

# A Voting Argumentation Framework: Considering the Reasoning behind Preferences

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**Abstract:** One of the most prominent ways to reach an acceptable collective decision in normal group settings is the employment of routines and methods of social choice theory. The classical social choice setting is the following: each agent involved in the decision expresses her preferences about a given set of alternatives in the form of a linear order on them. Then, the group’s aggregated decision is the outcome of the application of a voting rule to the input’s preferences. However, there are instances where social choice on its own cannot provide proper solutions. For example, there are decision problems where the outcome has to be based on the reasoning behind agents’ preferences, rather than the unjustified preferences itself. Hence, our research motivation is the practical case where agents’ rationale is needed for the decision outcome. In this paper, we explore how the agents’ rationale can be formulated inside the classical voting setting. Therefore, we propose a decision-making procedure based on argumentation and preference aggregation which permits us to explore the effect of reasoning and deliberation along with voting for the decision process. We quantify the deliberation phase by defining a new voting argumentation framework, that uses vote and generic arguments, and its acceptability semantics based on the notion of pairwise comparisons between alternatives. We prove for these semantics some theoretical results regarding well-known properties from Argumentation and Social Choice Theory.

## 1 INTRODUCTION

Collective decision-making in the context of multi-agents systems is a well-studied problem where many possible research approaches have been proposed in the literature for solving it. An overview of the research approaches can be found in this survey (Bulling, 2014). The procedure followed to reach a group decision is a complex task, in which there are many parameters that affect decision makers judgement. The key to making a “good” collective decision is knowing and hence, the agents (decision-makers) should have full knowledge on the different parameters that entail collective decisions.

In the decision-making literature it is widely believed that in order to confirm that the chosen decision outcome is the best one, the decision makers should believe that this is the best outcome, and have reasons to believe this. Using social choice theory we secure the first condition: agents express their individual preferences on the decision outcomes and voting

methods provide the means in order for agents to believe that their aggregated preference (outcome) is the best, i.e., fairest according to their preferences. Our motivation comes from fulfilling the second condition and in order to do that we have to take into account the reasoning behind the preferences and thus, deliberation and argumentation play an important role. Hence, the scope of this paper is to fulfill the central decision-making problem, which is to help decision makers produce “better” collective decision outcomes. “Better” decisions is a very broad term and the goal of many research papers on this domain. In this paper, we will study on how collective decision-making is helped with the intersection of argumentation along with deliberation in social choice theory. Thus, we will focus on a social choice theoretic approach for multi-agent decision making enriched with an argumentation framework.

Social choice theory can be applied to multi-agent systems (Endriss, 2014) where voting can provide the classical means for aggregating the individual agents’

preferences into a collective decision. In the original setting we have a set of *agents* and a set of *alternatives*. Each agent expresses her preference as a total order over a set of alternatives, and then the group's preference is computed from the individual preferences using a *voting rule*. A more analytic description of the voting problem and the social choice fundamentals can be found in the Handbook of Computational Social Choice (Brandt et al., 2016).

As previously mentioned, we believe that collective decision making should also rely on the reasoning the agents provide when expressing their preferences. In order to strengthen this view we can mention here that it is common in many occasions that agents lie in expressing preferences in favor of specific alternatives when it is not possible to justify their reasons. The same has also been noticed even if agents provide reasoning but there is no deliberation phase. One such example are the reviews and ratings agents provide in sites like Google, Amazon, etc., where in some cases the percentage of fake/questionable reviews for a category of alternatives/products can reach 67%<sup>1</sup>. Hence, it is logical to assume that the agents should not only give their preferences but provide also a reasoning about their preferences so that one can debate. Therefore, it is reasonable to search for a way to interpret the preferences and the reasoning behind them and an argumentation framework (Besnard and Hunter, 2008; Dung, 1995) seems to be a rational approach to do that. Hence, in order to fulfill a collective decision mechanism that considers both reasoning and deliberation we propose a decision-aiding procedure which combines argumentation with computational social choice.

Argumentation theory is widely used in the multi-agent decision-making context, e.g., (Amgoud and Prade, 2009; Gao et al., 2016; Fan and Toni, 2014) due to its ability for reasoning with incomplete and conflicting information (such as differences in opinions). An argumentation framework is based on the construction, the exchange and the evaluation of interacting arguments, where various semantics are defined in the literature to assess the acceptability of sets of arguments. Hence, an argumentation framework where agents provide arguments with their preferences can “correct” the “false” or “fake” information that can appear. The way to measure the “false” information included in agents’ preferences is by introducing the notion of *attacking power* of arguments. It is a function that quantifies the attacking strength of

<sup>1</sup>[https://www.washingtonpost.com/business/economy/how-merchants-secretly-use-facebook-to-flood-amazon-with-fake-reviews/2018/04/23/5dad1e30-4392-11e8-8569-26fda6b404c7\\_story.html](https://www.washingtonpost.com/business/economy/how-merchants-secretly-use-facebook-to-flood-amazon-with-fake-reviews/2018/04/23/5dad1e30-4392-11e8-8569-26fda6b404c7_story.html)

argument(s) exposed during the deliberation phase towards an argument stating an agent’s preference. For example, if a preference argument of an agent is attacked by many arguments which are revealed during deliberation that is most likely to mean that this preference is not truthful and thus its power for the collective decision should be reduced. Therefore, we introduce a method which takes into account the attacking power of the deliberation phase to reach a collective decision.

Concluding, it is our belief that enhancing the collective decision-making procedure with a voting argumentation framework can benefit the procedure in the following ways. First, agents’ justifications for preferences, which are depicted in the construction of the argumentation framework, can provide the reasoning which can serve as the rational explanation of the collective decision. Second, an argumentation framework can model the deliberation phase prior to the application of voting for making a group decision. This modelling permits us to construct a preference profile that is “justified”, since it refers to the agents’ preferences and their justifications. The justified preference profile is a type of structured profile which is the outcome of a pre-voting debate phase that consists of a deliberation procedure where agents reveal their preferences and justifications. The objective is to fairly aggregate the justified viewpoints of the agents and hence, the justified preference profile can be reported to a voting rule for computing the decision outcome.

**Our Work.** Seminal to our research is the work of (Bisquert et al., 2017) which first presented an argumentation framework based on agents’ preferences for the voting problem from a *qualitative* perspective. Based on the notions of this paper we present a novel *quantitative* procedure by designing a special kind of Argumentation Framework, the Voting Argumentation Framework ( $\mathcal{VAF}$ ) and its corresponding semantics, which are called pairwise comparison semantics. The proposed semantics take into account the deliberation<sup>2</sup> phase in terms of quantifying the attacking power of arguments on the justification of the arguments produced by agents’ preferences. We then compute the acceptability of the vote arguments which define the new profile, called the justified preference profile, that takes into account the justification and deliberation phase. The justified preference

<sup>2</sup>It should be noted that, similarly to (Bisquert et al., 2017), our work does not address deliberation protocols such as (Kok et al., 2011) directly. Indeed, we assume that the argumentation framework is obtained as a result of an unspecified deliberation protocol. This allows to abstract the approach from the representation of the deliberation itself.

profile is the outcome of a quantitative argumentation framework and its semantics and contains now all the “corrected” preferences of the agents. Thus, a voting rule can be applied to aggregate these preferences, which gives us the motivation to study social choice theoretic properties for the justified preference profile and prove under which conditions they can be satisfied. Finally, we look into properties that  $\mathcal{VAF}$  and its corresponding semantics should satisfy from an argumentative perspective.

