4-valued Logic for Agent Communication with Private/Public Information Passing

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Abstract: Thus far, the agent communication has often been modeled in dynamic epistemic logic, where each agent changes his/ her belief, restricting the accessibility to possible worlds in Kripke semantics. Prior to the message passing, in general, the sender should be required to believe the contents of the message. In some occasions, however, the recipient may not believe what he/ she has heard since he/ she may not have enough background knowledge to understand it or the information may be encrypted and he/ she may not know how to decipher it. In this paper, we generalize those messages that require special knowledge as private information and formalize that the recipient does not change his/ her belief receiving such private messages. Then, we distinguish the validity of the information from the belief change of the recipient; that is, even though the communication itself is held and the information is logically contradictory to his/ her original belief, the recipient may not change his/ her belief. For this purpose, we employ 4-valued logic where each proposition is given 2 (usual true and false) times 2 (private or public information or not) truth value.

1 INTRODUCTION

In van Ditmarsch et al. (2008), there have been distinguished the following difference in agent communication.

- public announcement: every agent receives the same information.
- whisper: other agents notice there happens an information transmission among others but the contents cannot be seen.
- channel: one to one communication: other agents cannot notice there has been an information transmission.

In addition, in this work we would distinguish the public/ private message passing, that is, the recipient cannot read nor understand what is written. The most probable case is that the information is meaningless for the recipient because he/she does not have enough background knowledge to understand it, *e.g.*, the message might be written in an unknown foreign language. The second most probable case is that the message is encrypted and the recipient cannot decipher it; in the latter case a simple tip or a password may suffice to read it. In either way, we can generalize these cases into a category, that is, *private* information. Here, we distinguish the following two categories.

- the contents of the message is only privately understood.
- the contents of the message is publicly understood.

In this paper, we distinguish these two, introducing 4-valued logic; that is, we distinguish if the message passing is successful and the recipient surely has received the message (T/F), and if his/her belief is affected even though the message might contradict to the belief of the recipient, since the agent could not decipher the contents. Here, the communication may fail in three cases shown in Figure 1.

In the following Section 2, we summarize the fundamental mechanism of belief change by dynamic epistemic logic (DEL), that is by the accessibility restriction in Kripke semantics. In Section 3, we survey the history and application of 4-valued logic. In Section 4, we revise the belief change by private/ public information passing, and show the recursion axiom to the ordinary DEL, that is sound and complete. Finally in Section 5, we summarize our contribution.

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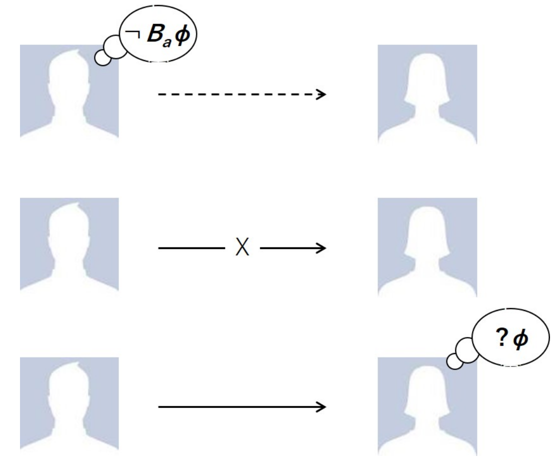

Figure 1: Three different miscommunication; top: the sender does not believe the contents of the information, middle: there is no channel between two agents, bottom: the recipient cannot decipher the contents.

2 SEMI-PRIVATE ANNOUNCEMENT IN DEL

Hatano et al. (2015) showed a modal epistemic language which has formalized agents' belief and channels.

2.1 Syntax

Let $\mathsf{PROP} = \{p, q, \ldots\}$ be a finite set of propositional variables and $\mathsf{G} = \{a, b, \ldots\}$ a finite set of agents. The language is generated by the following Backus-Naur form:

$$\alpha ::= p \mid \neg\alpha \mid \alpha \vee \alpha \mid c_{ab}$$

$$\varphi ::= \alpha \mid \mathsf{B}_a\,\alpha \mid [\alpha\downarrow^a_b]\varphi \mid \neg\varphi \mid \varphi \vee \varphi$$

where $p \in \mathsf{PROP}, a \in \mathsf{G}, b \in \mathsf{G}$ and α is an objective (non-modal) formula.

Here, c_{ab} means "There is a channel from agent a to agent b", $\mathsf{B}_a\,\varphi$ means "agent a believes φ". $[\alpha\downarrow^a_b]\varphi$ will be defined in section 2.3.

2.2 Semantics

A Kripke model $\mathfrak{M}$ is a tuple:

$$\mathfrak{M} = (W, R_{\mathsf{G}}, C_{\mathsf{G}}, V)$$

where W is a non-empty set of worlds, G is a non-empty set of agents, $R_{\mathsf{G}} = \{R_a \mid a \in \mathsf{G}\}$ and $R_a \subset W \times W$ is an accessibility of agent a on W, $C_{\mathsf{G}} = \{C_{ab} \mid a \in \mathsf{G}, b \in \mathsf{G}\}$ and $C_{ab} \subseteq W$ is a channel relation, and $V : Prop \to P(W)$ is a valuation function. Here, $C_{aa} = W$ for all $a \in G$ because each agent must have a channel to itself.

