Determining Equilibrium Staffing Flows in the Canadian Department of National Defence Public Servant Workforce

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Abstract: In large workforces, such as the Public Servant workforce of the Canadian Department of National Defence, many staffing actions (hires, departures, promotions, transfers) are actioned each year. Estimating the number of such actions that will take place in the department's various units is relevant to the assignment of Human Resource support capacity. The rates at which staffing actions are required is also indicative of the health of workforce occupational segments. This paper presents a method for estimating the number of staffing actions that will be required to maintain a workforce to a state of equilibrium. It also shows how that method was practically implemented in support of Strategic Human Resource Planning for Canada's Department of National Defence civilian workforce.

1 BACKGROUND

The Department of National Defence (DND) and Canadian Armed Forces make up the largest Canadian federal government department. In addition to uniformed Regular and Reserve Force members, approximately 25,000 Public Servants are employed in the department.

The model described in this paper was developed at the request of DND's Assistant Deputy Minister (Human Resources – Civilian), who has a need to monitor and predict the number of staffing actions taking place within the department's Public Servant workforce. Staffing actions are the movements of personnel in and out of departmental positions. As such, they include hiring, departures, promotions and transfers.

Monitoring and predicting staffing actions is of interest for two reasons. First, it informs the assignment of Human Resources support capacity to where it is needed (having the right number of Human Resources staff to process staffing actions in each unit). Secondly, the volume of staffing actions informs occupational health assessments across the department (e.g. high attrition, high transfer, or low promotion rates could be indicators of unhealthy workforce dynamics).

Workforce dynamics have often been investigated through simulation, such as Markov modelling (Mitropoulos, 1983; Georgiou, 1999), or Discrete Event Simulation (Okazawa, 2013; Zegers and Isbrandt, 2010). Instead, we take the approach of explicitly solving for staffing flows in a personnel system under an assumption of equilibrium. This is similar to (Bartholomew, 1969), but extended to consider both push and pull staffing actions. It is also similar to (Doumic et al, 2016), which represents staffing flows as differential equations, but focuses on workforce age profiles as the determinant of flows, rather than the occupation and level studied in this paper. In the case of a rigid military personnel system, additional control over recruitment (always into the lowest rank), promotion policy, terms of service, etc. allow for direct optimization, such as in (Marquez and Nelson, 1996 or Filinkov et al, 2011). Explicit optimization is not as helpful in more fluid Public Service workforces, so we take the approach of describing the flows in the system, an endeavour that Human Resource Planners find useful in its own right. This paper describes the processes developed to model and report the Public Servant staffing flows of the DND.

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2 STAFFING ACTIONS

Canadian Public Servants are classified according to the *Occupational Group Structure*, which is composed of Groups, Classifications and Sub-groups. For ease of presentation, we will refer to the various sub-divisions of the structure as occupations. Each occupation is then broken down into levels, which correspond to successive salary ranges and seniority.

In examples, this paper refers to the Financial Management Classification (denoted FI). It is broken down into four levels, FI01 being the junior level, and FI04 the senior.

Public Servants hold positions at a set occupation and level. Figure 1 depicts various staffing actions that could result in an employee moving in or out of the FI03 occupation-level. The employee can be promoted to FI04, or less commonly demoted to FI02 or FI01, or could transfer to any other occupation (the transfer to Administrative Services (AS07) is shown as it is the most common destination for FI03 employees). An FI03 employee could also depart the DND workforce completely, for retirement or another reason. Figure 1 also shows the similar movements into FI03 (external hiring, promotion, demotion and transfer).

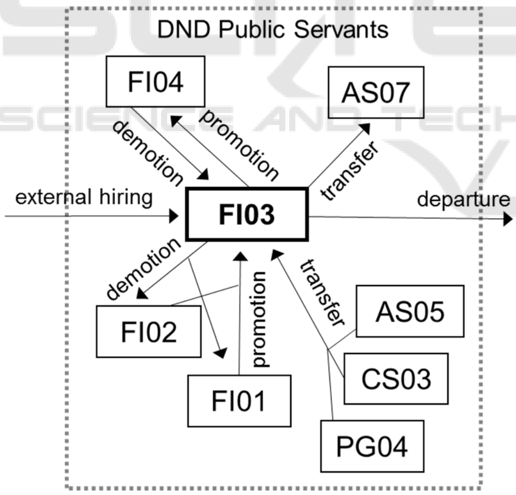

Figure 1: Staffing actions for the FI03 occupation-level.