**Related Work.** The intersection of argumentation and deliberation in social choice theory for “better” decision outcomes is a recently developed field. As already noted “better” can be defined in many ways. For example, in the work of (List et al., 2013) “better” refers to outcomes where majority cycling can be bypassed through single-peakedness. They show that deliberation can protect against majority cycles by presenting an empirical test using real data and use-cases from Deliberative Polls. The results show that the preferences after deliberation phase are closer, in terms of a particular proximity function, to being single-peaked. On the same path, i.e., decision outcomes based on deliberation that avoid majority cycling and related paradoxes, (Bisquert et al., 2017) design a qualitative model based on an argumentation framework that is built from the justifications of the agents pairwise preferences. They use Dung’s “preferred” semantics to compute extensions which provide the justified preference profile. The justified preference profile is computed through a qualitative method where the strength of votes, i.e, the number of times that a pairwise comparison appears in the original preference profile, is not taken into account for the aggregation. Hence, this method focuses on cases where the decision outcome is independent of the aggregation of agents’ preference relations. In our approach we instead focus on cases where the strength of the votes are included in the collective decision. They prove that the justified preference profile permits a type of structured preferences where the Condorcet paradox is avoided. Another work for exploring the effect of deliberation and its benefits in voting is the paper of (Fain et al., 2017). The authors in this paper present a sequential deliberation protocol, seen from a game-theoretic perspective, where agents negotiate in pairs and collaboratively propose outcomes that appeal to both of them. They describe a method where the space of preferences is defined by a median graph and prove that sequential deliberation is 1.208-approximate to the optimal social cost on such graphs. They also study the Pareto-efficiency property from a social choice perspective and prove that the outcome

of sequential deliberation is ex-post Pareto-efficient on a median graph, i.e., that there is no other alternative that has at most that social cost for all agents and strictly better cost for one agent.

There is also some significant research on the intersection of Social Choice and Argumentation which is proximately related to this work. Most of the works towards this research study the problem from an argumentative perspective and deal with collective argumentation. The studied problem refers to aggregating individual argumentation frameworks to a collective one. The aggregation mechanisms provided to compute the collective frameworks rely on social choice, which provides the means to accomplish that. A nice example towards this research direction is the work of (Coste-Marquis et al., 2007) for merging argumentation systems. They focus on scenarios in which some agents are able to consider arguments not known by other agents and disagree on the attack relation. A three-step process is proposed where in the first one, each attack relation is consensually expanded to become a partial system over the set of arguments. In the second step the merging is done by generating a class of argumentation systems that are at the shortest “distance” of the ones in the profile. In the final step the acceptable arguments are selected. Another example towards this direction is the research of (Delobelle et al., 2016) where specific merging operators based on extensions, though in combination with a framework-wise merging process. The authors study the generation of the Argumentation Framework resulting from a merging process. There has been also notable research on the combination of Judgement Aggregation, which is one of the main problems in Social Choice, and Argumentation where Pigozzi et al. have extensively studied the problem. In (Pigozzi, 2006) the author proposes an aggregation procedure, called argument-based, for the case in which the outcome is a set of arguments, combining features of premise and conclusion-based procedures. (Pigozzi and van der Torre, 2007) study the same problem looking for axioms that characterize the aggregation procedure and conditions such that the Condorcet Paradox is avoided. They show that a condition called premise independence of irrelevant propositional alternatives guarantees the existence of consistent ways of aggregating judgments. (Caminada and Pigozzi, 2011) employ an argumentation approach to judgment aggregation which satisfies standard judgment aggregation postulates and also avoids the problem of individual agents having to become committed to a group judgment that is in conflict with their own preferences. Another prominent work combining Argumentation and Voting is the one of (Leite and Mar-

tins, 2011) where they propose Social Abstract Argumentation Framework, which is based on Dung’s Abstract Argumentation Framework but also incorporates social voting. They propose a class of semantics for the Social Abstract Argumentation Framework and prove some important properties regarding Social Networks. An explorative survey for collective argumentation is provided by (Bodanza et al., 2017).

On a related research path, recently, there has been some work on the reasoning behind preferences but not from an argumentative and social choice perspective. For example, (Dietrich and List, 2011) propose a reason-based theory of rational choice where agents’ preferences are determined by their motivating reasons and clarify the relationship between deliberation for reasons and for rational choices. Another work on the same path is the one of (Pedersen et al., 2015) where they develop a modal logic for reasoning about preferences that depends on a set of motivational properties and also show that reasoning systems and algorithms developed for modal logic can be employed for reasoning about reason-based preferences.

## 2 PRELIMINARIES

In the following section, we are defining several notions and notations that will be used later in the paper.

### 2.1 Social Choice Theory

We consider a set of  $N = \{1, \dots, n\}$  agents and a set of alternatives  $A$ ,  $|A| = m$ . Each agent  $i \in N$  has preference relations ( $\succ$ ) over the alternatives denoted with  $x \succ_i y$  which means that agent  $i$  prefers alternative  $x$  to  $y$ . We define that each irreflexive preference relation satisfies transitivity, antisymmetry and comparability and hence, the set of all the preference relations for agent  $i$  produces a linear (strict total) order  $\succ_i$  on  $A$ , i.e., the ranking of agent  $i$  over the alternatives. Let  $\mathbb{L}_A$  be the set of linear orders over  $A$ . A *preference profile*  $\succ_{PP} = \langle \succ_1, \dots, \succ_n \rangle \in \mathbb{L}_A^n$  is a collection of the linear orders for all the agents. A *voting rule* is a mapping  $f : \mathbb{L}_A^n \rightarrow 2^A \setminus \{\emptyset\}$  from preference profiles to nonempty subsets of alternatives, which designates the winner(s) of the election. For two candidates  $x, y \in A$ , and  $\succ_{PP} \in \mathbb{L}_A^n$ , alternative  $x$  beats  $y$  in a *pairwise comparison* if  $|\{i \in N : x \succ_i y\}| > n/2$ , that is, if a (strict) majority of agents prefer  $x$  to  $y$ . The winner according to the *Condorcet method* (Condorcet, 1785), i.e. the *Condorcet winner*, is an alternative that beats every other alternative in a pairwise comparison. The *Condorcet paradox* as defined by (Black, 1958) (also known as voting paradox or the paradox

of voting) is a situation in which the application of the Condorcet method to a preference profile can lead to a voting cycle, and hence a Condorcet winner can not be declared. A *voting cycle* occurs when we have 3 alternatives  $x, y, z$  such that  $|\{i \in N : x \succ_i y\}| > n/2$ ,  $|\{i \in N : y \succ_i z\}| > n/2$ , and  $|\{i \in N : z \succ_i x\}| > n/2$ .<sup>3</sup>

### 2.2 Argumentation

In order to be general with regards to the deliberation step, we build upon the abstract argumentation framework proposed by (Dung, 1995):

