The Process of Intraconnection and Interconnection in Mathematical Problem Solving based on Stages of Polya

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Abstract: One of the factors that inhibit the success of students in constructing the problem-solving process is that students are not able to identify the type of mathematical connection that should be built in the problem-solving process. Therefore, the purpose of this study is to discuss the types of mathematical connections that occur in the stages and between stages of Polya. Identification of the type of connection in each stage and between stages of solving the Polya problem is defined as intraconnection and mathematical interconnection. The purposive sampling technique was used to select two students who had a tendency to productive connective thinking with complete connective thinking networks. Worksheets and recordings of the three students' thinking are analyzed using a qualitative descriptive approach. In the intraconnection process can be described the formation of a network of understanding connections, hierarchical connections, connections if so, equivalent representation connections, and procedural connections. Whereas in the interconnection process there is the formation of network connection planning, syntax or plan execution, and connection evaluation. The conclusion of the research results is the formation of five connection networks in the intraconnection process and three connection networks in the interconnection process.

1 INTRODUCTION

The problem-solving process requires establishing a connection between stages problem solving, as an effort to find solutions based on knowledge owned (Xenofontos & Andrews, 2014). The strategy of finding solutions to problems scientifically involves estimating, observing, analyzing information and forming results (Hong & Diamond, 2012). This strategy involves a problem-solving process that simultaneously develops students' skills in high-level thinking, one of which is to build mathematical connections (Hou, 2011). Students who have the tendency of productive connective thinking can always generalize their ability to establish mathematical connections at each stage of problem-solving, especially the solving of Polya's problems. But what inhibits students from being able to construct Polya problem solving is the inability of students to identify mathematical connections that occur within and between each Polya stage.

Through identification of the mathematical connection process of students who tend to produce productive thinking in each stage or between stages of problem-solving, Polya can know the ideas built by students when linking mathematical concepts. So that the results of this identification can be a reference for teachers to overcome student difficulties in establishing connections. Students can take advantage of connections in problem-solving, so they do not have to rely on their memory alone to remember too many isolated concepts and procedures when doing problem-solving processes (Hung & Lin, 2015). Students only need to know the relevant concepts in mathematics that can be used in other domains. To fulfill this goal students must have knowledge about connections in each stage and connections between stages in problem-solving according to Polya. This indicates that it is very important to identify the connection process that occurs in the Polya problem-solving process.
Exploring the connection process that occurs in each stage and between stages of solving Polya’s problem, is expected to lead to a positive attitude towards mathematics so that the students’ awareness and thinking will be more open to mathematics, not only focused on the particular material being studied (Hendriana, Slamet, & Sumarmo, 2014) Identifying the type of connection in stages and between stages of solving Polya’s problem, is expected to help students find the right strategy in solving mathematical problems, especially the application of mathematics in everyday life. At the same time improve mathematical connection capabilities, so that it can be used in the development and improvement of mathematics learning processes. The practical implications of the results of this study are expected to add to the scientific repertoire, especially the application of mathematical connections in solving mathematical application problems in everyday life so that in the end the essence of the mathematics learning objectives can be achieved.

Previous studies only looked at the mathematical connections that occurred in the general problem-solving process, either through assignments or through the learning process. (Eli, Mohr-Schroeder, & Lee, 2013; Hendriana et al., 2014; L. Suominen, 2015; Mhloko, Venkat, & Schfer, 2012a; Stylianou, 2013) identification of connections in verbal problem solving has been done but has not been described in detail the types of connections that occur in the stages and between stages of solving the problem Polya. So that this paper is directed to describe how connections are in the problem-solving process of students who tend to have productive connective thinking or have complete connective thinking networks? This study aims to describe the specific connection that occurs in the Polya problem-solving process for students who tend to productive connective thinking or have a complete network of connective thinking. Includes connections at each stage, as well as connections between stages in Polya’s steps which are categorized as mathematical interconnections and intracconnections.

2 REVIEW LITERATURE

Mathematical connections as a relation of several concepts or ideas, whether the relationship of concepts or ideas in mathematics and between one mathematical unity with other disciplines (Jaijan, 2012; Ozturk & Guven, 2016) Therefore mathematical connections should enable students to (1) recognize and use connections between mathematical ideas, (2) understand how mathematical ideas are interconnected and construct one another, (3) recognize and apply mathematics in an outside context mathematics. (Hsu & Silver, 2014).