There is another type of staffing action that is not depicted in Figure 1 and that can be important to Human Resources Planning – lateral transfers (e.g. transfers to a different FI03 position elsewhere in the DND). These can constitute a substantial portion of overall staffing turnover, but lie outside the model presented in this paper. If needed, lateral transfers can easily be considered separately from the model presented in this paper.

This paper aims to quantify the expected annual flows in and out of occupation-levels. To do so, it relies on historical data describing past flows. In practice, we have access to annual workforce snapshots and counts of staffing actions, tallied by occupation and level.

3 THE DEPARTMENTAL WORKFORCE

Let $\mathbf{w} = [w_1, w_2, \ldots, w_n]$ be a workforce, where each w_i is the number of employees of a given occupation and level (with the subscript denoting the occupation-level pair). Our goal is to estimate the number and type of staffing actions that should be expected each year when the workforce is at equilibrium.

Each year, some employees depart $\mathbf{w}$, change level through promotion, and transfer between occupations. Then, new employees are hired to fill the gaps. This paper presents an approach to estimating the number of such staffing actions that are expected to occur in a workforce at equilibrium, from historical data that are not necessarily from a system at equilibrium.

Note that equilibrium is not necessarily the most relevant state to be modelled from the perspective of Human Resources Planning. It is explored in this paper as the relevant base case, in the absence of more definite Human Resources plans. Incorporating planned growth or reductions into the model that is developed below is trivial, if such plans exist. In any case, when assessing the health of the occupation, equilibrium flows a useful indicator of what would be required to maintain the status quo.

4 ATTRITION

This section describes how to apply workforce attrition rates. The process used to estimate attrition rates from historical data will be described later. Let a_i be the historically observed annual attrition rate among employees of occupation-level i. Here, attrition is defined as departing $\mathbf{w}$ altogether. If there are w_i employees at a given occupation-level at the beginning of the year, and assuming no hiring or movements between occupation-levels, we expect that $w_i \cdot a_i$ of them will have departed by year end, and therefore, that $w_i \cdot (1 - a_i)$ will remain.

However, attrition does not only apply to the employees who are initially in the workforce. Employees externally hired in the course of the year are also subject to the attrition rate. If h_t employees are hired at date t (expressed a proportion of the year), we expect that $h_t \cdot (1-a_i)^{1-t}$ of them will remain at year end.

If we knew the precise dates on which new hires will join the workforce, we would use this information to determine the expected attrition among hires. However, we do not generally know the dates of future staffing actions – we likely only have annual staffing targets. In order to apply attrition to future hires, we need to make an assumption about how they will be distributed in time. In the absence of other information, we assume, as in (Okazawa, 2007; Fang and Bender, 2008), that they will be uniformly spread. Thus, among the h_i hires at occupation-level i over a given year, we expect that

$$h_i \int_0^1 (1-a_i)^{1-t}\,dt = h_i \frac{-a_i}{\ln(1-a_i)} \equiv h_i \cdot \gamma_i \qquad (1)$$

will remain at the end of the year (γ_i is introduced for ease of notation in follow-on equations). Attrition among employees coming in or out of w_i through promotion or transfer can be similarly handled.

5 STAFFING FLOWS

We now turn our attention to the balance of annual flows in and out of w_i. These are depicted in Figure 2.

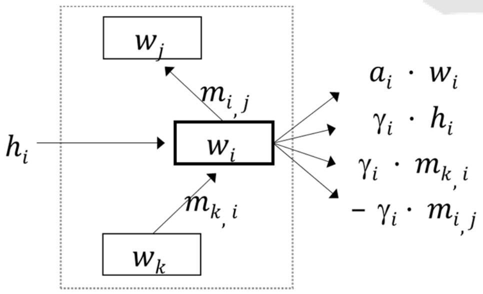

Figure 2: Staffing flows in and out of w_i.