Given any model $\mathfrak{M}$, any world $w \in W$ and any formula φ, we define the satisfaction relation inductively as follows:

$\mathfrak{M}, w \models p$	iff	$w \in V(p)$
$\mathfrak{M}, w \models \mathsf{c}_{ab}$	iff	$w \in C_{ab}$
$\mathfrak{M}, w \models \neg\varphi$	iff	$\mathfrak{M}, w \not\models \varphi$
$\mathfrak{M}, w \models \varphi \vee \psi$	iff	$\mathfrak{M}, w \models \varphi$ *or* $\mathfrak{M}, w \models \psi$
$\mathfrak{M}, w \models B_a\varphi$	iff	$\forall u \in W((w,u) \in R \to \mathfrak{M}, u \models \varphi)$

Here, we say φ is valid on $\mathfrak{M}$ if $\mathfrak{M}, w \models \varphi$ for any $w \in W$, and φ is valid in a class of Kripke models if φ is valid on any $\mathfrak{M}$ in the class. Then, it is clear that in Table 1, all of the axioms are valid and all of the rules preserve validity on $\mathfrak{M}$.

Table 1: Hilbert-style Axiomatization $\mathbf{K_c}$ of Static Logic.

(**Taut**)	φ, φ is a tautology.
($\mathbf{K_B}$)	$B_a(\varphi \to \psi) \to (B_a\varphi \to B_a\psi)$ $(a \in G)$
(**Selfchn**)	c_{aa} $(a \in G)$
(**MP**)	From φ and $\varphi \to \psi$, infer ψ
$\mathbf{Nec_B}$	From φ, infer $B_a\varphi$ $(a \in G)$

2.3 Semi-private Announcement

We often use public announcement to express the communication between agents. However, in general, most of the announcements are made between a group of agents, so that only the agent in the group can get the message, while others cannot know what they are talking. This kind of announcement is called semi-private announcement (Sano and Tojo (2013)).

Here, we use the dynamic operator $[\varphi\downarrow^a_b]$, which means "after the agent a sent a message φ to the agent b via a channel", to express the semi-private announcement. Then, $[\varphi\downarrow^a_b]\psi$ stands for 'after the agent a sent a message φ to the agent b via a channel, ψ holds". We provide the semantic of $[\varphi\downarrow^a_b]\psi$ on a Kripke model $\mathfrak{M} = (W, R_{\mathsf{G}}, C_{\mathsf{G}}, V)$ as follows:

$$\mathfrak{M}, w \models [\varphi\downarrow^a_b]\psi \quad \text{iff} \quad \mathfrak{M}^{\varphi\downarrow^a_b}, w \models \psi$$

where $\mathfrak{M}^{\varphi\downarrow^a_b} = (W, R'_{\mathsf{G}}, C_{\mathsf{G}}, V)$ and $R'_i \in R'_{\mathsf{G}}$ is defined as:

- If $i = b$, for all $x \in W$,

$$R'_b(x) := \begin{cases} R_b(x) \cap [\![\varphi]\!]_{\mathfrak{M}} & \text{if } \mathfrak{M}, x \models c_{ab} \wedge \mathsf{B}_a\,\varphi \\ R_b(x) & \text{otherwise.} \end{cases}$$

- Otherwise, $R'_i := R_i$.

Here, $[\![\varphi]\!]_{\mathfrak{M}}$ is called the truth set of φ in $\mathfrak{M}$, which is defined as follows:

$$[\![\varphi]\!]_{\mathfrak{M}} = \{w \in W \mid \mathfrak{M}, w \models \varphi\}$$

Semantically speaking, $[\varphi\downarrow_b^a]$ revises agent b's belief when agent a believes φ, and there is a channel from a to b. Otherwise, agent b's belief will not be restricted (Barwise and Seligman (1997)). And it is easy to see that others than b will not revise their belief while they don't get the message φ. Here, all of the agents are considered as believable and receivable, while they can only tell the truth and they receive any message made by others (Seligman et al. (2011)).

In the syntax including $[\varphi\downarrow_b^a]\psi$, ψ is valid on the class of all finite Kripke models iff ψ is a theorem in $\mathbf{K_c}[\cdot\downarrow_\mathbf{b}^\mathbf{a}]$ of Table 2 as follows:

Table 2: Hilbert-style Axiomatization $\mathbf{K_c}[\cdot\downarrow_\mathbf{b}^\mathbf{a}]$.

In addition to all the axioms and rules of K_c, we add:

$[\varphi\downarrow_b^a]p$	$\leftrightarrow$	p
$[\varphi\downarrow_b^a]c_{cd}$	$\leftrightarrow$	c_{cd}
$[\varphi\downarrow_b^a]\neg\psi$	$\leftrightarrow$	$\neg[\varphi\downarrow_b^a]\psi$
$[\varphi\downarrow_b^a]\psi\wedge\chi$	$\leftrightarrow$	$[\varphi\downarrow_b^a]\psi\wedge[\varphi\downarrow_b^a]\chi$
$[\varphi\downarrow_b^a]B_c\psi$	$\leftrightarrow$	$B_c\psi(c\neq b)$
$[\varphi\downarrow_b^a]B_b\psi$	$\leftrightarrow$	$(c_{ab}\wedge B_a\varphi\rightarrow B_b(\varphi\rightarrow\psi))\wedge$ $(\neg(c_{ab}\wedge B_a\varphi)\rightarrow B_b\psi)$
$(\mathbf{Nec}_{[\varphi\downarrow_\mathbf{b}^\mathbf{a}]})$		From ψ, infer $[\varphi\downarrow_b^a]\psi$

The prove is shown in Hatano et al. (2015).

