Bragg Grating Solitons in a Dual-core System with Separated Bragg Grating and Cubic-quintic Nonlinearity

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Abstract: We analyze the stability of solitons in a semilinear dual-core system where one core is linear with a Bragg grating and the other core is uniform and has cubic-quintic nonlinearity. It is found that there exist three spectral gaps in the model's linear spectrum. The quiescent soliton solutions are found by means of numerical techniques. It is found that the soliton solutions exist only in both the upper and lower bandgaps. Two distinct and disjoint families of solitons (i.e. Type 1 and Type 2 solitons) are found in the upper and lower bandgaps that are separated by a border. Stability of solitons are analyzed numerically. The stability analysis shows that stable Type 1 solitons may only exist in a part of the upper bandgap. Type 2 solitons in both upper and lower gaps are found to be unstable.

1 INTRODUCTION

Fiber Bragg Gratings (FBGs) have found use in numerous applications such as filtering, dispersion compensation, and sensing (Kashyap, 1999; Krug et al., 1995; Loh et al., 1996; Litchinitser et al., 1997; Cao et al., 2012). As a result of the variation of refractive index, the FBGs possess a distinctive feature, namely the existence of a bandgap in their linear spectrum within which linear waves do not propagate (Kashyap, 1999). Additionally, the coupling between transmitted and reflected waves in an FBG gives rise to an effective dispersion that can be 10^6 times stronger than the chromatic dispersion of silica (de Sterke and Sipe, 1994; Eggleton et al., 1997). At sufficiently high intensities, Bragg grating solitons can be generated in FBGs through a balance between the effective dispersion of the FBG and the nonlinearity of the medium (de Sterke and Sipe, 1994).

The generation and observation of Bragg grating (BG) solitons have been of great interest mainly because of their potential applications in generation of slow light and development of novel optical delay lines and memory elements. Consequently, BG solitons have been studied extensively both theoretically (Aceves and Wabnitz, 1989; Christodoulides and Joseph, 1989; Malomed and Tasgal, 1994; Barashenkov et al., 1998; De Rossi et al., 1998; Neill and Atai, 2006) and experimentally (Eggleton et al., 1996; de Sterke et al., 1997; Eggleton et al., 1999) in Kerr nonlinear media. To date, BG solitons propagating at 23% of speed of light in the medium have been reported (Mok et al., 2006). Theoretical studies have shown that BG solitons may exist in other optical structures and nonlinearities such as dual-core fibers (Atai and Malomed, 2000; Atai and Malomed, 2001b; Mak et al., 1998; Atai and Baratali, 2012), photonic crystal waveguides (Monat et al., 2010; Neill and Atai, 2007; Atai et al., 2006), waveguide arrays (Mandelik et al., 2004; Tan et al., 2009; Dong et al., 2011), and cubic-quintic nonlinear medium (Atai and Malomed, 2001a; Dasanayaka and Atai, 2010).

Nonlinear couplers with non-identical cores (particularly semi-linear ones) possess better switching characteristics than the standard dual-core fibers (Atai and Chen, 1992; Atai and Chen, 1993; Bertolotti et al., 1995). The presence of Bragg grating in one or both cores in such dual-core fibers will potentially lead to novel switching and slow light devices. Therefore, in this paper, we study the stability of zero-velocity solitons in a semilinear dual-core system where one core is linear with a Bragg grating written on it and the other one is uniform and has cubic-quintic nonlinearity.

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2 THE MODEL AND ITS LINEAR SPECTRUM

We consider the propagation of light in a dual-core fiber where one core is linear and has a Bragg grating and the other one is a uniform fiber with cubic-quintic nonlinearity. The mathematical model for such a system can be expressed as follows:

$$
\begin{aligned}
&iu_t + iu_x + \left[|v|^2 + \frac{1}{2}|u|^2\right] u - q\left[\frac{1}{4}|u|^4 + \frac{3}{2}|u|^2|v|^2 + \frac{3}{4}|v|^4\right] u + \phi = 0,\\
&iv_t - iv_x + \left[|u|^2 + \frac{1}{2}|v|^2\right] v - q\left[\frac{1}{4}|v|^4 + \frac{3}{2}|v|^2|u|^2 + \frac{3}{4}|u|^4\right] v + \psi = 0,\\
&i\phi_t + ic\phi_x + u + \lambda\psi = 0,\\
&i\psi_t - ic\psi_x + v + \lambda\phi = 0.
\end{aligned} \tag{1}
$$

Here, u and v are the forward-propagating and backward-propagating waves in the nonlinear core and their counterparts in the linear core are represented by ϕ and ψ, respectively. q is the parameter that controls the strength of the quintic nonlinearity. The coupling coefficient between the forward- and backward-propagating waves induced by Bragg gratings in the linear core is denoted by λ and c is the group velocity for this core. The linear coupling coefficient between the two cores and the group velocity in the nonlinear core are set equal to 1.