First are the flows that are internal to $\mathbf{w}$ – flows between occupation-levels. Let $m_{i,j}$ be the annual movement of employees from w_i to w_j. Figure 2 only shows one such flow into w_i, and one out, but such flows in and out can potentially exist between w_i and all of the other levels. The inflow into w_i from outside $\mathbf{w}$ corresponds to external hiring, and is denoted h_i.

Figure 2 also shows four types of attrition (external outflows) which we will now describe in turn. Basic attrition from w_i is obtained by applying the relevant attrition rate as $a_i \cdot w_i$. However, we must also account for the effect of hiring and internal movements on attrition. As described in Equation (1), $\gamma_i \cdot h_i$ is the attrition among new hires. Similarly, there can be attrition among employees that flow into w_i from other occupation-levels. If these flows are assumed to occur uniformly throughout the annual period, then the situation is as in Equation (1), and the attrition from these inflows is $\gamma_i \cdot m_{k,i}$ for each k. Finally, $-\gamma_i \cdot m_{i,j}$ accounts for the reduction in attrition out of w_i as a result of the $m_{i,j}$ outflow. Note that the last three attrition flows from Figure 2 simply amount to applying the rate γ_i to the net inflow.

Now, if the headcount w_i is to remain unchanged from year to year, all of the annual flows in and out of w_i must exactly balance out. Thus,

$$\begin{aligned} 0 = h_i + \sum_j m_{j,i} - \sum_j m_{i,j} - a_i w_i \\ -\gamma_i h_i - \gamma_i \sum_j m_{j,i} + \gamma_i \sum_j m_{i,j} \end{aligned} \qquad (2)$$

which simplifies to

$$0 = h_i + \sum_j m_{j,i} - \sum_j m_{i,j} - \frac{a_i}{1-\gamma_i} \cdot w_i \qquad (3)$$

6 PUSH STAFFING FLOWS

In order to use Equation (3) to determine staffing flows, further assumptions are necessary. We will use the two types of internal flows described by (Bartholomew et al, 1991) and that we believe adequately describe most real world staffing phenomena: push and pull flows.

Push flows are those that can be described as simple rates of outflow. In the Canadian Public Service, these are mainly a certain subset of the promotions. For example, employees in *incumbent-based* occupations are promoted solely based on the achieved level of professional development (rather than as the result of competitions for openings at the senior levels). Royal Military College Professors are an example of such a subset of the DND workforce. For these employees, promotions are adequately approximated by a constant annual flow rate from one level to the next – that is, a fixed proportion of employees are promoted from a given occupation-level each year.

Another example of a push staffing flow is the movement of employees out of apprentice levels. Apprenticeships are of a fixed duration. Promotion out of an apprentice level is automatic, and not dependent on a vacancy at the higher level. At equilibrium, a fixed proportion of apprentices are therefore promoted annually. Demotions due to unsatisfactory performance, although less common, can also be modelled as push flows.

Let $q_{i,j}$ be the annual rate at which employees from w_i move to w_j through push flows. Then, we expect $q_{i,j} \cdot w_i$ employees to thus flow each year. In practice, we observe $q_{i,j}$ in historical data.

7 PULL STAFFING FLOWS

Pull flows are those whereby a vacancy in w_i is filled by pulling in employees from other occupation-levels, or through external hiring. As in most workforces, the majority of DND internal staffing flows are best described as pull flows. Most pulls are the result of staffing competitions. A vacancy is advertised, and filled by selecting from a pool of applicants who may come from lower levels (promotions), other occupations (occupational transfers) or external hiring. The volume of the resulting movements is not a multiple of the size of the originating labour pools. Many demotions are also the result of pull processes, when employees actively seek positions at a lower level for various personal reasons.

In practice, some positions are also filled through lateral moves, by employees who are in other positions at the same occupation-level, but these are ignored here. As previously mentioned, such lateral transfers can be considered separately, outside the model described in this paper.

We will assume that pull flows occur in fixed proportion from each source. For example, annually, it could be that 50% of FI03 vacancies are typically filled by promoting FI02 employees, 20% by transfers from AS05, and 30% by external hiring.

