad (3)$$

$$\sum_{i\in I} x_{hik} \le 1; h \in H, k \in K \quad (4)$$

$$\sum_{j\in I, j\neq i} x_{ijk} + x_{ih_k k} = \sum_{j\in I, j\neq i} x_{jik} + x_{h_k ik} = \sum_{l\in L} y_{ikl};\ i \in I, k \in K \quad (5)$$

$$\sum_{i\in I} \sum_{l\in L} d_{il} y_{ikl \le} Cap^v;\ \ k \in K \quad (6)$$

$$\sum_{l\in I} \sum_{k\in K_h} \sum_{l\in L} D\, d_{il} y_{ikl} \le Cap_h^{CD} t_h;\ h \in H \quad (7)$$

$$\sum_{h\in H} \sum_{k\in K_h} \sum_{j\in I} x_{hjk} \le |K_0| \quad (8)$$

$$1 \le u_{ik} \le |I|;\ i \in I, k \in K \quad (9)$$

$$u_{ik} - u_{jk} + |I|x_{ijk} \leq |I| - 1;$$

$$i, j \in I, k \in K, i \neq j \quad (10)$$

$$x_{ijk} \in \{0,1\};\ (i,j,k) \in A \quad (11)$$

$$y_{ikl} \in \{0,1\};\ i \in I, k \in K\ \ , l \in L \quad (12)$$

$$t_h \in \{0,1\};\ h \in H \quad (13)$$

The objective (1) is to maximize the profit, which is the profit generated by serving customers minus the cost of establishing the distribution centers and the cost of transportation of the routes. Restrictions (2) ensure that each customer can only be visited once. Restrictions (3) define that the times a vehicle enters a distribution center is equal to the times it leaves it. Restrictions (4) ensure that each vehicle can only make one route. Restrictions (5) determine the connectivity of each route by determining the assignment of customers to each vehicle. Restrictions (6) limit the capacity of the vehicles and (7) ensure that the demand covered by each distribution center does not exceed its capacity. Restriction (8) limits the number of vehicles available. Restrictions (9) and (10), are Miller-Tucker-Zemlin sub-tour elimination constraints published by Miller et al. (1960), and restrictions (11-13) make the decision variables binary.

4 SOLUTION METHODS

Ahmadi-Javid et al. (2018), proposed the MILP model of polynomial size previously described to be able to solve small instances. This was programmed in CPLEX 12.3. Several major instances were run which were stopped after a few hours in order to obtain a feasible result, although the global optimum was not reached. To improve these results, Ahmadi-Javid et al. (2018), proposed a branch-and-price heuristic by previously creating an exponential size formulation of grouped sets using the decomposition of Dantzig-Wolfe, which simplifies the problem by dividing it into a master problem and several sub-problems. This model was programmed in C++.

4.1 Heuristic

As an alternative to solving this problem, a two-phase heuristic is proposed that aims to create sub-problems to decrease and divide the number of variables and restrictions in each phase. Previously, the MILP was programmed in LINGO to be able to verify the correct interpretation.

4.1.1 First Phase

The first phase consists of creating routes with the minimum possible demand, that is, with the highest level of markup taking into account the capacity restriction of each vehicle and distribution centers, but without taking into account the number of vehicles available. This phase is started by selecting the distribution center with the lowest sum of the distance between the nearest potential customers. Starting with the previously selected distribution center, a subgroup is created using the nearest neighbor algorithm. The stopping criterion for the heuristic of the nearest neighbor is executed when the sum of the minimum demands (markup 6 or 11) of the selected customers exceeds the capacity of the vehicle of that route, or the capacity of the distribution center taking account the demand covered by the routes previously assigned to that center.

Then the previously created group is taken to run a small instance with the MILP model, where it is established that only one vehicle is available to obtain a route. Due to the small number of variables, there is an optimal global solution for that combination, selecting the best route and the level of markup for each customer. Since the model in MILP is programmed to serve only those customers that are profitable, the customers that are not part of the result are taken into account for the pre-grouping of another possible route and the others that were assigned are eliminated. The creation of possible routes ends when all customers have been assigned to some route, when the capacities of all the distribution centers are to be exceeded by the sum of covered demands of the routes assigned to them, or when there are no longer profitable customers to attend.