3 4-VALUED MODAL LOGIC

In classical logic, a proposition has only two possible truth values, which are usually called true and false. In other words, if a proposition is not true, it is false, and vice versa. However, sometimes a proposition is not true, while it is not false. For example, we cannot say that the sentence "There is no alien in the universe" is true, while we cannot say it is false. So we can see sometimes two possible values are not enough.

To resolve this problem, many-valued logic whose proposition has more than two truth values has been studied. The sum of possible truth values can be three, four, any natural number more than three, and even infinite. Malinowski (2014) added a new possible truth value **Undefined**, to mark indeterminacy of some proposition. Bočvar(Ciucci and Dubois (2013)) added a truth degree 0.5, whose reading as "meaningless" or "senseless". Cattaneo and Nisticò (1989) use a structure $\langle\Sigma,\ 0,\ \leq,\ ',\ {}^\sim\rangle$ to express the third value. Łukasiewicz provided an infinite-valued logic that considered the truth value as a real number between 0 and 1. In Łukasiewicz logic, the number of truth value shows the probability that the formula is false (Giles (1976)). In this paper, we consider 4-valued logic with Belnapian truth values(Odintsov and Wansing (2010)).

In 4-valued logic, the four values are usually called true, false, neither and both. Belnap considered the valued as follows:

- the value of p is **True**(**T**) means that the computer is told that p is true.
- the value of p is **False**(**F**) means that the computer is told that p is false.
- the value of p is **Neither**(**N**) means that the computer is not told anything about p.
- the value of p is **Both**(**B**) means that the computer is told that p is both true and false(perhaps from different sources, or so on).

Odintsov and Wansing (2017) shows that 4-valued logic can also be used in modal logic, whose operators includes $\Box$ and $\Diamond$.(Blackburn et al. (2002))

3.1 Language and Truth-table

To define the language of Belnap-Dunn Modal Logic $BK^\Box$, first we consider the language $L^\Box$ where

$$L^\Box := \{\wedge,\vee,\rightarrow,\bot,\sim,\Box\}$$

where $\sim$ stands for "strong negation" and other operators are defined as follows:

$$\neg\varphi := \varphi\rightarrow\bot,\ \varphi\leftrightarrow\psi := (\varphi\rightarrow\psi)\wedge(\psi\rightarrow\varphi)$$

$$\Diamond\varphi :=\sim\Box\sim\varphi,\ \varphi\Leftrightarrow\psi := (\varphi\leftrightarrow\psi)\wedge(\sim\varphi\leftrightarrow\sim\psi)$$

Then, we can define the language $BK^\Box$ as the least $L^\Box$-logic containing the following three groups of axioms:

- Axioms of classical propositional logic in the language $\vee,\wedge,\rightarrow,\bot$.
- Strong negation axioms:

$$\sim(p\wedge q)\leftrightarrow(\sim p\vee\sim q)$$

$$\neg(p\rightarrow q)\leftrightarrow(p\wedge\sim q)$$

$$\sim(p\vee q)\leftrightarrow(\sim p\wedge\sim q)$$

$$\sim\sim p\leftrightarrow p,\ and\ \sim\bot$$

- Modal axioms:

$$\Box(p\rightarrow q)\rightarrow(\Box p\rightarrow\Box q)\quad \neg\sim\Box p\leftrightarrow\Box\neg\sim p$$

The truth-tables are shown in Table 3,4,5.

Table 3: Truth-table of "$\sim$".

φ	$\sim\varphi$
T	**F**
F	**T**
B	**B**
N	**N**

Table 4: Truth-table of "$\wedge$".

$\wedge$	**T**	**F**	**B**	**N**
T	**T**	**F**	**B**	**N**
F	**F**	**F**	**F**	**F**
B	**B**	**F**	**B**	**F**
N	**N**	**F**	**F**	**N**

Table 5: Truth-table of "$\vee$".

$\vee$	**T**	**F**	**B**	**N**
T	**T**	**T**	**T**	**T**
F	**T**	**F**	**B**	**N**
B	**T**	**B**	**B**	**T**
N	**T**	**N**	**T**	**N**

We can represent the elements of $\mathbf{T}, \mathbf{F}, \mathbf{B}, \mathbf{N}$ as pairs (a,b), where $a,b \in \{0,1\}$, then $\mathbf{T}, \mathbf{F}, \mathbf{B}, \mathbf{N}$ can be written as follows: (Odintsov and Wansing (2010))

$$\mathbf{T} = (1,\ 0), \mathbf{F} = (0,\ 1), \mathbf{N} = (0,\ 0), \mathbf{B} = (1,\ 1).$$

