

Container Yard Allocation under Uncertainty

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Abstract: This paper investigates allocation of space in storage yard to export containers under uncertain shipment information. We define two types of stacks: one is called the dedicated stack and the other is called the shared stack. Since containers meant for the same destination are assigned to dedicated stacks in the same block, no re-handling is required for containers in dedicated stacks. However, containers in shared stacks have different destinations; re-handling is required. We develop a two-stage stochastic recourse programming model for determining an optimal storage strategy, called the dual-response storage strategy. The first-stage response, regarding the allocation of containers to dedicated stacks, is made before accurate shipment information becomes available. The second-stage response, regarding allocation of additional containers to shared stacks, is taken after realization of stochasticity. Then, the unused spaces in the yard area can be released for other purposes. Computational results are provided to demonstrate the effectiveness of the proposed dual-response storage strategy obtained from the stochastic model.

1 INTRODUCTION

Containers were first used for international sea transportation in the 1950s, and the proportion of containerized items has been steadily increasing since then. Today, over 60% of the world deep sea cargo is transported in containers, where some routes, especially between economically strong and stable countries, are containerized up to 100% (Steenken et al., 2004). Containers are standardized steel boxes in three lengths, 20, 40 and 45 feet, and 8 feet wide and either 8.5 or 9 feet high. This standardization offers advantages of simplified discharging and loading of containers, protection against weather and pilferage and improved process of scheduling and controlling facilities, etc. Container terminals are places where containerized cargo is temporarily stored, before being shipped to the destination. The increased volume of container shipments has resulted in increased demand for seaport container terminals, container logistics and management, and the related technical equipment. Heightened competition between seaports, especially between geographically close ports, is a result of this development.

The container storage area in the terminal is usually separated into rectangular regions, called blocks, which are further segmented into rows, bays and tiers. The width of a block is typically divided

into several rows, one for trucks that interact with yard cranes and others for storing containers. Blocks are divided along their length into bays. Each bay is made up of several container stacks of a certain height (3 ~ 6 tiers). Containers are stored one on top of another to form a stack. The blocks are usually separated into areas allocated for export, import, special (such as reefer, dangerous, overweight/over width), and empty containers (Steenken et al., 2004).

A vessel normally visits a sequential list of ports, called the shipping route. A number of containers are discharged from the vessel to the port terminals along its shipping route. The locations occupied by these import containers on the vessel become available for loading new export containers from the terminals to the vessel. Export containers are assigned to specific locations on the vessel such that they can be easily discharged when the vessel arrives at the ports where they are to be discharged. However, for container terminals, decisions are different from the vessels. Containers pass through a terminal in three ways: imported, exported and transhipped. Import containers arrive in batches, in vessels, and leave the terminal by truck or rail, while export containers normally come at the terminals one-by-one, by trucks, in a random manner, and leave by vessels. Transhipped containers arrive and leave terminals in vessels. In practice, accurate shipment information,

particularly for export containers, is usually difficult to obtain because of the uncertainty involved in the process of delivering the containers to the yard (for example, truck delay and urgent shipment requirement, etc.). Yard managers are increasingly challenged by limited yard capacities, and the uncertain and dynamic information involved in the decision-making process. Therefore, yard managers need new methodologies to help them make better decisions about allocation of yard space.

In this paper, we assume that yard managers can obtain uncertain shipment information from their customers regarding destinations and quantity of containers to be shipped. The yard managers have to determine the storage yard plan before accurate shipment information is available. One of the methods that the yard managers adopt in practice is to place the containers heading for the same destination in the same block. Therefore, the number of blocks is equal to the number of destinations/ports, where the containers are to be discharged. The advantages of this strategy is that containers can be easily loaded from the yard to the ship without re-handling. However, some spaces may not be occupied at all because of the uncertain shipment information. This is particularly true when the possibility of high shipment demand is low. In this paper, it is assumed that the yard is divided into different blocks, and each block has the same size/capacity. We conceptually divide each block into two portions: a set of dedicated stacks and a set of shared stacks. Containers in dedicated stacks within the same block have the same destination (i.e. they will go to the same port). Containers in shared stacks have different destinations. Therefore, rehandling or reshuffling may be required in shared stacks. Rehandling happens when containers placed on the top of the required one have to be removed first. Rehandling is one of the most unproductive operations in the yard area. The workload at the terminals can be significantly reduced if no or limited number of rehandling occurs. However, containers assigned to dedicated stacks can be loaded to the ship sequentially, without the need of rehandling. It is noted that dedicated and shared stacks are not divided physically. In each block, there are two portions: one is for dedicated stacks and the other is for shared stacks. In addition, each block has a special stack to be used for re-handling containers in shared stacks; this stack can store no more than one container so that other containers in shared stacks of this block can be temporarily placed in the stack during the process of re-handling.