Let f_i be the number of employees that must be pulled into w_i from all sources each year, so as to maintain the headcount. Notice that f_i must not only offset the gap in w_i that would be left by attrition and net push flows, but it must also offset any gap left by any other pull flows that move individuals from this occupation-level to others. Let $r_{i,j}$ be the proportion of f_j that is to come from $m_{i,j}$ (such that $m_{i,j} = r_{i,j} \cdot f_j$). As with $q_{i,j}$, observation of historical flows provides a reasonable value of representative values of $r_{i,j}$ for the model.

Then, h_i is the portion of f_i that is not filled by the internal ($m_{i,j}$) pull flows, and so

$$h_i = f_i - \sum_j \left(r_{i,j} \cdot f_i\right) \tag{4}$$

Now, each $m_{i,j}$ may be described as either the result of a push or a pull flow (or both), such that

$$m_{i,j} = r_{i,j} \cdot f_j + q_{i,j} \cdot w_i \tag{5}$$

In general, flows between two given occupation-levels are designated as either push or pull, but not both, such that at least one of either $q_{i,j}$ or $r_{i,j}$ is zero.

8 REQUIRED STAFFING ACTIONS

We now have all the building blocks in place. We can determine, under a equilibrium assumption, the expected annual personnel flows for our workforce. This is done by substituting Equations (4) and (5) into Equation (3), which gives

$$\begin{aligned} 0 = f_i &- \sum_j \left(r_{i,j} \cdot f_i\right) \\ &+ \sum_j \left[r_{j,i} \cdot f_i + q_{j,i} \cdot w_j\right] \\ &- \sum_j \left[r_{i,j} \cdot f_j + q_{i,j} \cdot w_i\right] - \frac{a_i}{1-\gamma_i} \cdot w_i \end{aligned} \tag{6}$$

Equation (6) can be rearranged and simplified as

$$\begin{aligned} &f_i - \sum_j \left(r_{i,j} \cdot f_j\right) = \\ &\frac{a_i}{1-\gamma_i} \cdot w_i + \sum_j \left[q_{i,j} \cdot w_i - q_{j,i} \cdot w_j\right]_i \end{aligned} \tag{7}$$

Equation (7) defines a system of n linear equations in the n variables $f_1, f_2, \dots, f_n$. Let $\mathbf{k}$ be the column vector built from the values, for each i, on the right side of Equation (7), $\mathbf{R}$ be the matrix of $r_{i,j}$ coefficients, $\mathbf{f}$ be the column vector of f_i variables, and **I** the identity matrix. Then, Equation (7) can be re-written as

$$(\mathbf{I}-\mathbf{R})\cdot\mathbf{f}=\mathbf{k} \tag{8}$$

Finally, Equation (8) can be solved for **f** as

$$\mathbf{f}=(\mathbf{I}-\mathbf{R})^{-1}\cdot\mathbf{k} \tag{9}$$

Having the f_i values from Equation (9), the number of staffing actions of each type that should be expected annually is obtained using Equations (4) and (5). Finally, each $m_{i,j}$ flow can be labeled as either promotion, demotion or occupational transfer, for the purpose of reporting the results to Human Resource Planners.

9 ATTRITION ESTIMATION

The above derivation starts with attrition rates (a_i). We must now explain how we obtain these rates. Many approaches exist for attrition forecasting, such as time series analysis, models obtained through regression, or various flavours of simulation. With military workforces, forecasts based on Years of Service demographics are a common approach, as justified by (Fang and Bender, 2011). As shown in (Calitoiu and Vincent, 2017), a number of other workforce attributes can also be predictors of DND Public Servant attrition. However, in this paper, we will limit ourselves to the estimation of past attrition rates by occupation-level, and assume that these are representative of the rates that will apply in the future.

Attrition within an occupation-level can vary substantially from year to year. Here, we will look for the rates that are representative of observations over a multi-year period.

Let n be the number of years of data to be used in determining a_i. Also let $w_i(0)$ be the headcount at occupation-level i at the beginning of the multi-year period, and $w_i(n)$ at the end. Although we know that the attrition rate has varied up and down over the multi-year period, we are looking for an effective annual rate a_i that explains the transition from $w_i(0)$ to $w_i(n)$, given all other observed flows (hiring, promotions and transfers).