(Businskas, 2008) which explains the types of mathematical connections is a process that occurs in the minds of learners. Earlier (Hiebert, J., & Carpenter, n.d, 1999) explained that structured networks such as spider webs, where points or vertices can be considered as the pieces of information represented, and the series between them as connections. This indicates that new knowledge is built on existing knowledge, or a mathematical connection must be established between pre-existing schemes so that unknown mathematical ideas can be understood by the learner. A connection exists in every part of mathematics and the learners must engage in building activities or identifying such connections and recognizing the coherent nature of mathematics that includes: multiple representations, problem solving, verification, modeling and application of mathematics in the real world (Hsu & Silver, 2014).

The mathematical connection is one of the standard curriculum of elementary and middle school mathematics learning (Hendriana et al., 2014). In order to make the process of solving the problem, must first understand the problem and to be able to understand the problems must be able to make connections with related topics. Brunner(Permana & Sumarmo, 2007) states that there is no concept or operation in mathematics that is not connected with other concepts or operations in a system, because of a fact that the essence of mathematics is something that is always associated with something else. This indicates that when students connect mathematical ideas, their understanding is deeper and more lasting, and they will see mathematics as a whole (Hsu & Silver, 2014).

In general, the connection is the relationship between ideas, concepts or procedures (Businskas, 2008). In the problem-solving process, students will connect ideas, concepts, or procedures to understand problems, plan strategies, resolve problems as planned, and re-examine the results obtained. In these four stages, students will certainly engage in mathematical activities that require them to build connections between existing knowledge and new ideas that not known. Therefore, the mathematical connections in this study will be observed from two perspectives i.e. connections that occur in every stage of Polya, and the connections that occur between these stages.
In this research, will be observed connection process that happened in problem solving step according to Polya namely:

2.1 Understanding the Problem

The first step is to understand the problem, the student may not be able to solve the problem correctly, if not understand the problem given. Students should be able to show the parts of the principle of the problem, the question, the known, the prerequisites.

2.2 Planning a Solution

This second step relies heavily on student experience in solving problems. In general, the more varied their experiences are, the more creative the students tend to be in preparing a problem-solving plan. Understanding the problem for a solving plan may be long and tortuous. The ultimate success of solving problems is the idea of a plan. This idea may appear gradually, or after a failed experiment and doubt may occur suddenly, as a “brilliant idea”. A good idea can be based on previous experience or knowledge.

2.3 Solving Problems to Plan

To think about a plan, understanding the idea of completion is not easy. The teacher should ask firmly to the student to check each step, by asking Are you sure that step is right?

2.4 Checking Back Results Obtained

A good student, when he or she has got a problem solving and written down an answer neatly, he will check again the results obtained. Teachers can ask students with questions: Can you check the results? Can you check the argument? To provide challenges and satisfaction in solving problems ask Can you get results in different ways?

The process of thinking examined in this study relates to the association of ideas that arise when establishing mathematical connections in the process of solving Polya problems. (Holyoak, K.J and Morisson, 2012) explains that building a mathematical connection involves three cognitive processes, namely building new ideas from previous ideas, building relationships among topics in mathematics itself, and applying mathematical ideas to other sciences or everyday life. (Susanti, 2015) states that connective thinking is a process of thinking in making the association between mathematical ideas when connecting mathematical concepts.

Furthermore, Susanti classifies connective thinking into 3 categorization which is simple connective thinking, semi-productive connective thinking, and productive connective thinking.

In this study will be focused on the type of connection that is formed on the network of complete or productive thinking in the solving of Polya problem solving. Thinking productively connective is the ability to think in building many connections from relevant ideas which arose based on the information provided, then formed a generalization to conclude the general rule until the formation of a knowledge reconstruction. (Susanti, 2015). Therefore, in this research, will be identified connection process that occurs in complete connective thinking network or productive in Polya problem solving process. The connection process that occurs will be identified in the intraconnection and interconnection process in solving Polya problem.

Some research related to establishing mathematical connections in problem solving has been widely practiced. Stylianou., D, 2013 examines the mathematical connections in the troubleshooting process of high school students. In his research, he describes the connection between the justification process and the representation. Jaijan & Loipha, 2012 establishes a mathematical connection with an open-ended transformation, which is through open-ended problem solving. Next (Angeli & Valanides, 2012) looks at how connections of epistemological beliefs and student reasoning when thinking about problem-solving ill-structure. Open-ended or ill-structured problems often arise in real-world situations. However, students’ awareness to use mathematical connections in problem solving, in particular, solves the problem of mathematical applications in the world real low (Baki, Çatlıoğlu, Coştu, & Birgin, 2009)