**Definition 1.** *[Argumentation framework (Dung, 1995)] An argumentation framework (AF) is a pair  $(\mathbf{A}, \mathbf{R})$ , where  $\mathbf{A}$  is a finite nonempty set of arguments and  $\mathbf{R}$  is a binary relation on  $\mathbf{A}$ , called attack relation. Let  $\mathcal{A}, \mathcal{B} \in \mathbf{A}$ ,  $\mathcal{A} \mathbf{R} \mathcal{B}$  means that  $\mathcal{A}$  attacks  $\mathcal{B}$ .*

**Definition 2.** *[Ranking-based semantics (Bonzon et al., 2016)] Given an AF  $(\mathbf{A}, \mathbf{R})$ , a ranking-based semantics  $\sigma$  associates a ranking  $\succeq^\sigma$  on  $\mathbf{A}$ .  $\succeq^\sigma$  is a preorder (a reflexive and transitive relation) on  $\mathbf{A}$ . For  $a, b \in \mathbf{A}$   $a \succeq^\sigma b$  means that  $a$  is at least as acceptable as  $b$ .*

**Definition 3.** *[Path, attackers and defenders (Bonzon et al., 2016)] Given an AF  $(\mathbf{A}, \mathbf{R})$ , and  $\mathcal{A}, \mathcal{B} \in \mathbf{A}$ . A path  $p_{\mathcal{B}, \mathcal{A}}$  is a sequence  $s = \langle a_0, \dots, a_n \rangle$  of arguments where  $a_0 = \mathcal{A}$ ,  $a_n = \mathcal{B}$  and  $\forall i < n$  we have that  $(a_{i+1}, a_i) \in \mathbf{R}$ . The length of the path is denoted by  $\ell_p$ . Note that  $\ell_p = n$ . A defender of an argument  $\mathcal{A}$  is an argument situated at the beginning of an even-length path leading to  $\mathcal{A}$ . Respectively, an attacker of  $\mathcal{A}$  is an argument situated at the beginning of a path of odd length. We denote the multiset of defenders and attackers of  $\mathcal{A}$  by  $Def_n(\mathcal{A}) = \{b \mid \exists p_{\mathcal{B}, \mathcal{A}}, \ell_p \in 2\mathbb{N}\}$  and  $Att_n(\mathcal{A}) = \{b \mid \exists p_{\mathcal{B}, \mathcal{A}}, \ell_p \in 2\mathbb{N} + 1\}$  respectively. The direct attackers of  $\mathcal{A}$  are the arguments in  $Att_1(\mathcal{A})$ . An argument  $\mathcal{A}$  is defended if  $Def_2(\mathcal{A}) \neq \emptyset$ .*

Based on these notions, we can now present the Voting Argumentation Framework combining the strengths of social choice and argumentation.

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<sup>3</sup>We are assuming an odd number of agents.

### 3 THE VOTING ARGUMENTATION FRAMEWORK

#### 3.1 Construction of the Voting Argumentation Framework

In order to take advantage of the reasoning capabilities of AF in social choice we define our model by constructing a special Argumentation Framework adapted for social choice and voting. In the following we describe the construction of this specialized framework, which we will call *Voting Argumentation Framework*, i.e.,  $\mathcal{VAF}$ .

We are going to distinguish between two types of arguments: “vote” arguments and “generic” arguments. We describe their role and the attacks that can occur between them in the following paragraphs.

##### 3.1.1 Vote Arguments

A vote argument  $\mathcal{A}_{r_{i,j}}$  represents the argument which considers the total order, i.e., ranking  $r_i$  produced by agent  $j$  and the justification provided by this agent for each of the pairwise comparisons included in her total order. We denote by  $\mathbf{A}_\mathcal{R} = \{\mathcal{A}_{r_{i,j}}, \forall j \in N\}$  the set of all the vote arguments and by  $\mathbf{A}_{\mathcal{R}_{xy}}$  the set of vote arguments where the preference relation  $x \succ y$  occurs in the ranking  $r_i$ .

Vote arguments cannot be attacked by other vote arguments, since it is natural to assume that for each distinct agent  $j$  a different vote argument is produced representing her preferences. Hence, two different votes given by two different agents are not considered inconsistent.

**Example 1.** Assume a decision problem with three agents  $\{v_1, v_2, v_3\}$  and three alternatives  $\{c_1, c_2, c_3\}$ . The agents after a deliberation phase provide the following preferences along with a justification:

- $v_1: c_1 \succ c_2 \succ c_3$
- $v_2: c_2 \succ c_3 \succ c_1$
- $v_3: c_3 \succ c_1 \succ c_2$

These preferences and their justifications are respectively represented by vote arguments  $\mathbf{A}_\mathcal{R} = \{\mathcal{A}_{r_{1,v_1}}, \mathcal{A}_{r_{2,v_2}}, \mathcal{A}_{r_{3,v_3}}\}$ .

##### 3.1.2 Generic Arguments

Generic arguments represent the deliberation phase and regroup all the other possible arguments that can arise during a debate. In particular, those arguments are able to attack other generic arguments and vote

arguments. Indeed, giving a reason contradicting a preference  $x \succ y$  of agent  $j$  triggers an attack on the vote argument  $\mathcal{A}_{r_{i,j}}$ . We denote this generic argument as a direct attacker of  $\mathcal{A}_{r_{i,j}}$ , i.e.,  $da_{r_{i,j}} \in Att_1(\mathcal{A}_{r_{i,j}})$ . The generic argument, denoted by  $g_{r_{i,j}}^k$ , which is situated in the beginning of a path leading to  $\mathcal{A}_{r_{i,j}}$  can either attack or defend a vote argument  $\mathcal{A}_{r_{i,j}}$ . Note that even if the premise of an argument  $g_{r_{i,j}}^k$  is the same as argument  $g_{r_{i',j'}}^k$ , we consider them as different arguments as they correspond to the vote of a different agent, i.e.,  $j$  versus  $j'$ . Hence, a generic argument  $g_{r_{i,j}}^k$  can not have a path leading to two different vote arguments. By  $k$  we denote the index of generic arguments attacking/defending  $\mathcal{A}_{r_{i,j}}$ . We denote by  $G$  the set of all the generic arguments and by  $G_{r_{i,j}}$  the set of generic arguments attacking  $\mathcal{A}_{r_{i,j}}$ .

**Example 1 (cont.).** Assume now that the agents enunciated eight generic arguments  $G = \{g_{r_{1,v_1}}^1, \dots, g_{r_{1,v_1}}^8\}$ ,  $i \in [1, 3]$ . Figure 1 presents the argumentation framework  $(\mathbf{A}_\mathcal{R} \cup G, \mathbf{R})$ , where  $\mathbf{R}$  is represented by the arrows between the arguments.

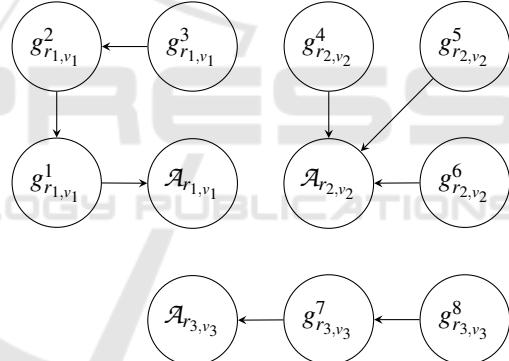


Figure 1: Voting argumentation framework of Example 1.