Under the presentation above, we can get the form of twist-operators following:

$$(a,b) \vee (c,d) = (a \vee c, b \wedge d)$$

$$(a,b) \wedge (c,d) = (a \wedge c, b \vee d)$$

$$\sim (a,b) = (b,a), (a,b) \to (c,d) = (a \to c, a \wedge d)$$

3.2 Semantics

A **BK**-model is a tuple $\mathfrak{M} = (W,R,V)$ where W is a non-empty set of worlds, $R \subset W \times W$ is an accessibility relation on W, and $V : Prop \times W \to \mathbf{T}, \mathbf{F}, \mathbf{B}, \mathbf{N}\}$ is a valuation function. It will be convenient to have another definition closed to the standard Kripke model, so we assign functions $v^+, v^- : Prop \to 2^W$ defined as follows instead of V: (Odintsov and Wansing (2017))

$$v^+(p) = \{w | V(p,w) \in \{\mathbf{T}, \mathbf{B}\}\}$$

$$v^-(p) = \{w | V(p,w) \in \{\mathbf{F}, \mathbf{B}\}\}$$

For a **BK**-model $\mathfrak{M} = (W,R,v^+,v^-)$, we define $\models^+$ and $\models^-$ between the worlds of $\mathfrak{M}$ and formulas as follows: (Odintsov and Wansing (2017), Odintsov and Wansing (2010))

$$\begin{array}{lcl}
\mathfrak{M}, w \models^+ p & \Leftrightarrow & w \in v^+(p) \\
\mathfrak{M}, w \models^- p & \Leftrightarrow & w \in v^-(p) \\
\mathfrak{M}, w \models^+ \varphi \wedge \psi & \Leftrightarrow & \mathfrak{M}, w \models^+ \varphi \text{ and} \\
 & & \mathfrak{M}, w \models^+ \psi \\
\mathfrak{M}, w \models^- \varphi \wedge \psi & \Leftrightarrow & \mathfrak{M}, w \models^- \varphi \text{ or} \\
 & & \mathfrak{M}, w \models^- \psi \\
\mathfrak{M}, w \models^+ \varphi \vee \psi & \Leftrightarrow & \mathfrak{M}, w \models^+ \varphi \text{ or} \\
 & & \mathfrak{M}, w \models^+ \psi \\
\mathfrak{M}, w \models^- \varphi \vee \psi & \Leftrightarrow & \mathfrak{M}, w \models^- \varphi \text{ and} \\
 & & \mathfrak{M}, w \models^- \psi \\
\mathfrak{M}, w \models^+ \varphi \to \psi & \Leftrightarrow & \mathfrak{M}, w \models^+ \varphi \Rightarrow \\
 & & \mathfrak{M}, w \models^+ \psi \\
\mathfrak{M}, w \models^- \varphi \to \psi & \Leftrightarrow & \mathfrak{M}, w \models^+ \varphi \text{ and} \\
 & & \mathfrak{M}, w \models^- \psi \\
\mathfrak{M}, w \not\models^+ \bot & & \text{always} \\
\mathfrak{M}, w \models^- \bot & & \text{always} \\
\mathfrak{M}, w \models^+ \sim \varphi & \Leftrightarrow & \mathfrak{M}, w \models^- \varphi \\
\mathfrak{M}, w \models^- \sim \varphi & \Leftrightarrow & \mathfrak{M}, w \models^+ \varphi \\
\mathfrak{M}, w \models^+ \Box\varphi & \Leftrightarrow & \forall u \in W((w,u) \in R \Rightarrow \\
 & & \mathfrak{M}, u \models^+ \varphi) \\
\mathfrak{M}, w \models^- \Box\varphi & \Leftrightarrow & \exists u \in W((w,u) \in R \text{ and} \\
 & & \mathfrak{M}, u \models^- \varphi) \\
\mathfrak{M}, w \models^+ \Diamond\varphi & \Leftrightarrow & \exists u \in W((w,u) \in R \text{ and} \\
 & & \mathfrak{M}, u \models^+ \varphi) \\
\mathfrak{M}, w \models^- \Diamond\varphi & \Leftrightarrow & \forall u \in W((w,u) \in R \Rightarrow \\
 & & \mathfrak{M}, u \models^- \varphi)
\end{array}$$

If we use a pair (a,b) to express the value of formula "φ" in world w according to the previous subchapter, the basis definition of $\models^+$ and $\models^-$ can also be written as follows:

$$\begin{array}{lcl}
\mathfrak{M}, w \models^+ \varphi & \Leftrightarrow & a = 1 \\
\mathfrak{M}, w \models^- \varphi & \Leftrightarrow & b = 1
\end{array}$$

4 4-VALUED LOGIC FOR MULTI-AGENT COMMUNICATION

Consider two people Ann and Bill, who are chatting on the Internet. Ann learned a new dance and she believes that her dance is very good, so she wants to tell Bill it. Then she sends a video of her dance to Bill. However, Bill doesn't get the message that Ann's dance is good. The possible reasons are as follows:

- The Internet is not connected.
- Bill's computer is too old to watch the video.

so he cannot get the message to revise his belief. Also, it can be explained in other ways. For another example, let agents a and b be two companies. p means that "a is faced with bankruptcy". Obviously, if a and b are opponents, they won't tell it to each other if they believe p or the negation of p. Such p can be seen as a private proposition.

Hatano et al. (2015) can express the disconnection by channels, but it cannot show the other situation.

Here, we use a pair (a,b) to express the value of a proposition φ. ($a \in \{T,F\}, b \in \{0,1\}$)

- $\varphi : (T,1)$ means "φ is true and public."
- $\varphi : (T,0)$ means "φ is true and private."
- $\varphi : (F,1)$ means "φ is false and public."
- $\varphi : (F,0)$ means "φ is false and private."