3 METHOD

The purposive sampling technique is used to select two students to research students. The two students of the FA and the AM were selected based on the results of a written test conducted by thinking a load and semi-structured interviews. Students with a network of productively connective thinking are selected to obtain a complete picture of the connection process that occurs in each stage and between stages in solving Polya problems. The work and recording of think aloud of the two students were analyzed by the qualitative descriptive approach. The semi-structured interview process was conducted to deepen the analysis of the connection process that occurred at the
stage of understanding, planning, implementation of the plan, and evaluation and connection process between the four stages to obtain the conclusion of the research results. The problem-solving sheet used in collecting data is as follows:

“Four students will take part in an innovative work competition. For that, a fee of Rp. 900,000.00 is required. Because each has a different financial condition, the amount of each student’s contribution is not the same. Student A contributes half the contribution of three other students. Student B contributed one-third of the contributions of three other students. Student C contributes a quarter of the contribution of the other three students. Student D will be.”

4 RESULT AND DISCUSSION

Connections that occur in each stage and between the stages in the problem-solving step according to Polya in this study are categorized as intra-connection and interconnection. The description of the intra-connection and interconnection of the two students is explained as follows: At the stage of understanding, FA students demonstrate their ability to identify each element that is known and asked if the problem given. Every student needs a different time to understand the problem given. This understanding arises after students write down the elements that are known and asked in the question. The process can be seen from the results of transcripts of interviews with FA students as follows:

R : What can you understand after reading the questions given?
FA: here there are four students who take part in the competition, namely students A, B, C, and D
R : What will be done by the four students?
FA: The four students will contribute to the innovative work competition.
R : What are the contributions of the four students?
FA: Its contribution, namely student A contributed half of the contribution of three other students, student B contributed one third of the contribution of three other students, student C contributed a quarter of the contribution of three other students and the total cost was 900,000
R : What is student D?
FA: what is asked in the question is the contribution of student D?

Based on the results of the transcript of interviews with FA students it was found that in order to understand the problem, the FA students identified the elements known and asked in the questions. The FA is able to identify concepts that will be used as the initial idea to develop a plan for solving the problem given. The process indicates the ability of FA students to connect each element known and asked through connection understanding. (Tasni, Nurfaiddah, 2017) examines the barriers of productive connective thinking of students in solving mathematical (Tasni & Susanti, 2016) problems and finds that one of the factors that inhibits students’ ability to think productively is the inability to establish complete connections at the stage of understanding. However, in the conditions shown by the FA students, he was able to establish a complete connection at the understanding stage, so he was able to plan better at the planning stage. In the interconnection process students make connections of understanding while if observed in the intra-connection aspects students make planning connections. explains that connection understanding, that is, connections that are built based on the ability of the subject to identify the elements that are known and asked in the question, to find out the concepts and procedures that will be used as a settlement strategy. It is also explained by (Hsu & Silver, 2014) that the ability of students to recognize connections is directly related to mathematical understanding.

At the planning stage, the first student FA reviews what is known and asked in the question then attempts to translate it into a mathematical equation. In this process, FA students plan to use the concept of comparison to formulate an equation that shows the amount of money from each student. Next, the FA students think to define the solution of the equation using the two-variable linear equation system concept, by selecting the elimination and substitution methods to determine the value of each variable. In the intra-connection process students make hierarchical connections, (Tasni & Susanti, 2016) explains that hierarchical connections namely connections are built on a hierarchical relationship between two concepts or one concept is a component of another concept. This condition can be observed when students use the concept of comparison to formulate a mathematical model of the problem being solved. Furthermore, students use procedural connections by selecting the elimination and substitution methods to determine the value of each variable. As (Nakamura, 2014) explained that knowledge can be built through the construction of hierarchical concepts in mathematics.

The second student, AM, developed a more mature settlement plan. In this case, AM students plan to use the maximum number concept to form the general equation form of the total costs that must be
spent by the four students based on the problem given, then use the concept of comparison to damage the mathematical equation of each statement in the problem. In this condition AM students make connections if it is, that is by building a connection to the question questions that want a lot of contributions from wrong attacks and the maximum amount of costs needed to participate in innovative work activities. As explained by (Mhloko, Venkat, & Schfer, 2012b) that doing mathematics with reasoning, students must look at the eye or the relationship between hypothesises and conclusions. To simplify the form the equations that have been compiled, AM Students use the concept of fractions. The process is reflected in the work of AM students as follows:

Figure 1: Work Results I of AM Student.

The following are the results of AM students' aloud transcripts in designing problem solving.