In order to determine the spectral regions where solitons may exist, it is essential to analyze the linear spectrum of the model. Substituting $u, v, \phi, \psi \sim \exp(ikx - i\omega t)$ into the linearized version of Eqs. (1) results in the following dispersion relation:

$$
\omega^4 - \left[\left(1 + c^2\right)k^2 + \left(2 + \lambda^2\right)\right]\omega^2 + \left(ck^2 - 1\right)^2 + \lambda^2 k^2 = 0. \tag{2}
$$

The dispersion relation (2) is identical to that of Ref. (Atai and Malomed, 2001b). For $c = 0$, Eq. (2) gives rise to three disjoint gaps (Atai and Malomed, 2001b). When $\lambda > \dfrac{1}{\sqrt{2}}$, the gaps are given by

$$
\begin{cases}
\lambda < \omega < \sqrt{1 + \dfrac{\lambda^2}{4}} + \dfrac{\lambda}{2};\\[2ex]
-\left(\sqrt{1 + \dfrac{\lambda^2}{4}} - \dfrac{\lambda}{2}\right) < \omega < \sqrt{1 + \dfrac{\lambda^2}{4}} - \dfrac{\lambda}{2};\\[2ex]
-\left(\sqrt{1 + \dfrac{\lambda^2}{4}} + \dfrac{\lambda}{2}\right) < \omega < -\lambda,
\end{cases}
$$

and for $\lambda < \dfrac{1}{\sqrt{2}}$, the gaps are as follows:

$$
\begin{cases}
\left(\sqrt{1 + \dfrac{\lambda^2}{4}} - \dfrac{\lambda}{2}\right) < \omega < \sqrt{1 + \dfrac{\lambda^2}{4}} + \dfrac{\lambda}{2};\\[2ex]
-\lambda < \omega < \lambda;\\[2ex]
-\left(\sqrt{1 + \dfrac{\lambda^2}{4}} + \dfrac{\lambda}{2}\right) < \omega < -\left(\sqrt{1 + \dfrac{\lambda^2}{4}} - \dfrac{\lambda}{2}\right).
\end{cases}
$$

For $\lambda = \dfrac{1}{\sqrt{2}}$, the three gaps combine into a single gap defined by $-\sqrt{2} < \omega < \sqrt{2}$. Examples of dispersion diagrams for different values of λ are shown in Fig. 1.

In the case of $c \neq 0$, the characteristics of the dispersion diagrams change significantly. More specifically, the upper and lower gaps of the dispersion diagram overlap with one branch of the continuous spectrum. This means that only the central gap remains a genuine one (Atai and Malomed, 2001b).

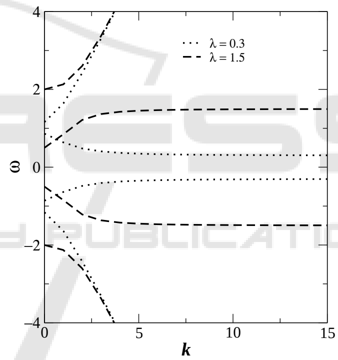

Figure 1: Linear spectrum for Eqs. (1) at $c = 0$ for different values of λ.