#### 3.2 A Quantitative Model for Decision Making based on the Voting Argumentation Framework

In the following we use  $\mathcal{VAF}$  in order to present a quantitative model for social choice and decision making problems that takes into account the reasoning behind the preferences. We start by defining our proposition for a new kind of semantics for computing the acceptability of the vote arguments. We call them Pairwise Comparison Semantics and can be seen as a kind of ranking-based semantics specially adapted to fit in the voting setting. The intuition is the same though, as each vote argument has a degree which denotes its strength, and hence the level of acceptability

in the outcome. We are then computing a new preference profile taking into account the strength and the acceptability of the vote arguments. This new profile will be thus “justified” as it is based on the outcome of the voting argumentation framework.

### 3.2.1 Pairwise Comparison Semantics

Let  $P_{r_{i,j}}$  the set of paths  $p_{g, \mathcal{A}_{r_{i,j}}}$  of attacks starting from a generic argument  $g \in G$  and leading to vote argument  $\mathcal{A}_{r_{i,j}}$ . The attacking power of a generic argument  $g_{r_{i,j}}^k$ , which is the starting point of a path  $p_{g_{r_{i,j}}^k, \mathcal{A}_{r_{i,j}}}$ , on a vote argument  $\mathcal{A}_{r_{i,j}}$  is denoted by  $ap(g_{r_{i,j}}^k)$  and is computed by the following formula.

$$ap(g_{r_{i,j}}^k) = \begin{cases} \frac{1}{\frac{m \cdot (m-1)}{2}} & \text{if an odd-length path } p_{g_{r_{i,j}}^k, \mathcal{A}_{r_{i,j}}} \text{ exists} \\ 0 & \text{otherwise} \end{cases}$$

**Example 1** (cont.). The attacking powers of the generic arguments that are the starting nodes of the paths attacking the vote arguments are the following.

$$\begin{aligned} ap(g_{r_{1,v_1}}^3) &= \frac{1}{3} \\ ap(g_{r_{2,v_2}}^4) &= ap(g_{r_{2,v_2}}^5) = ap(g_{r_{2,v_2}}^6) = \frac{1}{3} \\ ap(g_{r_{3,v_3}}^8) &= 0 \end{aligned}$$

Note that when the attacking power of a generic argument is 0 then this argument is defending the vote argument. The intuition for computing the attacking power function by the above formula is that in each vote argument there are  $\frac{m \cdot (m-1)}{2}$  pairwise comparisons between the alternatives, where  $m$  is the number of the alternatives. We assume that each generic argument refers to the justification of one pairwise comparison each time. Hence, the weight of each vote argument is reduced by  $\frac{1}{\frac{m \cdot (m-1)}{2}}$  when it is attacked by a generic argument. What we are actually interested in is the effect a generic argument has on the weight of the vote argument. Hence, the reason for denoting positive attacking power on a generic argument only if there exists a path of odd length starting from it, is that having such a path affects the weight of the vote argument as there exists an active direct attacker on the vote argument. In the case where there exists an even path means that the direct attacker is not active so the attacking power of the path is 0. Note that according to Dung’s preferred semantics an even path would defend the direct attacker of a vote argument even if an odd path exists. In our case though,

the semantics we use are not binary as Dung’s, i.e., an argument can either be included or excluded from an extension, but instead we propose semantics where the level of acceptability of an argument depends on its weight, which is reminiscent of ranking semantics (Bonzon et al., 2016). Hence, having an odd path is sufficient for the vote argument to decrease its acceptability value. It is reasonable to assume here that if one wants to define the effect of the deliberation phase, which is reflected through the generic arguments, in a different way then the attacking power function should be changed.

The attacking power of the set of generic arguments attacking  $\mathcal{A}_{r_{i,j}}$ , denoted by  $ap(G_{r_{i,j}})$ , is the sum of all the generic arguments attacking it, hence  $ap(G_{r_{i,j}}) = \sum_k ap(g_{r_{i,j}}^k)$ .

**Example 1** (cont.). The attacking power of the set of generic arguments attacking the vote arguments is the following for each one of them.

$$\begin{aligned} ap(G_{r_{1,v_1}}) &= \frac{1}{3} \\ ap(G_{r_{2,v_2}}) &= 1 \\ ap(G_{r_{3,v_3}}) &= 0 \end{aligned}$$

Since our goal is to design an AF towards social choice and voting we care about joining together the vote arguments  $\mathbf{A}_{r_i}$  that correspond to the same total order  $r_i$  rather than the single vote argument  $\mathcal{A}_{r_{i,j}}$  produced by agent  $j$  itself. We call *unification (coalition)* of arguments this joining of arguments  $\mathcal{A}_{r_{i,j}}$ , who have the same ranking  $r_i$ , into a meta-argument  $\mathbf{A}_{r_i}$  with higher weight. We define as  $ap(G_{r_i}) = \sum_j ap(G_{r_{i,j}})$  the attacking power of the generic arguments attacking  $\mathbf{A}_{r_i}$ . The attacking power of the whole deliberation phase is defined as  $ap(G) = \sum_{r_i} ap(G_{r_i})$ .

The weight of each vote argument is initially 1, hence  $w(\mathcal{A}_{r_{i,j}}) = 1$ . A total order  $r_i$  can appear  $|r_i|_\succ$  times in the preference profile  $\succ$ , which means that  $|r_i|_\succ$  agents have a preference order  $|r_i|$ . It is easy to see that if we sum up the initial weights of all the vote arguments expressing votes with total order  $r_i$ , then we get the number of appearances of  $|r_i|_\succ$  in the preference profile. Hence,  $|r_i|_\succ = \sum_j w(\mathcal{A}_{r_{i,j}})$ , where  $j$  denotes an agent voting for  $r_i$ .

For each vote argument  $\mathcal{A}_{r_{i,j}}$  we define its degree for the acceptability semantics as

$$d(\mathcal{A}_{r_{i,j}}) = \max \{0, w(\mathcal{A}_{r_{i,j}}) - ap(G_{r_{i,j}})\}.$$

**Example 1** (cont.). The weight of each vote argument is initially 1, hence  $w(\mathcal{A}_{r_{1,v_1}}) = w(\mathcal{A}_{r_{2,v_2}}) = w(\mathcal{A}_{r_{3,v_3}}) = 1$ . For each vote argument we compute its degree:

$$\begin{aligned} d(\mathcal{A}_{r_1,v_1}) &= 1 - \frac{1}{3} = \frac{2}{3} \\ d(\mathcal{A}_{r_2,v_2}) &= 1 - 1 = 0 \\ d(\mathcal{A}_{r_3,v_3}) &= 1 - 0 = 1 \end{aligned}$$