Since only containers in the shared stacks require

re-handling, the number of containers that require re-handling in each block is limited. Therefore, managers reserve only one stack in each block for re-handling. However, the traditional sharing strategy, in which all containers are mixed up, may require more than a stack in each block for re-handling, since re-handling happens frequently. Sometimes, a stack in each bay is reserved for re-handling in practice because frequent movement within a block might cause safety concerns.

Although the concept of separate dedicated and shared portions has already been used in some terminals, yard managers are increasingly facing the challenge of determining the split between dedicated and shared stacks under uncertain shipment information. Steenken et al. (2004) state that the need for optimization of container terminal operations has become an important issue in recent years. In this paper, we propose a dual-response storage strategy to deal with uncertain shipment information for export containers. At the first stage, before the accurate shipment information is available, yard managers need to make the first response by determining how the dedicated stacks in each block should be allocated for storage of containers. At the second stage, when the uncertain shipment information is realized, yard managers need to respond to the situation by determining the size of the shared stacks in each block to store extra containers. As a result, spaces still left in the blocks will be free for use.

The main problem considered in this paper is to determine the optimal size of spaces to be reserved for the dedicated stacks, as well as the shared stacks, such that the total operational cost can be minimized. In order to obtain an optimal dual-response storage strategy, we formulate a two-stage stochastic recourse programming model. The rest of the paper is organized as follows. Section 2 provides the literature review on storage management at container terminals and stochastic modelling for allocation problems at container terminals. Section 3 provides notations and definitions for modelling the storage problem. Section 4 presents a two-stage stochastic model for storage management under uncertainty. Section 5 shows computational results and analysis. The final section gives the conclusions of this paper and recommendations for future research.

2 LITERATURE REVIEW

Due to the growing importance of maritime transportation, operations of sea container terminals have received increasing attention from researchers

(Froyland et al., 2008). Excellent review papers about detailed descriptions and classification of operations of container terminals are provided by Vis and De Koster (2003), Steenken et al. (2004), and Stahblock and Voß (2008). The problem to be discussed in this paper belongs to the category of storage yard management in the literature. Storage management addresses the assignment of locations to containers, which includes space allocated to containers moving into and out of storage yards as well as reshuffles/rehandles (Froyland et al., 2008). Chen (1999) investigates yard operations in Taiwan, Hong Kong, Korea and UK. Kim and Park (2003) study storage space allocation for export containers. A mixed-integer programming model is proposed for efficient utilization of storage space and efficient loading. Two heuristics approaches are designed for solving this problem. Lee and Hsu (2007) propose an integer programming model for the container pre-marshalling problem with a vessel and a rail mounted gantry crane. Jin et al. (2016) study the daily storage yard manage problem arising in maritime container terminals, which integrates the space allocation and yard crane deployment decisions together with the consideration of container traffic congestion in the storage yard. Lin and Chiang (2017) investigate the storage space allocation problem at a container terminalgantry crane in Taiwan. A decision rule-based heuristic is proposed. Extant literature on storage management problems has mainly discussed deterministic situations where all accurate information is available at the time when decisions are made. Unfortunately, storage planning based on available (at the time of decision-making) information seldom matches the real situation because container delivery is a stochastic process that cannot be exactly foreseen (Steenken et al., 2004). Zhen et al. (2011) propose stochastic programming models for managing container terminal operations. A meta-heuristic approach is proposed to solve the above problem in large-scale real environments. Zhen (2013) exams sstorage allocation in transshipment hubs under uncertainties and proposes a real-time decision support system (DSS) which can act as an ultimate solution for coping with uncertainties in yard storage allocation process.