3 SOLITON SOLUTIONS AND THEIR STABILITY

To obtain stationary zero velocity soliton solutions for the system, we first substitute $\{u(x,t), v(x,t), \varphi(x,t), \psi(x,t)\} = \{U(x), V(x), \Phi(x), \Psi(x)\}\, e^{(-i\omega t)}$ into Eqs. (1). Upon simplification and invoking the symmetry conditions from quiescent solitons i.e. $U = -V^*$ and $\Phi = -\Psi^*$, we arrive at the following system of ordinary differential equations:

$$
\begin{aligned}
\omega U + iU_x + \frac{3}{2}|U|^2 U - \frac{5}{2}q|U|^4 U + \Phi = 0,\\
\omega\Phi + ic\Phi_x + U - \lambda\Phi^* = 0.
\end{aligned} \tag{3}
$$

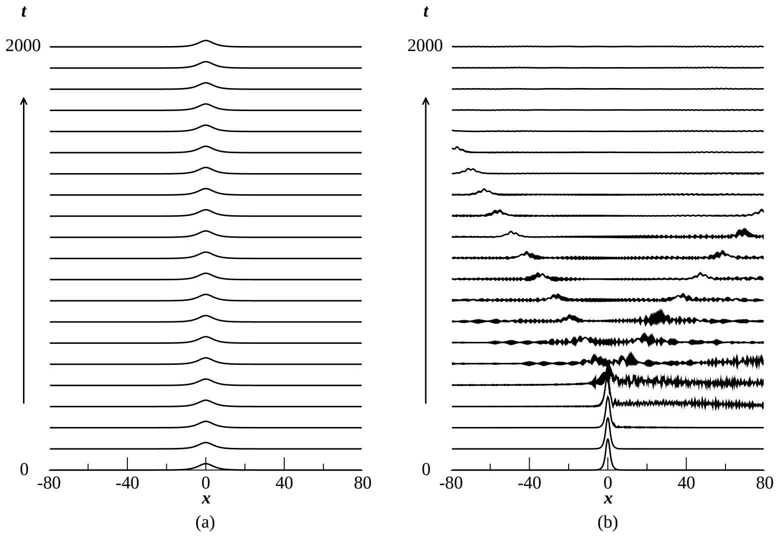

Figure 3: Examples of propagation of Type 1 zero-velocity solitons. (a) a stable soliton in the upper gap for $\lambda = 1$, $c = 0$, $\omega = 1.6$ and $q = 0.13$ and (b) an unstable soliton in the lower gap for $\lambda = 1$, $c = 0$, $\omega = -1.41$ and $q = 0.16$. Only the u components are shown here.

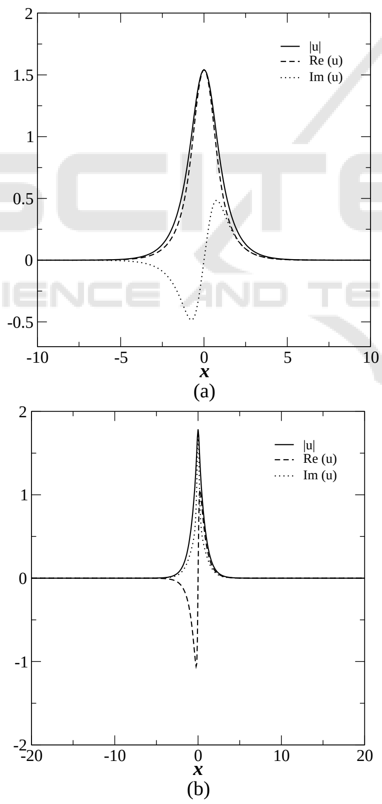

Figure 2: Soliton solutions at $c = 0$ for (a) Type 1 soliton in the upper gap at $\lambda = 1.2$, $\omega = 1.56$, $q = 0.12$ and (b) Type 2 soliton in the lower gap at $\lambda = 1.2$, $\omega = -1.5$, $q = 0.5$.

Eqs. (3) can be solved numerically using the relaxation algorithm to obtain soliton solutions. The results of the numerical analysis show that soliton solutions exist only in the upper gap and the lower gap. Moreover, in each gap (i.e. upper and lower gap), there exist two different and disjoint families of solitons (henceforth referred to as Type 1 and Type 2). The Type 1 and Type 2 families differ in their phase structure and amplitude. In particular, Type 2 solitons are characterized by a sharp nonsingular peak. Fig. 2 displays examples of Type 1 and Type 2 solitons in the upper and lower bandgaps.