### 3.2.2 Computing the Justified Preference Profile under $\mathcal{VAF}$ and Pairwise Comparison Semantics

It is possible to compute the set of “coherent preferences”, i.e., the justified preference profile, by using the defined semantics on the voting argumentation framework. The above mentioned semantics define the acceptability degree of each vote argument in the justified preference profile  $\succ_{JP}$  under  $\mathcal{VAF}$ . For simplicity and onwards, when we refer to  $\succ_{JP}$  computed by  $\mathcal{VAF}$  and the pairwise comparison semantics, we will use just the  $\succ_{JP}$  symbol. In order to build the  $\succ_{JP}$  we take into account the degree of each vote argument. For each total order  $r_i$  we compute the utility/acceptability  $uv(r_i)$  value to denote its strength in the  $\succ_{JP}$ . For the computation we take into account the acceptability degrees of each of the vote arguments  $\mathcal{A}_{r_i,j}$  that refer to this total order ( $r_i$ ), i.e., the degree of meta-argument  $\mathbf{A}_{r_i}$  after the unification. Hence  $uv(r_i) = d(\mathbf{A}_{r_i}) = \sum_{\forall j} d(\mathcal{A}_{r_i,j})$ . It is easy to verify that  $uv(r_i) = |r_i|_{\succ} - ap(G_{r_i})$ . The number of times a ranking  $r_i$  appears in the justified preference profile  $\succ_{JP}$  is the ratio of the utility value of  $r_i$  over the sum of all the utility values multiplied by the total number of agents  $n'$  in  $\succ_{JP}$ . Hence,

$$|r_i|_{\succ_{JP}} = n' \cdot \frac{uv(r_i)}{\sum_{\forall r_i} uv(r_i)}.$$

The total number of agents in the  $\succ_{JP}$  is computed as following. Note that  $N'$  is the set of agents in the  $\succ_{JP}$ .

$$n' = \begin{cases} \sum_{\forall r_i} uv(r_i) & \text{if } \forall uv(r_i) \in \mathbb{N} \\ \frac{\sum_{\forall r_i} uv(r_i)}{gcd(uv(r_i), \forall r_i \in N')} & \text{if } \forall uv(r_i) \in \mathbb{Q} \setminus \mathbb{N} \end{cases}$$

When it is clear in the context and for notation simplicity, we refer to  $gcd$  for denoting the  $gcd(uv(r_i), \forall r_i \in N')$ , which is the greatest common divisor of all the utility values  $uv(r_i)$  that belong to the set of agents  $N'$  in the  $\succ_{JP}$ .

The need to have a integer number of agents leads to considering the multiplication of  $\sum_{\forall r_i} uv(r_i)$  by the  $gcd$ .

**Example 1** (cont.). In the following, we compute the justified preference profile  $\succ_{JP}$  by defining the number of times a ranking  $r_i$  appears in it. For each total order  $r_i$  we compute the utility  $uv(r_i)$  taking into account its acceptability degree. Hence, we have that  $uv(r_1) = \frac{2}{3}, uv(r_2) = 0, uv(r_3) = 1$ . Therefore,  $\sum_{\forall r_i} uv(r_i) = \frac{5}{3}$  and  $gcd(1, 0, \frac{2}{3}) = \frac{1}{3}$ . The total number of agents  $n'$  in  $\succ_{JP}$  is  $n' = \frac{5/3}{1/3} = 5$ . Therefore,

$$\begin{aligned} |r_1|_{\succ_{JP}} &= 5 \cdot \frac{2/3}{5/3} = 2 \\ |r_2|_{\succ_{JP}} &= 5 \cdot \frac{0}{5/3} = 0 \\ |r_3|_{\succ_{JP}} &= 5 \cdot \frac{1}{5/3} = 3 \end{aligned}$$

That means we have 2 agents with ranking  $c_1 \succ c_2 \succ c_3$  and 3 agents with  $c_3 \succ c_1 \succ c_2$  in the  $\succ_{JP}$ .

## 4 PROPERTIES OF $\mathcal{VAF}$ UNDER PAIRWISE COMPARISON SEMANTICS

In this section, we are going to study some desirable properties that should be satisfied by any voting argumentation framework. Due to space restriction, we omitted some proofs which can be found at the following link: <https://www.dropbox.com/s/195fb6r8h73lmoe/icaart-app.pdf?dl=0>.

### 4.1 $\mathcal{VAF}$ Desirable Properties from an Argumentative Perspective

We will study the properties presented in the literature of argumentation and ranking-based semantics that make sense to be satisfied by a Voting Argumentation Framework and its corresponding semantics. An overview of the desirable properties for ranking-based semantics can be found in (Bonzon et al., 2016). We believe that these properties should be taken into account when one wants to compute the social choice outcome. Under this perspective, we slightly change the definitions of the properties of Cardinality and Defense Precedence. Hence, we call the modified properties as Weak Cardinality Precedence and Weak Defense Precedence.

**Definition 4** (Weak Cardinality Precedence). *The greater the number of direct attackers for a vote argument, the weaker the level of acceptability of this argument. Given two vote arguments  $\mathcal{A}, \mathcal{B} \in \mathbf{A}_{\mathcal{R}}$  s.t.  $d(\mathcal{A}) \neq 0$  and  $d(\mathcal{B}) \neq 0$ ,*

$$|\text{Att}_1(\mathcal{A})| < |\text{Att}_1(\mathcal{B})| \Rightarrow \mathcal{A} \succ \mathcal{B}.$$

**Theorem 1.** *The justified preference profile computed by  $\mathcal{VAF}$  under pairwise comparison semantics ( $\succ_{\mathcal{JP}}$ ) satisfies Weak Cardinality Precedence.*

*Proof.* The degree  $d(\mathcal{A})$  of an argument  $\mathcal{A}$  defines the level of its acceptability in the framework. Thus, it suffices to show that given two vote arguments  $\mathcal{A}, \mathcal{B} \in \mathbf{A}_{\mathcal{R}}$  s.t.  $d(\mathcal{A}) \neq 0$  and  $d(\mathcal{B}) \neq 0$ , if  $|Att_1(\mathcal{A})| < |Att_1(\mathcal{B})|$  then  $d(\mathcal{A}) > d(\mathcal{B})$ .

If  $|Att_1(\mathcal{A})| < |Att_1(\mathcal{B})|$  then  $ap(G_{\mathcal{A}}) < ap(G_{\mathcal{B}})$ . The degree of a vote argument  $\mathcal{Y}$  is given by

$$d(\mathcal{Y}) = \max \{0, w(y) - ap(G_{\mathcal{Y}})\}.$$

Since  $d(\mathcal{A}) \neq 0$  and  $d(\mathcal{B}) \neq 0$ , we have that  $d(\mathcal{A}) = w(\mathcal{A}) - ap(G_{\mathcal{A}})$  and  $d(\mathcal{B}) = w(\mathcal{B}) - ap(G_{\mathcal{B}})$ . Also, we have that  $w(\mathcal{A}) = w(\mathcal{B}) = 1$ . Summing up, we have that  $d(\mathcal{A}) > d(\mathcal{B})$ .  $\square$

**Definition 5** (Weak Defense Precedence). *For two vote arguments with the same number of direct attackers, a defended argument is ranked higher than a non-defended argument. Given two vote arguments  $\mathcal{A}, \mathcal{B} \in \mathbf{A}_{\mathcal{R}}$  s.t.  $d(\mathcal{A}) \neq 0$  and  $d(\mathcal{B}) \neq 0$ ,*

$$\begin{aligned} |Att_1(\mathcal{A})| &= |Att_1(\mathcal{B})|, \\ Def_2(\mathcal{A}) &\neq \emptyset \text{ and } Def_2(\mathcal{B}) = \emptyset \end{aligned} \Rightarrow \mathcal{A} \succ \mathcal{B}.$$

**Theorem 2.** *The justified preference profile computed by  $\mathcal{VAF}$  under pairwise comparison semantics ( $\succ_{\mathcal{JP}}$ ) satisfies Weak Defense Precedence.*

*Proof.* It suffices to show that given two vote arguments  $\mathcal{A}, \mathcal{B} \in \mathbf{A}_{\mathcal{R}}$  s.t.  $d(\mathcal{A}) \neq 0$  and  $d(\mathcal{B}) \neq 0$ , if  $|Att_1(\mathcal{A})| = |Att_1(\mathcal{B})|$ ,  $Def_2(\mathcal{A}) \neq \emptyset$  and  $Def_2(\mathcal{B}) = \emptyset$  then  $d(\mathcal{A}) > d(\mathcal{B})$ .