For simplicity of presentation, we first define $b_i(y)$ as the sum of all net flows into w_i during year y (i.e. h_i plus all $w_{k,i}$, minus all $w_{i,j}$). Then, we are looking for the attrition rate a_i that explains the final headcount as

$$\begin{aligned} w_i(n) = {} & (1-a_i)^n\cdot w_i(0) \\ & + \sum_{y=1}^{n}\left[\gamma_i(1-a_i)^{n-y}\cdot b_i(y)\right] \end{aligned} \tag{10}$$

In the summation within Equation (10), the net inflow $b_i(y)$ into w_i during the y^{th} year is reduced, through attrition by a factor of γ_i during year y, and by a factor of $(1-a_i)$ in each subsequent year. As in Equation (1), it is the assumption that $b_i(y)$ occurred uniformly over the annual periods y that results in the γ_i factor. In general, Equation (10) must be solved numerically for a_i.

To simplify the implementation, we take the first order Taylor expansion of γ_i around zero, resulting in the approximation

$$\gamma_i=\frac{-a_i}{\ln(1-a_i)}\approx 1-\frac{a_i}{2}=\frac{1}{2}+\frac{1-a_i}{2} \tag{11}$$

and by substituting into Equation (10),

$$\begin{aligned} w_i(t) = {} & (1-a_i)^t\cdot w_i(0) \\ & + \sum_{y}\left[(1-a_i)^{t-y}\frac{b_i(y)}{2}\right] \\ & + \sum_{y}\left[(1-a_i)^{t-y+1}\frac{b_i(y)}{2}\right] \end{aligned} \tag{12}$$

Now Equation (12) is a polynomial in a_i. Its root is the attrition rate that we are seeking. Further, notice that Equation (12) is equivalent to what we would have obtained, had we assumed half of the net annual flow $b_i(y)$ taking place at the start of the year, and half at the end of the year (instead of assuming uniform intake and simplifying with the Taylor expansion). Looking for a parallel from the field of Finance, note that the problem of measuring the effective attrition rate from historical data is closely analogous to the problem of measuring the rate of return for an investment fund (with external cash flows taking the place of external personnel flows). Equation (12) is equivalent to the formula that, in Finance, describes the internal rate of return for an investment given an initial cash inflow of $w_i(0)$, additional cash inflows or outflows of $b_i(y)/2$ at the beginning and end of each year, and a single outflow $w_i(n)$ at the end of the multi-year period. Since internal rate of return calculations are commonly used in Finance, numerical implementations are widely available (e.g. the `XIRR` function in Microsoft Excel).

Table 1: Example of expected staffing actions in the FI occupation.

OCC–LEVEL	HEAD–COUNT	EXPECTED STAFFING								TOTAL TURNOVER (RATE)	TURNOVER HISTORY
		INFLOW				OUTFLOW					
		HIRE	PROMO	DEMOTE	TRANS	LEAVE DEPT	PROMO	DEMOTE	TRANS		
FI01	108	17.0	-	0.0	4.6	10.3	9.8	-	1.6	21.7 (20%)	10% - 39%
FI02	189	14.8	9.8	0.0	1.9	16	9.2	0	1.4	26.6 (14%)	6% - 31%
FI03	147	5.9	9.2	0.3	1.5	14.2	2.5	0	0.2	16.9 (11%)	4% - 22%
FI04	49	2.1	2.5	-	0.0	3.7	-	0.3	0.6	4.6 (9%)	5% - 17%

10 APPLICATION

Table 1 presents a small subset of a staffing analysis conducted in DND. It shows expected staffing flows for the four levels of the Financial Management (FI) occupation. The flows are broken down into hires, promotions, demotions, transfers, and departures. As this is based on an equilibriummodel, the sum of all inflows equals the sum of all outflows – this sum is highlighted as the total turnover. Turnover is an indication of the number of staffing actions that will need to be performed by Human Resources staff – it is thus useful to the allocation of personnel management resources. Turnover, and its components also describes the dynamics of an occupation, and is related to its health. Very high or very low rates would warrant management attention, and potentially, corrective measures.

Essentially, the method described in this paper takes large quantities of historical Human Resources data, and derives the personnel flows that would be expected if the workforce were at equilibrium. These flows describe the workforce's dynamics, and Table 1 then presents these flows in a way that can be readily assimilated by Human Resources Planners.