In order to determine the stability of the quiescent BG solitons, we have conducted a numerical stability analysis by solving Eqs. (1) using symmetrized split-step method (Agrawal, 2013). The results suggest that for a given λ, stable Type 1 solitons exist only in a certain region of the upper band gap. Additionally, Type 2 solitons are unstable in both the upper and lower gaps. Figs. 3 and 4 show examples of the propagation of Type 1 and Type 2 solitons in the upper and lower gaps. A noteworthy feature of these figures is that Type 2 solitons are highly unstable and decay into radiation very quickly. The results of the stability analysis for $\lambda = 1$ are presented in Fig. 5.

4 CONCLUSIONS

We study the existence and stability of the Bragg grating solitons in a dual-core system where one core is linear and is equipped with a Bragg grating and the other one is uniform with cubic-quintic nonlinearity. for $c = 0$, the linear spectrum of the system has three distinct gaps which merge into one gap when the coupling coefficient between the core is equal to $1/\sqrt{2}$. In the case of $c \neq 0$, the upper and lower gaps overlap

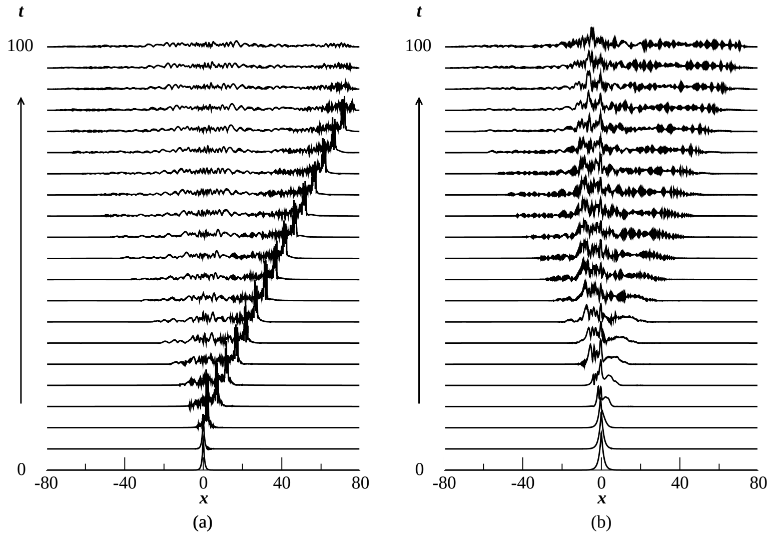

Figure 4: Examples of propagation of Type 2 zero-velocity solitons. (a) an unstable solution in the upper gap for $\lambda = 1$, $c = 0$, $\omega = 1.21$ and $q = 0.63$ and (b) an unstable solution in the lower gap for $\lambda = 1$, $c = 0$, $\omega = -1.47$ and $q = 0.38$. Only the u components are shown here.

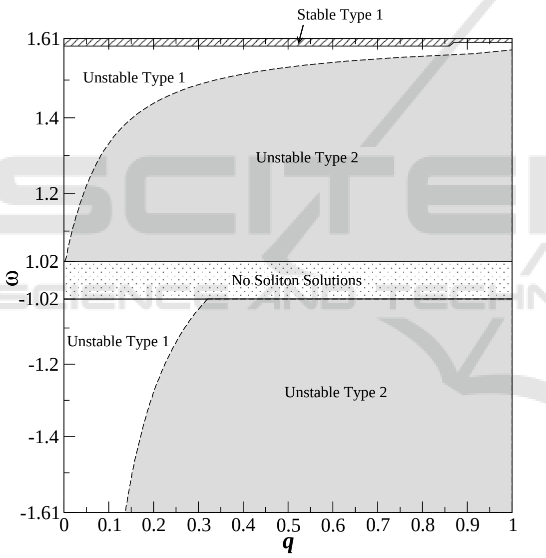

Figure 5: Stability diagram for $c = 0$ and $\lambda = 1$ in the (q, ω) plane. The dashed curves are the borders separating the Type 1 and Type 2 families of solitons.

with one branch of the continuous spectrum. Therefore, only the central gap will be a genuine bandgap. The soliton solutions for the systems are found using the relaxation algorithm. It is found that there are no soliton solutions in the central bandgap. On the other hand, there exist two different and disjoint families of solitons, namely Type 1 and Type 2, in the upper and lower bandgaps.

Stability of the solitons are investigated by means of numerical techniques. It is found that the Type 2 solitons in both upper and lower bandgaps are unstable and decay into radiation. The results also suggest that stable Type 1 solitons may only exist in a certain region within the upper bandgap.

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