Let  $x = |Att_1(\mathcal{A})| = |Att_1(\mathcal{B})|$  and  $|Def_2(\mathcal{A})| = y$ , with  $x, y > 0$ . Note that  $d(\mathcal{B}) \neq 0$  and  $|Def_2(\mathcal{B})| = 0$ . Hence, the degree of  $\mathcal{B}$  is  $d(\mathcal{B}) = w(\mathcal{B}) - ap(G_{\mathcal{B}})$ , where  $ap(G_{\mathcal{B}}) = x \cdot \frac{1}{m \cdot (m-1)}$ . Also, note that  $d(\mathcal{A}) \neq 0$ , and hence the degree of  $\mathcal{A}$  is  $d(\mathcal{A}) = w(\mathcal{A}) - ap(G_{\mathcal{A}})$ , where  $ap(G_{\mathcal{A}}) = (x-y) \cdot \frac{1}{m \cdot (m-1)}$ . From the above conditions we have that  $ap(G_{\mathcal{B}}) > ap(G_{\mathcal{A}})$ , and since  $w(\mathcal{A}) = w(\mathcal{B}) = 1$ , we conclude that  $d(\mathcal{A}) > d(\mathcal{B})$ .  $\square$

## 4.2 $\mathcal{VAF}$ Desirable Properties with Respect to Social Choice

In this section we study the properties that should be satisfied by a Voting Argumentation Framework and its corresponding semantics from a social-choice theoretic scope.

### 4.2.1 Justified Preference Profile Consistency with Respect to AF

The first property refers to the relation the Voting Argumentation framework should have with respect to the produced justified preference profile. We will show that the following property is satisfied when pairwise comparison semantics are applied to  $\mathcal{VAF}$  for computing the justified preference profile.

**Definition 6** (Justified Preference Profile consistency with respect to AF). *If a unification of vote arguments  $\mathbf{A}_{r_i}$  is stronger than  $\mathbf{A}_{r_j}$ , i.e., the degree of  $\mathbf{A}_{r_i}$  is higher than  $\mathbf{A}_{r_j}$ , then the corresponding total order  $r_i$  should appear more times than the total order  $r_j$  in the justified preference profile.*

**Example 1** (cont.: Justified Preference Profile consistency with respect to AF). *Assume that another vote argument  $\mathcal{A}_{r_{1,v_4}}$  is added to the previous voting argumentation framework. Figure 2 presents this new  $\mathcal{VAF}$ .*

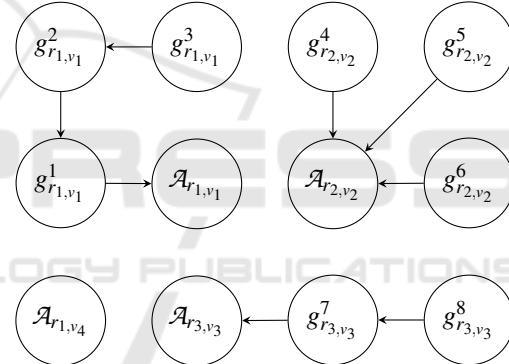


Figure 2:  $\mathcal{VAF}$  exemplifying the Justified Preference Profile consistency with respect to AF property.

We can now recompute the justified preference profile  $\succ_{\mathcal{JP}}$  the same way as before. We have that  $d(\mathbf{A}_{r_1}) = d(\mathcal{A}_{r_{1,v_1}}) + d(\mathcal{A}_{r_{1,v_4}}) = \frac{2}{3} + 1 = \frac{5}{3}$ ,  $d(\mathbf{A}_{r_2}) = 0$ ,  $d(\mathbf{A}_{r_3}) = 1$ . Therefore,  $\sum_{\forall r_i} uv(r_i) = \frac{5}{3} + 1 = \frac{8}{3}$  and  $gcd(1, 0, \frac{5}{3}) = \frac{1}{3}$ . The total number of agents  $n'$  in  $\succ_{\mathcal{JP}}$  is  $n' = \frac{8/3}{1/3} = 8$ . Therefore,

$$\begin{aligned} |r_1|_{\succ_{\mathcal{JP}}} &= 8 \cdot \frac{5/3}{8/3} = 5 \\ |r_2|_{\succ_{\mathcal{JP}}} &= 8 \cdot \frac{0}{8/3} = 0 \\ |r_3|_{\succ_{\mathcal{JP}}} &= 8 \cdot \frac{1}{8/3} = 3 \end{aligned}$$

Hence, now  $r_1$  is the ranking with the highest degree and it appears more times in  $\succ_{\mathcal{JP}}$  so the property is satisfied.

**Theorem 3.** *The justified preference profile computed by  $\mathcal{VAF}$  under pairwise comparison semantics ( $\succ_{JP}$ ) satisfies Justified Preference Profile consistency with respect to AF.*

*Proof.* Let us suppose that a unification of vote arguments  $\mathbf{A}_{r_i}$  is stronger than  $\mathbf{A}_{r_j}$ , and hence the degree  $d(\mathbf{A}_{r_i}) > d(\mathbf{A}_{r_j})$  or  $uv(r_i) > uv(r_j)$ . The corresponding total order  $r_i$  appears in the  $\succ_{JP}$ ,  $|r_i|_{\succ_{JP}} = n' \cdot \frac{uv(r_i)}{\sum_{\forall r} uv(r)}$  times while  $|r_j|_{\succ_{JP}} = n' \cdot \frac{uv(r_j)}{\sum_{\forall r} uv(r)}$ . Given that  $uv(r_i) > uv(r_j)$ , we conclude that  $|r_i|_{\succ_{JP}} > |r_j|_{\succ_{JP}}$ .  $\square$

#### 4.2.2 Social Choice Profile Consistency

The second property that we are going to check is a new but fundamental property that quantitative voting argumentation frameworks should have. We call it the Social choice profile consistency. The intuition behind this property is that if there is no deliberation phase for the preferences of the agents then the outcome of the Voting Argumentation Framework, i.e., the justified preference profile should be the same as the outcome of the original social choice profile, since there is no new information or conflicts expressed by arguments. More formally, this property can be defined as follows.