3 NOTATIONS AND DEFINITIONS

3.1 Known Parameters

It is assumed that there are a total of N destinations, indexed by n . Since the yard area is equally divided into N blocks according to containers' destination, n also represents the index of blocks. Each block consists of dedicated and shared stacks. Containers in dedicated stacks (within the same stack) have the same destination, while containers in shared stacks may have different destinations. Parameters R , B , H represent the maximum numbers of rows, bays and tiers for any block. C represents the maximum capacity of yard. Since each block has $(H-1)$ spaces for reshuffling, we have:

$$C = N*[R*B*(H-1)].$$

It is also assumed that the total capacity of the storage yard is adequate to accommodate the total quantity of export containers under any possible scenario. If the total capacity of the yard is fully used, further demand from customers will be either rejected, or handled by other terminal operators. The cost of handling these extra containers is not considered in our model.

3.2 Stochastic Parameters

$\xi = \xi(\omega)$ is a random vector, which represents the stochastic nature of arrivals of export containers. $\omega \in \Omega$, (Ω, F, P) is a probability space. Let $p(\omega)$ represent the probability density function of ω , $p(\omega) \geq 0$, for all $\omega \in \Omega$, $\int p(\omega)d\omega = 1$.

$$\mathbf{d}(\xi) = (d_1(\xi), d_2(\xi), \dots, d_N(\xi)),$$

is a random vector, where $d_n(\xi)$ represents the random shipment demand in the n th destination ($n = 1, 2, \dots, N$).

3.3 Decision Variables

Under uncertain shipment information, yard managers have to decide how many containers should be assigned to the dedicated stacks in each block. Therefore, we have a set of decisions to be taken without accurate information on the stochastic demand. These decisions are called the first stage decisions, which are represented by vector:

$$\mathbf{x} = (x_1, x_2, \dots, x_N),$$

where x_n represents the quantity of containers to be assigned to the dedicated stacks in the n th block ($n = 1, 2, \dots, N$). When the full information is received on realization of random vector ξ , the second stage ac-

tions are taken, which are represented by vector:

$$(y(\xi)) = y_1(\xi), y_2(\xi), \dots, y_N(\xi)),$$

where $y_n(\xi)$ represents the quantity of container to be assigned to the shared stacks in the n th block ($n = 1, 2, \dots, N$).

3.4 Costs

The unit cost of handling containers in the dedicated and shared stacks plays an important role in the model. We use the following parameters to represent some crucial factors that have impact on handling cost:

- α_n represents the average cost of assigning and holding a container in the n th block. Note that in some container terminals, the cost of using a container location in a dedicated stack can be bid by yard managers. It can also be considered as the unit cost of storing containers. Here, we write the coefficients α_n in a vector-form as $\boldsymbol{\alpha} := (\alpha_1, \alpha_2, \dots, \alpha_N)^T$.
- γ_n represents the average cost of moving a container in a dedicated stack in the n th block. The movement includes lifting the container, putting it on a trailer or an internal truck, and transporting it to the quay side. This cost is calculated by yard managers from the average movement cost for individual containers. We write the coefficients γ_n in a vector-form as $\boldsymbol{\gamma} := (\gamma_1, \gamma_2, \dots, \gamma_N)^T$.
- β represents the average cost of assigning and holding a container in the shared stack. In some container terminals, the cost of using a space in the shared stack can be bid by yard managers. It can also be regarded as the unit cost of storing a container in the shared stack. One of the simplest methods to estimate this parameter is to divide the annual cost of assigning and holding space in the shared stacks by the total number of all spaces used in the shared stacks in a year.
- λ represents the average cost of moving a container in the shared stack. The movement includes searching, re-handling, lifting of the container, and then loading it on a trailer or internal truck, and transporting it to the quay side. It is noted that the average cost of moving a container in the shared stack is significantly higher than the dedicated stack because it involves searching and re-handling, which does not occur in the dedicated stack. One of the natural ways to estimate this parameter is to divide the annual cost related to movements (such as searching, re-handling, lifting, releasing and transporting) of containers in the shared stacks by the total number of containers stored in the shared stacks

per year

4 A TWO-STAGE STOCHASTIC REOURSE PROGRAMMING MODEL FOR YARD STORAGE ALLOCATION UNDER UNCERTAINTY

4.1 A General Two-Stage Stochastic Model for Uncertain Yard Storage Allocation Problems

A two-stage stochastic recourse programming model for determining a dual-response storage allocation strategy is formulated as follows:

$$\min_{\boldsymbol{x}} \boldsymbol{a}^T \boldsymbol{x} + E_{\xi}(Q(\boldsymbol{x}, \xi))$$

subject to

$$0 \leq x_n \leq C/N, \text{ integer for } n = 1, 2, \dots, N, \quad (1)$$

Where $Q(\boldsymbol{x}, \xi)$ is the optimal solution of the second stage problem:

$$\begin{aligned} Q(\boldsymbol{x}, \xi) = & \min_{\boldsymbol{y}} \{ \beta e^T \boldsymbol{y}(\omega) + \gamma^T \min[\boldsymbol{x}, \boldsymbol{d}(\omega)] \\ & + \lambda e^T \max[0, \boldsymbol{d}(\omega) - \boldsymbol{x}] : e^T \boldsymbol{x} \\ & + e^T \boldsymbol{y}(\omega) \leq C; \boldsymbol{x}_n + \boldsymbol{y}(\omega) \\ & \geq \boldsymbol{d}_n(\omega); \boldsymbol{x}, \boldsymbol{y}(\omega) \geq 0, \text{ integer}, \omega \\ & \in \Omega, n = 1, 2, \dots, N \} \end{aligned}$$

In (1), $e = [1, 1, \dots, 1]^T$ represents an appropriate dimension. The first stage decisions (x_n) are independent of realization of the stochastic variable ξ . It means no matter what shipment demand is realized (i.e. how containers will arrive), the first-stage decisions remain the same. However, yard managers can make different responses (the second stage decision $y_n(\xi)$) for any shipment situation that might happen.

In (1), for vectors $\boldsymbol{u} = (u_1, u_2, \dots, u_N)^T$ and $\boldsymbol{v} = (v_1, v_2, \dots, v_N)^T$, functions $\min[\boldsymbol{u}, \boldsymbol{v}]$ and $\max[\boldsymbol{u}, \boldsymbol{v}]$ are defined in accordance with the following vector-forms:

$$\begin{aligned} \min[\boldsymbol{u}, \boldsymbol{v}] &:= (\min(u_1, v_1), \min(u_2, v_2), \dots, \min(u_N, v_N))^T \\ \max[\boldsymbol{u}, \boldsymbol{v}] &:= (\max(u_1, v_1), \max(u_2, v_2), \dots, \max(u_N, v_N))^T \end{aligned}$$

Here, we summarize explanations of the above model as follows. In the objective function, $\boldsymbol{a}^T \boldsymbol{x}$ is the total cost of assigning and holding containers in the dedicated stacks. $E_{\xi}(Q(\boldsymbol{x}, \xi))$ represents the

expectation of the overall cost caused by assigning, holding and moving containers, plus the movement cost in the dedicated stacks. The term $E\{\beta e^T y(\omega)\}$ represents the expectation of the cost caused by assigning and holding container space in the shared stacks. The term, $E\{\gamma^T \min[x, d(\omega)]\} = E\{\sum_{n=1}^N \gamma_n \min(x_n, d_n(\omega))\}$, denotes the overall expected cost of moving containers in the dedicated stacks with the value of $\min[(x_n, d_n(\omega))]$ being the number of containers stored in the dedicated stacks of the n th block. The term

$$E\{\lambda e^T \max[0, d(\omega) - x]\} := \\ E\{\lambda \sum_{n=1}^N \max(0, d_n(\omega) - x_n)\},$$

is the cost of moving containers stored in the shared stacks with the value of $\max(0, d_n(\omega) - x_n)$ being the number of containers placed in the shared stacks in the n th block.

Now, let us look at the constraints in (1). The constraint $0 \leq x_n \leq C/N$ ensures that the quantum of the dedicated stacks in each block does not exceed the maximum capacity of the block, and the first stage decision variables have to be non-negative integer. The constraints $e^T x + e^T y(\omega) \leq C; x_n + y(\omega) \geq d_n(\omega)$ ensure that the total capacity of the yard is not exceeded, and shipment demand has to be satisfied for any scenario. It means all containers are assigned to either the dedicated or shared stacks. The final constraint is the non-negative and integer requirement for all decision variables.

4.2 A Two-Stage Stochastic Model with Finite Scenarios for Uncertain Yard Storage Allocation Problems

In this subsection, we investigate a model (See (2)), which is a simplified two-stage recourse model in (1) for the uncertain storage problem. From the definitions of γ_n and λ in Section 3.4, we know that γ_n and λ represent the average cost of moving a container from the dedicated stacks and shared stack to vessels at the quay side. It is natural to assume that for planning purposes, both γ_n and λ are dependent on the basis of the overall capacity rather than the space used for each individual container. In real-world situations, both the labour costs (salary) and equipment costs (for example, the cost of purchasing and maintaining a vehicle) are almost fixed, even in the case, where no service is performed in the yard area. Compared to the cost of assigning and holding a container, other costs related to the movement (for example, fuel or gas used for lifting, trucking etc.) are also relatively small. Therefore, we only focus on the cost of assigning and holding in the yard area

(including the dedicated and shared stacks) in Model (1).