If φ is public, other agents can get this message, and if φ is private, others cannot revise their beliefs by this message.

4.1 Syntax

In this paper, we define a new kind of 4-valued logic different from **BK**-model.

Let $\mathsf{PROP} = \{p, q, \dots\}$ be a finite set of propositional variables and $\mathsf{G} = \{a, b, \dots\}$ a finite set of agents. The language is generated by the following Backus-Naur form:

$$\alpha ::= p \mid \neg\alpha \mid \alpha \wedge \alpha \mid c_{ab}$$

$$\varphi ::= \alpha \mid \mathsf{B}_a \alpha \mid^{pub} \alpha \mid [\alpha\downarrow^a_b]\varphi \mid \neg\varphi \mid \varphi \wedge \varphi$$

where $p \in \mathsf{PROP}, a \in \mathsf{G}, b \in \mathsf{G}$ and α is an objective (non-modal) formula.

Here, c_{ab} means "There is a channel from agent a to agent b". ${}^{pub}\alpha$ means "α is public". And $\mathsf{B}_a\alpha$ means "agent a believes α."

The truth-table of ¬ is as follows:

Table 6: Truth-table of ¬.

φ	$\neg\varphi$
$(T,1)$	$(F,1)$
$(T,0)$	$(F,0)$
$(F,1)$	$(T,1)$
$(F,0)$	$(T,0)$

The truth-table of ¬ is as follows. Here, we let ${}^{pub}\varphi$ be always public.

Table 7: Truth-table of ^{pub}.

φ	${}^{pub}\varphi$
$(T,1)$	$(T,1)$
$(T,0)$	$(F,1)$
$(F,1)$	$(T,1)$
$(F,0)$	$(F,1)$

For the 4-valued logic, we define ∧. Let φ and ψ be two proposition. Then the proposition $\varphi \wedge \psi$ is true if and only if φ is true and ψ is true. And $\varphi \wedge \psi$ is public if and only if φ is public and ψ is public. Table 8 shows the truth-table of ∧.

Table 8: Truth-table of ∧.

$\wedge$	$(T,1)$	$(T,0)$	$(F,1)$	$(F,0)$
$(T,1)$	$(T,1)$	$(T,0)$	$(F,1)$	$(F,0)$
$(T,0)$	$(T,0)$	$(T,0)$	$(F,0)$	$(F,0)$
$(F,1)$	$(F,1)$	$(F,0)$	$(F,1)$	$(F,0)$
$(F,0)$	$(F,0)$	$(F,0)$	$(F,0)$	$(F,0)$

We define the function ∨ as follows:

$$\varphi \vee \psi := \neg(\neg\varphi \wedge \neg\psi)$$

Table 9: Truth-table of ∨.

$\vee$	$(T,1)$	$(T,0)$	$(F,1)$	$(F,0)$
$(T,1)$	$(T,1)$	$(T,0)$	$(T,1)$	$(T,0)$
$(T,0)$	$(T,0)$	$(T,0)$	$(T,0)$	$(T,0)$
$(F,1)$	$(T,1)$	$(T,0)$	$(F,1)$	$(F,0)$
$(F,0)$	$(T,0)$	$(T,0)$	$(F,0)$	$(F,0)$

The truth-table of ∨ is shown in Table 9.

Here, we should take notice of the truth-table of ∨. In this paper, we define that if $\varphi \vee \psi$ is public if and only if φ is public and ψ is public. In other words, if φ is private, $\varphi \wedge \psi$ and $\varphi \vee \psi$ are all private even if ψ is public. It is because that if we cannot tell φ to others, anything related to φ like $\varphi \wedge \psi$ or $\varphi \vee \psi$ also cannot be told to others.

Finally, we define the "→" as follows:

$$\varphi \to \psi := \neg\varphi \vee \psi$$

The truth-table of → is shown in Table 10.

Table 10: Truth-table of →.

$\to$	$(T,1)$	$(T,0)$	$(F,1)$	$(F,0)$
$(T,1)$	$(T,1)$	$(T,0)$	$(F,1)$	$(F,0)$
$(T,0)$	$(T,0)$	$(T,0)$	$(F,0)$	$(F,0)$
$(F,1)$	$(T,1)$	$(T,0)$	$(T,1)$	$(T,0)$
$(F,0)$	$(T,0)$	$(T,0)$	$(T,0)$	$(T,0)$

Notice that $\varphi \to \psi$ is public only if φ is public and ψ is public, which is similar to the operator ∨.

4.2 Semantics

Here, we use Kripke semantics with our syntax. A Kripke model $\mathfrak{M}$ is a tuple:

$$\mathfrak{M} = (W, R_{\mathsf{G}}, C_{\mathsf{G}}, V)$$

where W is a non-empty set of worlds, G is a non-empty set of agents, $R_{\mathsf{G}} = \{R_a \mid a \in \mathsf{G}\}$ and $R_a \subset W \times W$ is an accessibility of agent a on W, $C_{\mathsf{G}} = \{C_{ab} \mid a \in \mathsf{G}, b \in \mathsf{G}\}$ and $C_{ab} \subseteq W$ is a channel relation, and $V : Prop \times W \to \{(T,1),(T,0),(F,1),(F,0)\}$ is the valuation function. In many cases it is convenient to replace the four-valued V by two function, so we assign functions $v^t, v^p : Prop \to 2^W$ defined as follows to express V:

$$v^t(p) = \{w | V(p,w) \in \{(T,1),(T,0)\}\}$$

$$v^p(p) = \{w | V(p,w) \in \{(T,1),(F,1)\}\}$$

Given any model $\mathfrak{M}$, any world $w \in W$, any agent $a, b \in \mathsf{G}$, and any formula φ, we define the satisfaction relation $\mathfrak{M}, w \models^t \varphi$ and $\mathfrak{M}, w \models^p \varphi$ as follows:

$$\begin{array}{lll}
\mathfrak{M}, w \models^t p & \text{iff} & w \in v^t(p) \\
\mathfrak{M}, w \models^t \varphi \wedge \psi & \text{iff} & \mathfrak{M}, w \models^t \varphi \textit{ and } \mathfrak{M}, w \models^t \psi \\
\mathfrak{M}, w \models^t \varphi \vee \psi & \text{iff} & \mathfrak{M}, w \models^t \varphi \textit{ or } \mathfrak{M}, w \models^t \psi \\
\mathfrak{M}, w \models^t \varphi \rightarrow \psi & \text{iff} & \mathfrak{M}, w \not\models^t \varphi \textit{ or } \mathfrak{M}, w \models^t \psi \\
\mathfrak{M}, w \models^t \neg\varphi & \text{iff} & \mathfrak{M}, w \not\models^t \varphi \\
\mathfrak{M}, w \models^t {}^{pub}\varphi & \text{iff} & \mathfrak{M}, w \models^p \varphi \\
\mathfrak{M}, w \models^t B_a\varphi & \text{iff} & \forall u \in W((w,u) \in R_a \rightarrow \\
 & & \mathfrak{M}, u \models^t \varphi) \\
\mathfrak{M}, w \models^t \mathsf{c}_{ab} & \text{iff} & w \in C_{ab}
\end{array}$$

$$\begin{array}{lll}
\mathfrak{M}, w \models^p p & \text{iff} & w \in v^p(p) \\
\mathfrak{M}, w \models^p \varphi \wedge \psi & \text{iff} & \mathfrak{M}, w \models^p \varphi \textit{ and } \mathfrak{M}, w \models^p \psi \\
\mathfrak{M}, w \models^p \varphi \vee \psi & \text{iff} & \mathfrak{M}, w \models^p \varphi \textit{ and } \mathfrak{M}, w \models^p \psi \\
\mathfrak{M}, w \models^p \varphi \rightarrow \psi & \text{iff} & \mathfrak{M}, w \models^p \varphi \textit{ and } \mathfrak{M}, w \models^p \psi \\
\mathfrak{M}, w \models^p \neg\varphi & \text{iff} & \mathfrak{M}, w \models^p \varphi \\
\mathfrak{M}, w \models^p {}^{pub}\varphi & & \textit{always} \\
\mathfrak{M}, w \models^p B_a\varphi & \text{iff} & \mathfrak{M}, w \models^p \varphi \\
\mathfrak{M}, w \models^p \mathsf{c}_{ab} & & \textit{always}
\end{array}$$

Here, $\mathfrak{M}, w \models^p B_a\varphi$ iff $\mathfrak{M}, w \models^p \varphi$ means that if φ is public, the message that agent a believes φ is also public and vice versa.

Also, let the value of φ in world w be (a, b), we can define $\models^t$ and $\models^p$ in the other way as follows:

$$\begin{array}{lll}
\mathfrak{M}, w \models^t \varphi & \text{iff} & a := T \\
\mathfrak{M}, w \models^p \varphi & \text{iff} & b := 1
\end{array}$$

Semantically speaking, in a model $\mathfrak{M}$, $\mathfrak{M}, w \models^t \varphi$ means φ is true in world w, and $\mathfrak{M}, w \models^p \varphi$ means φ is public in world w.

4.3 Multi-agent Communication

In this paper, we use the same dynamic operator $[\varphi\downarrow^a_b]$ as Hatano et al. (2015), which means "after agent a sends a message φ to agent b via a channel", and $[\varphi\downarrow^a_b]\psi$ means "after agent a sends a message φ to agent b via a channel, ψ holds". Here, the communication $[\varphi\downarrow^a_b]$ will success only if the following hold:

- There is a channel from agent a to agent b.
- Agent a believes the content of the message φ.
- The message φ is public.

In Hatano et al. (2015), all of the message is regarded as public message, which can be told to others. Here, we use 4-valued logic which can express whether a formula is public or not, so even if agent a believe φ and there is a channel from a to b, the communication will fail if φ is private, which is different from Hatano et al. (2015).

The semantics of $[\varphi\downarrow^a_b]\psi$ on a Kripke model $\mathfrak{M} = (W, R_\mathsf{G}, C_\mathsf{G}, v^+, v^-)$ is given as follows:

$$\begin{array}{lll}
\mathfrak{M}, w \models^t [\varphi\downarrow^a_b]\psi & \text{iff} & \mathfrak{M}^{\varphi\downarrow^a_b}, w \models^t \psi \\
\mathfrak{M}, w \models^p [\varphi\downarrow^a_b]\psi & \text{iff} & \mathfrak{M}^{\varphi\downarrow^a_b}, w \models^p \psi
\end{array}$$

where $\mathfrak{M}^{\varphi\downarrow^a_b} = (W, R'_\mathsf{G}, C_\mathsf{G}, v^t, v^p)$ and $R'_i \in R'_\mathsf{G}$ is defined as:

- If $i = b$, for all $x \in W$,

$$R'_b(x) := \begin{cases} R_b(x) \cap [\![\varphi]\!]_{\mathfrak{M}} & \text{if } \mathfrak{M}, x \models^t c_{ab} \wedge \mathsf{B}_a\varphi \\ & \textit{and } \mathfrak{M}, x \models^p \varphi \\ R_b(x) & \text{otherwise.} \end{cases}$$

- Otherwise, $R'_i := R_i$.