**Definition 7** (Social choice profile consistency). *The semantics of a Voting Argumentation Framework satisfies Social choice profile consistency if when there is no deliberation phase, i.e., no generic arguments attacking the vote arguments, then the justified preference profile should be the same as the original social choice profile.*

**Theorem 4.** *The justified preference profile computed by  $\mathcal{VAF}$  under pairwise comparison semantics ( $\succ_{JP}$ ) satisfies Social choice profile consistency.*

*Proof.* Let  $\succ$  be the social choice preference profile and  $\succ_{JP}$  the justified preference profile for the  $\mathcal{VAF}$  when the pairwise comparison semantics is applied. In order to prove that  $\mathcal{VAF}$  satisfies this property we have to show that each total order  $r_i \in \succ$  is also included in  $\succ_{JP}$  the same number of times.

Let  $r_i$  be a given ranking included in  $\succ$ . It suffices to show that  $|r_i|_{\succ} = |r_i|_{\succ_{JP}}$ , where  $|r_i|_{\succ}$  is the number of times  $r_i$  appears in  $\succ$  and  $|r_i|_{\succ_{JP}}$  the number of times it appears in  $\succ_{JP}$ .

Recall that

$$|r_i|_{\succ_{JP}} = n' \cdot \frac{uv(r_i)}{\sum_{\forall r} uv(r)}$$

Also, recall that  $uv(r_i) = \sum_{\forall j} d(\mathcal{A}_{r_{i,j}})$  and  $d(\mathcal{A}_{r_{i,j}}) = w(\mathcal{A}_{r_{i,j}}) - ap(G_{r_{i,j}})$ .

Note that  $|r_i|_{\succ} = \sum_{\forall j} w(\mathcal{A}_{r_{i,j}})$  under pairwise comparison semantics and that  $\sum_{\forall j} ap(G_{r_{i,j}}) = 0$  since there are no attacks on the preferences. Hence,  $uv(r_i) = |r_i|_{\succ}$ .

The number of agents  $n'$  in the  $\succ_{JP}$  is  $\sum_{\forall r} uv(r_i)$  since  $uv(r_i)$  equals to  $|r_i|_{\succ}$  and therefore each  $uv(r_i)$  belongs to the set of natural numbers  $\mathbb{N}$ . Hence,

$$|r_i|_{\succ_{JP}} = \frac{|r_i|_{\succ}}{\sum_{\forall r} uv(r_i)} \cdot \sum_{\forall r} uv(r_i) = |r_i|_{\succ}$$

$\square$

#### 4.2.3 Measuring the Effect of Deliberation on Avoiding Voting Cycles

In this section we are going to study the impact of a deliberation phase in social choice theory and especially on voting cycles, i.e., the Condorcet Paradox. The intuition that triggered this study is that if there exists an amount of ambiguous information in the preferences of the agents the deliberation phase can reveal it using the imposed arguments and “correct” the misinformation on the preferences of the agents that trigger voting cycles. In order to study this impact we are measuring the effect of a deliberation phase in producing a justified preference profile where there are no voting cycles. The metric that we are going to use to measure the impact is the number of arguments needed to attack preferences of the original profile so that no cycles exist. In other words, it is the attacking power of the generic arguments for total order  $r_i$ , i.e.,  $ap(G_{r_i})$ .

**Theorem 5.** *Given that for alternatives  $a, b, c \in A$ ,  $a$  beats  $b$  and  $b$  beats  $c$  in both  $\succ$  and  $\succ_{JP}$ , the justified preference profile computed by  $\mathcal{VAF}$  does not produce any voting cycle when the following conditions on the attacking power of the generic arguments hold.*

1.  $\sum_{rac \in \mathcal{R}_{ac}} ap(G_{rac}) \leq \sum_{rca \in \mathcal{R}_{ca}} ap(G_{rca})$ , when  $|\mathcal{R}_{ac}|_{\succ} > |\mathcal{R}_{ca}|_{\succ}$
2.  $\sum_{rac \in \mathcal{R}_{ac}} ap(G_{rac}) > \sum_{rca \in \mathcal{R}_{ca}} ap(G_{rca})$ , when  $|\mathcal{R}_{ac}|_{\succ} - |\mathcal{R}_{ca}|_{\succ} > \sum_{rac \in \mathcal{R}_{ac}} ap(G_{rac}) - \sum_{rca \in \mathcal{R}_{ca}} ap(G_{rca})$
3.  $\sum_{rac \in \mathcal{R}_{ac}} ap(G_{rac}) < \sum_{rca \in \mathcal{R}_{ca}} ap(G_{rca})$ , when  $|\mathcal{R}_{ac}|_{\succ} < |\mathcal{R}_{ca}|_{\succ} < \sum_{rca \in \mathcal{R}_{ca}} ap(G_{rca}) - \sum_{rac \in \mathcal{R}_{ac}} ap(G_{rac}) + |\mathcal{R}_{ac}|_{\succ}$

The above relations state that in order to avoid the cycle we must have, for 3 alternatives  $a, b, c \in A$  where  $a$  beats  $b$  and  $b$  beats  $c$  in  $\succ$  and  $\succ_{JP}$ , the following conditions:

1. The attacking power for the vote arguments  $\mathbf{A}_{\mathcal{R}_{ac}}$  is less or equal to the attacking power for the vote arguments  $\mathbf{A}_{\mathcal{R}_{ca}}$ , when the majority of agents prefer  $a$  over  $c$  in the preference profile  $\succ$ .

2. The attacking power for the vote arguments  $\mathbf{A}_{\mathcal{R}_{ac}}$  is greater than the attacking power for the vote arguments  $\mathbf{A}_{\mathcal{R}_{ca}}$ , when the difference in number of agents preferring  $a \succ c$  to  $c \succ a$  is lower bounded by the difference of the corresponding attacking powers.
3. The attacking power for the vote arguments  $\mathbf{A}_{\mathcal{R}_{ac}}$  is less than the attacking power for the vote arguments  $\mathbf{A}_{\mathcal{R}_{ca}}$ , when the majority of agents prefer  $c$  over  $a$  in the preference profile  $\succ$  and the difference in number of agents preferring  $c \succ a$  to  $a \succ c$  is upper bounded by the difference of the corresponding attacking powers.

### 4.3 $\mathcal{VAF}$ Attitude Towards Classical Social Choice Desirable Properties

In this section we explore the behaviour of the proposed approach towards classical desirable properties from the viewpoint of Social Choice. Apart from the properties that should be satisfied by a Voting Argumentation Framework and its corresponding semantics which are related to the Social choice outcome, we are also referring to classical desirable properties in order to further evaluate  $\mathcal{VAF}$  and pairwise semantics. We begin by studying the properties of homogeneity and monotonicity which both belong to the stability category as recognized by social choice theorists. The properties of this category are concerned with ensuring that the winning set remains the same when the changes in the preference profile arguably should not modify the winning set. Our goal here is to evaluate the effect of deliberation and argumentation in voting and investigate how the consideration of the deliberation phase along with voting can affect the outcome in respect with the social choice properties.

#### 4.3.1 Homogeneity

The first property discussed in this section is the one of homogeneity. A method is homogeneous if the replication<sup>4</sup> of the preference profile does not change the winning set of the alternatives. We prove the following positive result for our proposed method which ensures that replicating the original instance of the problem, i.e., the agents' preferences and the justifications, will have no effect on the decision output as long as the voting rule used for the aggregation of the justified preference profile satisfies homogeneity. This property is widely accepted among social choice

theorists since making an *exact copy* of the agents' preferences  $x$  times should not have an effect on the winning set.