In addition, we notice that one of the difficulties in solving the two stage stochastic programming model (See (1)) is the continuity of the scenario set Ω . There are three reasons for the difficulties caused: 1) It is almost impossible to obtain a continuous distribution of the uncertainty in a real-world process. However, in most of cases, a set of finite scenarios and approximate discretized distribution functions can be easily obtained from historical data; 2) In a real-world situation, the number of containers is normally finite, and hence it is reasonable to use a finite scenario model to capture uncertain situations that might happen in the future. 3) Even for the case where the number of scenarios is infinite and the distribution is available, it is difficult to integrate the expectation of the objective function, due to the complexity of the distribution function. Therefore, in this subsection, we proceed to analyse a variation of stochastic model with a finite scenario set. It is assumed that a support set Ω with finite number of scenarios, denoted by $\Omega = \{d^1, d^2, \dots, d^K\}$, where $d^k = \{d_1^k, d_2^k, \dots, d_N^k\}$, $k = 1, 2, \dots, K$, and K is the maximum number of scenarios. Note that this assumption holds true in the real problem, where the number of containers arrived is normally finite. We have the following notation related to a finite set of scenarios:

- k : Index different scenarios for demand. ($k = 1, 2, \dots, K$)
- p_k : Probability of scenario k . ($p_k \geq 0, k = 1, 2, \dots, K, \sum_{k=1}^K p_k = 1$)
- d_n^k : Realization of demand for containers in the n th destination under scenario k . ($n = 1, 2, \dots, N; k = 1, 2, \dots, K$)
- y_n^k : Number of containers in the shared stacks of the n th block under scenario k . ($n = 1, 2, \dots, N; k = 1, 2, \dots, K$)

We write the second stage decision variables y_n^k in vector-form, as $y^k := (y^1, \dots, y^k)^T$. Now, the two stage stochastic optimisation for the storage management problem in Model (1) can be equivalently reformulated as the following algebraic equivalent linear programming form:

$$\min a^T x + \sum_{k=1}^K p_k [\beta e^T y^k]$$

subject to

$$0 \leq x_n \leq C/N, \text{ for } n = 1, 2, \dots, N$$

$$e^T x + e^T y^k \leq C, \quad (2)$$

$$x_n + y_n^k \geq d_n^k, \text{ for } n = 1, 2, \dots, N; k = 1, 2, \dots, K$$

$$x_n, y_n^k \geq 0 \text{ and integer for } n = 1, 2, \dots, N; k = 1, 2, \dots, K$$

5 COMPUTATIONAL RESULTS AND ANALYSIS

In order to illustrate the effectiveness of the proposed two stage model in Section 4 for the uncertain storage management problem of container terminals, we use data provided by a container terminal in Hong Kong. Located at the mouth of the Pearl River with a deep natural harbour, Hong Kong is geographically and strategically important as a gateway to China and trans-shipment port for intra-Asian and world trade. Hong Kong is the largest container port serving southern China and one of the busiest ports in the world. Consider export containers stacking for a vessel visiting 10 ports for discharge of containers. There are 10 blocks reserved to hold the containers in the yard area. Each block has 6 rows and 8 bays. Since the maximum height for stacking is 5 tiers, saving $(5-1) = 4$ free spaces for re-handling, the maximum number of containers that can be accommodated is $C = 10*[6*8*5-4] = 2360$. It is assumed that the demand for containers is uncertain for different ports (See Table 1). In general, there are five different scenarios representing the trend for the shipment demand in the future. The demand quantities and likelihood of each scenario are estimated by a yard storage planner (Table 1).

Table 1: Known data.