Input data for the first phase:

I	Set of potential customers $i \in I$; coordinate x, coordinate y, initial demand
H	Set of Distribution Centers $h \in H$; coordinate x, coordinate y
Cap_veh	Capacity of vehicles
Cap_CD_h	Capacity of distribution centers
$Mark\_up\_lvls$	No. of markup levels = {*6,11*}

Calculations for the first phase:

1. Distances between all possible nodes ϵ M

   These distances are calculated with the Euclidean formula.

$$Distance = \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2} \quad (14)$$

2. Adjusted demands of all customers $i \in I$ for all markup levels $l \in L$,

$a = 1.10\ b = 1.5\ c = 0.25$

$$d_{il} = f(P_l) = di(a - bP_l)^{\frac{1}{v}} \quad , \quad P_l < a/b \quad (15)$$

3. Profit to cover customer demand $i \in I$ in the mark up $l \in L$

$$Profit_{ij} = ajdusted\ demand_{il} \ * p_{il} \quad (16)$$

4. Maximum number of customers to take into account for possible routes

$$Q = Mean\ sum\ of\ initial\ demands / capacity\ of\ vehicle \quad (17)$$

PHASE 1. HEURISTICS-CREATION OF ROUTES
START
Input: set customers, set distribution centers, vehicle capacity, capacity distribution centers, initial demand, markup levels set
r = 1
Do Until | I | = 0 or | H | = 0

1. **Select distribution center h using selection procedure Fig. 2**
2. **Create subgroup using subgroup creation procedure Fig.3**
3. **Solve MILP model, according to equations 1-13, with data from subgroup to generate r, with a single vehicle k**
4. **IF profit of r = 0, delete selected center h from H, return to step 1.**
5. Cap_CD_h **selected** $h \in H$= Cap_CD_h**selected- demand covered in route** r
6. **IF** $i \forall I$, **belongs to solution r, eliminate** $i\ from\ I$
7. **r=r+1**

LOOP
Output: set of routes R, profit of routes $U_r,\ r \in R$

Figure 1: Heuristic- creation of routes.

SELECTION PROCEDURE DISTRIBUTION CENTER
Input: coordinates set | I |, coordinates set | H |, Q
START

1. **Calculate Euclidean distances of $i \forall I, a\ h \forall H$ according to equation (14).**
2. IF **Q (eq. 17)** $\leq$ **|I|,** THEN **add distances of Q nearest customers to $h \forall H$,**

   ELSE **add all distances.**
3. **Select the center with the shortest distance added in step 2.**

Output: Selected center
END

Figure 2: Selection procedure for distribution centers.

SUBGROUP CREATION PROCEDURE
Input: coordinates set | I |, coordinates selected center, initial demand | I |, demand at mark level -up greater for set | I |, capacity distribution center selected
Accumulated capacity = 0
START

1) IF Cap_CD_h **selected** $\leq$ Cap_veh**,**

THEN Cap_veh **=** Cap_CD_h **selected**

DO WHILE Cap_CDh **selected <accumulated capacity**

1) **Create selected center distance matrix a** $i \forall I$
2) **Starting at selected center, select nearest neighbor according to algorithm.**
3) **Add nearest neighbor** $i \in I$ **to subgroup**
4) **Cumulative demand = cumulative demand + demand at the higher markup level of nearest neighbor from step 4 according to equation (15).**

LOOP
END
Output: subgroup set

Figure 3: Subgroup creation procedure.

4.1.2 Second Phase

The second phase consists of a model that aims to maximize the profit by selecting routes r created in phase 1 of the set R having as the sole restriction the number of vehicles available. The other restrictions of the problem are taken into account for the creation of said routes and subtracting the opening cost of each center if at least one route is selected in said center. This problem contains disjunctive constraints, since the binary variable that multiplies the cost of establishing a center takes the value of 1 when there is at least one selected route from that distribution center as shown in the constraint (20). The second phase was solved using Excel solver.

Input data for second phase:
r Set of possible routes assigned to CD_h $r \in R$, $h \in H$
H Set of distribution centers $h \in H$
$|K|$ Available vehicles
U_r profit of each route $g \in G$, $r \in R$
F_h Fixed cost of establishing distribution centers

Model Phase 2

From the output of phase 1 (Fig 1.) the following model is solved:

br binary which takes a value of
1 *if* route r is accepted
b_f *b*inary which takes a value of
1 if at least one DC h route is accepted

$Maximize$:

$$\sum_{r\in R} U_r b_r - \sum_{h\in H} F_h b_{fh}; \quad \forall\ r \in R,\ h \in H \tag{18}$$

Subject to

$$\sum_r b_r \leq |K| \qquad \forall\ r \in R \tag{19}$$

$$b_{fh} = \begin{cases} 1 & if\ \sum_r b_{rh} > 0;\ h \in H \\ 0 & if\ \sum_r b_{rh} = 0;\ h \in H \end{cases} \tag{20}$$

$$b_r = \{0,1\} \quad r \in R \tag{21}$$

$$b_{fh} = \{0,1\} \quad h \in H \tag{22}$$

The objective (18) is to maximize the total profit; that is, the profit of the selected routes minus the cost to open the distribution centers. Restriction (19) ensures that the accepted number of routes is equal to the number of vehicles available. Restriction (20) gives the value of 1 if at least one route assigned to the distribution center $h \forall H$ was accepted, or 0 if none was accepted. Restrictions (21-22) ensure that the variables of accepting a route and opening a distribution center are binary.