**Example 1** (cont.: Homogeneity). *Assume that the original profile along with its arguments is replicated 2 times. The following vote arguments are added:  $\mathcal{A}'_{r_1,v_1}, \mathcal{A}'_{r_2,v_2}, \mathcal{A}'_{r_3,v_3}, \mathcal{A}'_{r_1,v_4}$ . For  $\succ'_{JP}$  we have that  $d(\mathbf{A}_{r_1}) = d(\mathcal{A}_{r_1,v_1}) + d(\mathcal{A}'_{r_1,v_1}) + d(\mathcal{A}'_{r_1,v_4}) = \frac{10}{3}$ ,  $d(\mathbf{A}_{r_2}) = 0$ ,  $d(\mathbf{A}_{r_3}) = 2$ . Therefore,  $\sum_{\forall r_i} uv(r_i)' = \frac{16}{3}$  and  $gcd(2, 0, \frac{10}{3}) = \frac{2}{3}$ . The total number of agents is  $n'_{\succ'_{JP}} = \frac{16/3}{2/3} = 8$ . Therefore,  $|r_1|_{\succ'_{JP}} = 5$ ,  $|r_2|_{\succ'_{JP}} = 0$  and  $|r_3|_{\succ'_{JP}} = 3$ . Hence, the justified replicated profile is the same as the original justified profile. Therefore an application of a voting rule on both profiles will have the exact same winning ranking and the property is satisfied.*

**Theorem 6.**  $\mathcal{VAF}$  satisfies homogeneity if the voting rule used for the aggregation of  $\succ_{JP}$  satisfies also homogeneity.

#### 4.3.2 Monotonicity

The second property discussed in this section is the one of monotonicity. A method is monotonic if a winning alternative remains the winning one in the new profile which is created after she is moved upward in the preferences of some of the agents. We prove the following positive result for our proposed method which ensures that if an alternative who is a winner in an instance of the problem is moved upwards in the preferences of some agents then this will have no effect on the decision output of the new instance as long as (1) the voting rule used for the aggregation of the justified preference profile satisfies monotonicity, (2) the attacking power on the votes where the winner has a better position (compared to the original profile) is equal to the attacking power on the corresponding votes in the original profile. This property is also widely accepted among social choice theorists since improving the position of a winner alternative in some agents should not make him worse and should remain the winner.

**Theorem 7.**  $\mathcal{VAF}$  under pairwise semantics satisfies monotonicity if the voting rule used for the aggregation of  $\succ_{JP}$  satisfies monotonicity and the attacking power of the  $w$ -improvement votes is equal to the attacking power of the votes that change in the original preference profile.

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<sup>4</sup>Please note that, in our context, the replication concerns both the preference profile and the arguments referring to this preference profile.

### 4.3.3 Consistency Related to Majority

In this section, we present our results about classical properties coming from Social Choice Theory which in general concern the consistency between the outcomes of the given method and the majority rule.

**Condorcet Consistency.** One of the most meaningful properties is the Condorcet consistency. This property states that a method  $\mathcal{F}$  satisfies Condorcet consistency if whenever there is an alternative  $c$  who beats every other alternative in a pairwise comparison (i.e.,  $c$  is the dominant alternative) then  $c$  is the winner under  $\mathcal{F}$ .

**Theorem 8.**  *$\mathcal{VAF}$  under pairwise semantics satisfies Condorcet consistency if the voting rule used for the aggregation of  $\succ_{JP}$  satisfies Condorcet consistency and the following condition holds: for any alternatives  $a, c \in A$*

$$|\mathcal{R}_{ca}|_\succ - |\mathcal{R}_{ac}|_\succ > ap(G_{\mathcal{R}_{ca}}) - ap(G_{\mathcal{R}_{ac}})$$

where  $\mathcal{R}_{xy}$  denotes the number of agents that rank alternative  $x$  over  $y$  and  $ap(G_{\mathcal{R}_{xy}})$  denotes the attacking power of the generic arguments on the arguments of the agents that rank  $x$  over  $y$  and  $c$  is the dominant alternative.

The theorem states that Condorcet consistency is satisfied if the difference in the number of agents preferring the dominant alternative  $c$  to every other alternative  $a$  minus the number of agents preferring  $a$  to  $c$  is greater than the difference of the corresponding attacking power on the vote arguments.

**Invariant loss consistency.** This property is also based on the Condorcet's intuition and is similar to the Condorcet consistency. It states that a method  $\mathcal{F}$  satisfies Invariant loss consistency if whenever there is an alternative  $c$  who is beaten by every other alternative in a pairwise comparison then  $c$  cannot be the winner under  $\mathcal{F}$ .

**Theorem 9.**  *$\mathcal{VAF}$  under pairwise semantics satisfies Invariant loss consistency if the voting rule used for the aggregation of  $\succ_{JP}$  satisfies Invariant loss consistency and following condition holds: for any alternatives  $a, c \in A$*

$$|\mathcal{R}_{ca}|_\succ - |\mathcal{R}_{ac}|_\succ < ap(G_{\mathcal{R}_{ca}}) - ap(G_{\mathcal{R}_{ac}})$$

where  $\mathcal{R}_{xy}$  denotes the number of agents alternative  $x$  over  $y$  and  $ap(G_{\mathcal{R}_{xy}})$  denotes the attacking power of the generic arguments on the arguments of the agents that rank  $x$  over  $y$  and  $c$  is the alternative who is beaten by every other alternative (i.e, the Condorcet loser).

## 5 CONCLUSION AND FUTURE WORK

In this paper, we have proposed a method for group decision-making that is built upon the justified preferences of the agents. Our method is able to simulate real decision problems where the decision outcome relies on agents' preferences and their reasoning. Our design choices assume and comply with a deliberation phase in which a discussion is conducted among the agents of the group so that the preferences along with justifications are revealed. In order to do so, we introduced a voting argumentation framework and its corresponding semantics. We proposed a method for computing a new preference profile based on the acceptability semantics of the vote arguments. We proved several properties from an argumentative and social choice theoretic point of view on the so-called justified preference profile.

In terms of future work, we plan to extend our research towards the proposed modelling and characterize other properties of social choice and argumentation that are satisfied by  $\mathcal{VAF}$ . Also, another future step is to define another kind of acceptability semantics and explore its properties. For example, it would be interesting to design semantics that permit us to avoid the Condorcet paradox under any case. Furthermore, it is also appealing to design different kinds of semantics which are specially adapted to distinctive real decision problems. Indeed, with the semantics proposed in this paper, the generic arguments have the same strength, but there are cases where generic arguments attacking vote arguments are not of equal importance. It is therefore a motivation to design a semantics with graded strength of generic arguments. It is also interesting to investigate more on the properties from an argumentative perspective (such as the ones defined in (Bonzon et al., 2016)) for the proposed semantics and identify which ones are respected. Finally, it is left for future work a practical application of this method in real decision problems. It will be very interesting to design a real experiment and see how the proposed method works by measuring the satisfaction of the agents for the decision outcome when their justifications are taken into account compared to unjustified preferences where pure social choice methods are used.

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