The truth set $[\![\varphi]\!]_{\mathfrak{M}}$ is defined by:

$$[\![\varphi]\!]_{\mathfrak{M}} = \{ w \in W \mid \mathfrak{M}, w \models^t \varphi \}.$$

Semantically speaking, after $[\varphi\downarrow^a_b]$, agent b will revise his/ her belief if there is a channel from agent a to b, agent a believes the content of the message φ, and φ is public. Otherwise, agent b will not revise his/ her belief. Other agents than b will not change beliefs because they get no message.

Example : Here, we give an example to show the belief change after a semi-announcement. Consider a Kripke model $\mathfrak{M} = (W, R_\mathsf{G}, C_\mathsf{G}, v^t, v^p)$. (see Figure 2)

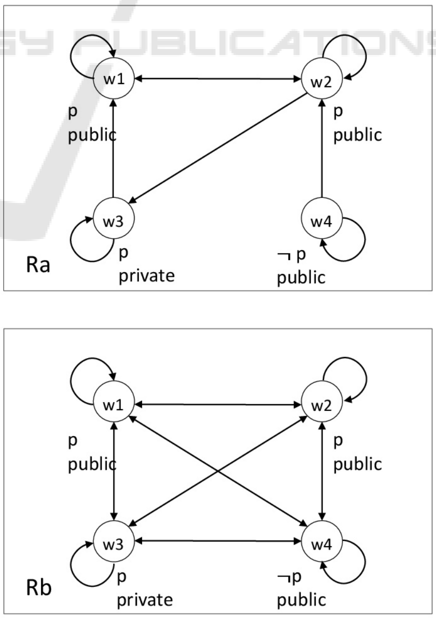

Figure 2: Accessibility relations of agents a and b.

Let $\mathsf{G} = \{a, b\}$, $W = \{w_1, w_2, w_3, w_4\}$, $R_a = \{(w_1, w_1), (w_1, w_2), (w_2, w_1), (w_2, w_2), (w_2, w_3), (w_3, w_1), (w_3, w_3), (w_4, w_2), (w_4, w_4)\}$, $R_b = W \times W$,

$C_{ab} = \{w_1, w_3, w_4\}$, $C_{ba} = \emptyset$, $C_{aa} = C_{bb} = W$, $v^t(p) = \{w_1, w_2, w_3\}$ and $v^p(p) = \{w_1, w_2, w_4\}$.

As the configuration above, p is true in w_1, w_2, w_3 and is false in w_4. p is public in w_1, w_2, w_4 and is private in w_3. According to the definition of $B_a\varphi$, we can see that agent a believe p in world w_1, w_2 and w_3, while not believing anything in world w_4. Agent b doesn't believe anything in any world. There are channels from agent a to b in w_1, w_2, w_4, while no channel exists in w_3.

Now, consider the model $\mathfrak{M}^{\varphi\downarrow^a_b}$ which shows the new accessibility relation after the action agent a sends the message φ to agent b.

- In world w_1, agent a believes p so a can send the message, there is a channel from agent a to b so the message can be sent to b, and as p is public so b can understand the message p. As the result, b will revise his/ her belief to believe p.
- In world w_2, agent a believes p so a can send the message, and as p is public so b can understand the message p. However, there isn't a channel from agent a to b so the message cannot be sent to b. So as the result, b won't revise his/ her belief.
- In world w_3, agent a believes p so a can send the message, and there is a channel from agent a to b so the message can be sent to b. However, as p is private so b cannot understand the message p. As the result, b won't revise his/ her belief.
- In world w_4, there is a channel from agent a to b so the message can be sent to b, and as p is public so b can understand the message p. However, agent a doesn't believe p so a cannot send the message. As the result, b won't revise his/ her belief.

We can see that after the action $[\varphi\downarrow^a_b]$ which means that agent a tells b the message p, agent b becomes to believe p only in world w_1. In other worlds, agent b doesn't change his/ her belief. So in the new model $\mathfrak{M}^{\varphi\downarrow^a_b} = (W, R'_{\mathsf{G}}, C_{\mathsf{G}}, v^t, v^p)$, $R'_b := W \times W / \{(w_1, w_3), (w_1, w_4)\}$ shown in Figure 3.

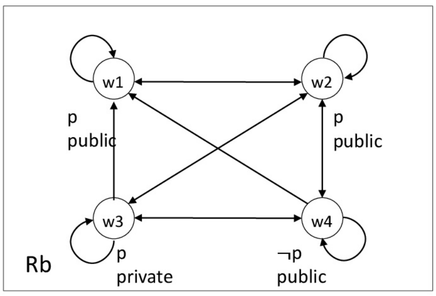

Figure 3: Accessibility relation of agent b after the announcement.