5 EXPERIMENTATION

In the instances used, the number of potential customers and the number of distribution centers available are first defined. Then the coordinates in x and y, and the initial demands of the customers. Then the capacities of the centers and the cost of establishing them are presented. Finally, the capacity of the vehicles, the available number and the cost per unit of distance are shown. For all instances, the base price $w = 5$ was established. Each instance was resolved with both levels of markup 6 and 11. To name the instances, the initial of the author of the instance was taken, followed by the number of available customers, the number of distribution centers and the level of markup. Table 1 shows the original names of the instances with their respective data. Since this problem is new, the instances were generated modifying LRP benchmark instances available in the literature. The original names of the instances are shown in Table 1.

Table 1: Instances.

Instance name	Original name	no. customers	mark up levels
Pe-12x2x6	Perl183-12-2	12	6
Pe-12x2x11			11
G-21x5x6	Gaskell67-21x5	21	6
G-21x5x11			11
G-22x5x6	Gaskell67-22x5	22	6
G-22x5x11			11
M-27x5x6	Min92-27x5	27	6
M-27x5x11			11
	Cap DC	Fixed cost	vehicle capacity
Pe-12x2x6	280	100	140
Pe-12x2x11			
G-21x5x6	15000	50	6000
G-21X5X11			
G-22X5X6	15000	50	4500
G-22X5X11			
M-27X5X6	9000	272	2500
M-27X5X11			

6 RESULTS

Table 2 summarizes the best solution of the objective value, and the number of variables for the eight instances in the four proven methods: the MILP in CPLEX, the branch-and-price heuristic by Ahmadi-Javid et al. (2018), the method in LINGO and the proposed two-phase heuristic. The last column shows the error percentage of the two-phase heuristic on the best solution found among the other methods.

For the smallest instance, the program was allowed to run until finding the global optimum; after 12 hours with 30 minutes, the overall optimum was obtained, seven hours after the model in CPLEX. In the results, a difference of 0.60 is shown, which may be due to decimals considered in each engine used. For all other instances, a limit of ten hours was established, and the program was interrupted in Lingo in order to find a feasible solution. For the two-phase heuristic it was not possible to measure the time, since a part was done manually in Excel, and another in LINGO.

Table 2: Results in objective value.

Instance name	CPLEX	B & B ALG	LINGO
Pe-12x2x6	71.08	71.08	71.68
Pe-12x2x11	96.66	96.66	87.9
G-21x5x6	17775.06	17859	17535.97
G-21x5x11	18290.59	18391.9	17595.64
G-22x5x6	8667.44	8927.72	8706.022
G-22x5x11	9097.83	9097.83	8873.885
M-27x5x6	2927.16	2927.16	2633.36
M-27x5x11	3543.58	3543.58	3207.085
	TWO PHASES	NO. OF VARIA BLES	% ERROR VS. BEST SOLUTIO N
Pe-12x2x6	65.28	1181	8.929%
Pe-12x2x11	91.09	1461	5.762%
G-21x5x6	17064.727	17168	4.447%
G-21x5x11	17520.293	19768	4.739%
G-22x5x6	8840.96	18368	0.972%
G-22x5x11	8936.687	21068	1.771%
M-27x5x6	2926.5927	24968	0.019%
M-27x5x11	3492.1874	28168	1.450%

For the percentage of error on the best solution found with the two-phase heuristic solution, it can be seen that the greater the number of variables, the percentage of error tends to decrease, behaving similarly when you have 6 or 11 associated markup levels.

On the other hand, the improvement due to increasing the number of markup levels is positive in all cases for all the methods used. However, there is no trend associated with the number of variables but rather, with another particularity of each instance, since the smallest instance and the largest one, have a much more significant increase than the two median-size instances.

7 CONCLUSIONS

The results of the heuristic were satisfactory. However, no result was better than that obtained in the heuristic proposed by Ahmadi-Javid et al. (2018). In spite of not being able to measure the time for the metaheuristic, it can be seen that a better result is obtained than in the MILP. As shown in Table 2, the percentage of errors decreases as the size of the instance increases. It may be that this method obtains better results with larger instances.

As in the reference article, implementing the differentiation of prices for each customer when the demands are price sensitive increases significantly to a greater number of markup levels. That is, this policy can increase company profits, however, it would be difficult to predict the behavior of the demands for each customer, making the problem less feasible for real cases.

For future research, we will code the heuristic in C++, in order to compare the execution times. Moreover, it is expected to analyze the effects of the sensitivity of the demand in the maximization of profit, to be able to apply a penalty according to those customers who should not be served.

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