Scenario	Port										Likelihood
	1	2	3	4	5	6	7	8	9	10	
1	200	150	270	200	240	220	180	150	160	140	0.1
3	200	250	160	245	280	180	180	120	250	240	0.3
4	240	180	270	240	260	250	180	200	250	210	0.1
5	200	100	250	260	240	230	200	240	270	260	0.2

The data in Table 1 pertain to a real situation. For simplicity, we assume the cost of assigning and holding a container in the dedicated stacks is 1 unit, while the cost in the shared stacks is 3.5 units. We run the two stage stochastic programming model in (1) with optimization software Xpress IVE. The results of yard space allocation are as shown in Table 2. The total cost of the dual-response storage plan is

2570.25 units. Table 2 gives the dual-response storage strategy. The row “Dedicated Stack” indicates the first-stage response, which is the predetermined number of containers for different destinations to be allocated to the dedicated stacks. The row “Shared Stack” under “Scenario k ” indicates the second-stage response, which is the reactive number of containers for different destinations to be allocated to the shared stacks according to the real demand under scenario k . From the results in Table 2, we can see that only 35 spaces in Block 2 and 10 spaces in Block 5 are required for storing containers in the shared stacks if Scenario 1 happens. The capacity for each block is 236. 235 spaces in Block 3 and 230 spaces in Block 5 are allocated to the dedicated stacks. Therefore, the managers can use 35 spaces in Block 1, 5 spaces in Block 4, and 5 spaces in Block 5 for storing extra containers in the shared stacks. As a result, the spaces that have not been allocated to either the dedicated or shared stacks are free for other purposes. For example, there are total 135 spaces left if Scenario 1 happens. By adopting the dual-response strategy, the managers do not need to hold all spaces until the containers are loaded into the ship. As soon as the managers have full information about the shipment demand, i.e. the stochastic demand is realized, the managers can make the corresponding response by deciding the size of the shared stacks and releasing the spaces that will not be required, simultaneously. Releasing the unused space is very important in practice because the unused spaces make no profit, which will potentially increase the total operations cost of the terminal. This is particularly true for terminals with limited yard space, like the Hong Kong container terminal.

Table 2: Yard Allocation under Uncertainty.

	Port									
	1	2	3	4	5	6	7	8	9	10
Dedicated Stack	200	210	235	230	230	230	180	195	235	235
Shared Stack										
Scenario 1	0	0	35	0	10	0	0	0	0	0
Scenario 2	20	40	65	0	0	0	0	0	5	15
Scenario 3	0	40	0	15	50	0	0	0	15	10
Scenario 4	40	0	35	10	30	20	0	5	15	0
Scenario 5	0	0	15	25	10	0	20	45	35	25

It is noted that when the number of containers cannot be divided by the number of tiers, which is five

in this example, we need to move some containers (less than five) from the dedicated stacks to the shared stacks so that re-handling is not required in the dedicated stacks. Whereas, if there are no dedicated stacks allocated, all containers will be mixed in the shared stacks. This is a traditional sharing strategy. We can still use the model proposed in Section 4 to obtain the solution for the traditional sharing strategy. Since no container is assigned to the dedicated stacks, $x_n = 0$, for $n = 1, 2, \dots, N$. We run this model again with optimization software Xpress IVE and obtain the cost of using the traditional sharing strategy as 7320.25 units. Compared with this traditional sharing strategy, the average savings in cost for the dual response-strategy proposed in this paper is 64.89%.

6 CONCLUSIONS

This paper investigates storage yard allocation problems for export containers under uncertainty. The yard storage is physically divided into blocks and each block is conceptually divided into dedicated and shared stacks. The dedicated stacks in the same block have the same destination/port. The shared stack has mixed containers, destined for different ports. As a result, no re-handling is required for containers stored in the dedicated stacks but containers in the shared stacks need re-handling. We propose the dual-response storage policy to decide how containers are allocated to the two different types of stacks under uncertain shipment information. At the first stage, when accurate shipment information is not available, we need to decide how containers are to be allocated to the dedicated stacks. At the second-stage, when the uncertainty is realized, we need to respond to the different possible shipment scenarios that might happen. The decision at the second stage includes determining how additional containers are allocated to the shared stacks. As only a small number of containers are allocated to the shared stacks, re-handling is significantly reduced. In addition, we develop the two-stage stochastic recourse programming model to obtain the optimal dual-response storage plan. The computational results show the effectiveness of the two-stage stochastic model for storage problems under uncertain shipment information. Compared with the traditional sharing strategy (in which all containers are mixed up) and the non-sharing strategy (in which no containers are mixed up), the dual-response storage strategy can significantly reduce operations cost and, therefore, enhance productivity of container terminals. Future research might consider a situation in which both yard

space and shipment demand are uncertain. In addition, how to precisely determine the placement of containers in the shared stacks is a potential area for future research. The yard storage problem for import containers under uncertainty is also a potential area to explore.

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