4.4 Hilbert-style Axiomatization

Here, we say φ is t-valid on $\mathfrak{M}$ if $\mathfrak{M}, w \models^t \varphi$ for any $w \in W$, and φ is t-valid in a class of Kripke models if φ is valid on any $\mathfrak{M}$ in the class. If we disregard whether a formula is public or private, the definition of t-valid is just the same as the definition of valid in chapter 2. So it is clear that in Table 11, all of the axioms are t-valid and all of the rules preserve validity on $\mathfrak{M}$. However, the concept p-valid defined in the same way has no meaning, for there isn't a tautology about the concept of public and private.

Table 11: Hilbert-style Axiomatization $\mathbf{K_c}$ of 4-valued logic.

(**Taut**)	$\vdash^t \varphi$, φ is a tautology.
($\mathbf{K_B}$)	$\vdash^t B_a(\varphi \to \psi) \to (B_a\varphi \to B_a\psi)$
(**Selfchn**)	$\vdash^t c_{aa}$
(**MP**)	From $\vdash^t \varphi$ and $\vdash^t \varphi \to \psi$, infer $\vdash^t \psi$
$\mathbf{Nec_B}$	From $\vdash^t \varphi$, infer $\vdash^t B_a\varphi$

Here, $a \in G$.

In our 4-valued logic, the following equivalence relations hold.

Table 12: Hilbert-style Axiomatization $\mathbf{K_c}[\cdot\downarrow^{\mathbf{a}}_{\mathbf{b}}]$ of 4-valued logic.

In addition to all the axioms and rules of $\mathbf{K_c}$, we add:

$[\alpha\downarrow^a_b]p$	$\leftrightarrow$	p
$[\alpha\downarrow^a_b]c_{cd}$	$\leftrightarrow$	c_{cd}
$[\alpha\downarrow^a_b]\neg\varphi$	$\leftrightarrow$	$\neg[\alpha\downarrow^a_b]\varphi$
$[\alpha\downarrow^a_b]^{pub}\varphi$	$\leftrightarrow$	${}^{pub}\varphi$
$[\alpha\downarrow^a_b]\varphi \wedge \psi$	$\leftrightarrow$	$[\alpha\downarrow^a_b]\varphi \wedge [\alpha\downarrow^a_b]\psi$
$[\alpha\downarrow^a_b]B_c\varphi$	$\leftrightarrow$	$B_c\varphi (c \neq b)$
$[\alpha\downarrow^a_b]B_b\varphi$	$\leftrightarrow$	$((c_{ab} \wedge B_a\alpha \wedge {}^{pub}\alpha) \to B_b(\alpha \to \varphi)) \wedge (\neg(c_{ab} \wedge B_a\alpha \wedge {}^{pub}\alpha) \to B_b\varphi)$
($\mathbf{Nec}_{[\alpha\downarrow^a_b]}$)		From $\vdash^t \varphi$, infer $\vdash^t [\alpha\downarrow^a_b]\varphi$

Prove: It is clear that equivalence relation of the first value of the pairwise truth hold except $[\varphi\downarrow^a_b]B_b\psi$, according to the prove in Hatano et al. (2015). The second value of the pairwise truth shows whether a formula is public or private, so the equivalence relations also hold with the second value of the pairwise truth. Therefore, it is easy to see that the value of left and right are the same except the line $[\varphi\downarrow^a_b]B_b\psi$.

Then, consider $[\varphi\downarrow^a_b]B_b\psi$, which means "after the announcement φ from agent a, agent b believes ψ". According to the definition, the communication successes for three conditions. First, c_{ab} is true, which means that there is a channel from a to b. Second, $B_a\varphi$ is true, which means that agent a believes φ. Third, φ is public, which is the same as ${}^{pub}\varphi$ is true. So if the formula $(c_{ab} \wedge B_a\varphi \wedge {}^{pub}\varphi)$ is true, the communication will succeed, and if the formula is false, the communication will fail. Therefore, the relation about $[\varphi\downarrow_b^a]B_b\psi$ is also an equivalence relation.

In our 4-valued logic, if we just look at the first value of the pairwise truth, which shows whether a formula is true or false, and ignore the operator "^{pub}" in the syntax, furthermore if we change "$\models^t$" into "$\models$" and disregard the "$\models^p$" in the semantic, this logic will be the same with the logic in Hatano et al. (2015), which has been proved to be complete and sound, So if we disregard whether a formula is public or private, the logic in this paper is also complete and sound.

5 CONCLUSION

We have shown a 4-valued logic which distinguishes the ordinary truth value of each proposition as well as the information is private or public. By private information transmission, since the recipient cannot read the contents he/ she does not change his/ her belief. This unsuccessful message passing corresponds to such practical situations that the information needs other background knowledge, password, deciphering protocol, and so on.

We have reconstructed the dynamic epistemic logic including the 4-valued logic, and have introduced the two kinds of negations, the truth tables for the logical connectives, their semantics, and its Hilbert-style axiomatization. Since the recursion axioms can reduce the formulae with dynamic operators to those without them, we can ensure the completeness and soundness if we disregard the second value of the pairwise truth.

In the current stage, our formalization may still have redundancy; in the case we need a password for the private information, the password itself would be formalized in the very similar way to the channel variables. However, our objective is to formalize the unsuccessful communication in general. Thus, we will further develop the distinction between miscommunication by lack of necessary information and that by unsuccessful message transmission in future.

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