Some of the values in Table 1 can be broken down further. In particular, transfer inflows (respectively outflows) lump together all the transfers from (respectively to) all other occupations. But the specific origins and destinations of transfers would be of interest to Human Resource Planners. Similarly, promotions and demotions include ordinary moves between adjacent levels, but also cases where an employee skips one or more levels in changing position (e.g. double promotions). Another breakdown that can be of interest is the separation of retirements from other sources of attrition, which can be estimated based on the historical split. In DND, highlighting how many of the hires are Canadian Armed Forces Veterans is also a relevant concern. Such breakdowns of the values in Table 1 are normally reported in additional supporting tables.

Table 1 also reports a turnover history in the right-most column. This is the range of turnover rates that have been observed in past years. We have found that reporting the turnover history provides empirical context regarding the possible variability of outcomes, in lieu of a derived confidence interval.

11 CONFIDENCE

This paper presents a method for deriving expected staffing flows within a workforce. As with any prediction, this must be accompanied by some measure of the confidence that we have in the result. The difficulty in this case is the absence of sufficient historical data to bound the prediction. The values of a_i, $q_{i,j}$ and $r_{i,j}$ are based on annual historical observation, but structural shifts in the workforce generally means that data from more than a few years ago is unlikely to be representative of the present or future. Thus, we cannot directly observe historical distributions of a_i, $q_{i,j}$ and $r_{i,j}$ that could be used to derive reliable prediction intervals for the forecasted staffing flows.

An alternative would be to fit a theoretically justifiable probabilistic models of a_i, $q_{i,j}$ and $r_{i,j}$ with the limited historical data that are available. Then, the distributions of a_i, $q_{i,j}$ and $r_{i,j}$ could determine prediction intervals for the reported equilibrium staffing flow estimates. Substantial analysis would however be required before this kind of probabilistic model could be correctly employed.

The derivation of appropriate prediction intervals to accompany the staffing flow estimates presented in this paper will be an area of future work. For the time being, we believe that historical ranges in the flows provide reasonable bounds for our expectations of future variability, and so this is what we have been reporting.

12 PRACTICAL CONSIDERATIONS

In practice, one of the most time consuming tasks in the application of the presented method has been the identification of historical exceptions that must be ignored in deriving the values of a_i, $q_{i,j}$ and $r_{i,j}$. In the past, many events have taken place that should be excluded in equilibrium staffing analyses. For example, in the DND, entire units have been transferred to other government departments, and some occupations have been restructured, or split from others, resulting in spikes in observed attrition or transfers. However, there has been no centralized record keeping of such changes. Careful outlier detection of the historical record has been necessary to identify the anomalies in the data record.

Another practical consideration has to do with the many very small occupation-levels in the Department of National Defence. It is unreasonable to model staffing actions in very small occupations where the historical record is insufficient to accurately describe typical flow rates. Also, in very small occupations, future events will not be averaged out over enough individuals to be predictable. Ignoring the small occupation-levels is not necessarily the best option. First, Human Resources Planners might prefer a poor estimate to none at all, but also, modelling the larger occupation-levels still requires consideration of their interactions with the smaller ones. The solution is to aggregate select occupation-levels, such as combining adjacent levels of an occupation. This is a delicate task, as the small occupation-levels should only be aggregated with similarly-behaving ones.

In order to determine when smaller occupation-levels should be aggregated, an understanding of the relationship between headcount and the width of prediction intervals around occupation-level estimates will be helpful. We would then keep aggregating until the confidence intervals become sufficiently narrow. So far, we have used 10 years of historical data, and groupings of no less than 10 employees – giving a minimum of 100 employee-years per segment. This is certainly on the low end of statistical relevance. Until a characterization of model confidence can be completed, we believe that reporting the historical ranges in staffing flows will provide sufficient context.

13 CONCLUSION

This paper presented a method that takes historical staffing data as input, and derives expected equilibrium staffing flows that have been valuable to Human Resource Strategic Planning in Canada's DND. The expected flows can inform the allocation of Human Resource capacity to process staffing actions, and can also contribute to assessments of occupational health by flagging occupation-levels with unusually high or low flows.

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