“Here I use the concept of maximum number, the concept of comparison and the concept of fractions. And to compile the equation model I will use the concept of comparison because the equation is still in fraction, so I will simplify it using the fraction concept”

Based on the results of AM student work and fragments of interview transcripts, it was found that students who used hierarchical relationships between fraction and comparison concepts. Hierarchy relationships occur when a concept is a component or contained in another concept. This was identified by the process carried out by AM students in compiling mathematical equations using the concept of comparison. This process is carried out by students to avoid the fraction of equations arranged, without changing the value of each equation. In this condition AM students make equal representation connections. In the previous study (Tasni & Susanti, 2016) explained that Equivalent Connection Representation, namely connections are built on concepts that are represented in different ways and forms but have the same value. In this study shows the equality of verbal representation to symbolic. The same thing was stated by (Businskas, 2008) that is in the same form is an equivalent representation.

At the stage of implementation of the plan, FA Students carry out the stages of implementing the plan according to the draft arranged in the previous stage. In the intra-connection process, FA students make procedural connections. (Businskas, 2008) explain that a concept can be a type of procedure or method used to connect when working with other concepts. Furthermore, in the interconnection process, FA students make syntax connections or implement plans. According to (Paper & Ribeiro, 2016) connection syntax is formed by using the basic nature of a concept to construct a new concept used in problem-solving.

The next completion step is that the FA student determines the value of variable D which is the core question of the question given. Based on the results of his work in determining the value of each variable and confirmation through the interview process can it is known that FA students use one of the concepts that has dependency logical to the other concepts. Or show a relationship if then between the two concepts. This was identified when FA students used logical reasoning in the process determine the values of variables A, B, C, and D. Where each equation is seen as a premise, while the results or values of variables are obtained from the process of elimination is a logical conclusion. (Mhloko et al., 2012b) explains that the characteristics of the connection if it is when students prove each guess and make conclusions based on facts previously known.

In the Evaluation Stage. Every student has different abilities in investigating the truth of the problem solving that has been done. At the evaluation stage the FA students focus on the question, is there another procedure that can be used to obtain the same answer. So that FA students believe that elimination and substitution procedures are the only way to determine the solution of each equation. FA students assume that another method, in this case the graphical method cannot be used because each equation that is composed contains four variables that cannot be described in dimension two. Therefore, FA students choose the substitution method as another procedure to verify the answers obtained. In the intra-connection process FA students make equal connection connections while simultaneously evaluating connections on the interconnection process. As explained in (Tasni & Susanti, 2016) that connection is Justification and Representation, that is a connection that is built when the subject evaluates the truth of the answers obtained, with the concepts
and procedures used. The process can be seen from the FA students’ think aloud transcripts as follows:

“To prove the truth of the answers I got, I will use a different method, namely the substitution method.”

Interconnection is the process of connection that occurs between each stage of Polya. In the interview process the FA and AM students showed a good understanding of the questions given. This was identified by their ability to write down the elements known and asked in the questions. With their good understanding, they are able to develop a problem solving strategy, which is to develop a mathematical model of the elements known in the problem. (Mackrell & Pratt, 2017) explains that by having adequate strategic knowledge, students will design appropriate strategies to solve problems. Through the understanding they have of the questions given, they are also able to identify the concepts that will be used.

In developing problem solving strategies. Among other things, FA students plan use the concept of a two-variable linear equation system to solve problems and the concept of comparison to form a mathematical model. (Plaxco & Wawro, 2015) explains that understanding in linear algebra can make it easier for students to do mathematical solutions.

Both FA and AM students make planning connections. Planning connection is a process of interconnection that occurs from the understanding stage to the planning stage, namely the idea of completion that appears in the minds of students after understanding the problem. This can be noted from its ability to identify the concepts to be used and compile resolution strategies. As shown in the FA student think aloud transcripts as follows:

“I simplify each equation that is formed by using the fraction concept, where to change the form of fractions into integers, I multiply it by the same number in the denominator, after the simple form I use the elimination and substitution methods to determine the value of each variable”

Based on the fragments of the interview transcript, it can be said that there are interconnections carried out by students from the planning stage to the completion stage of the settlement. This is identified by the ability of students to use the previously mentioned concepts to simplify the model of equality that has been compiled. In this case the researcher identifies the interconnection process that occurs from the planning stage to the implementation stage is the connection implementation plan (syntax connection). Connection implementation plan or syntax occurs because there is a relationship between the strategy designed and the implementation of the strategy. By compiling a mature resolution strategy, students will succeed in the process of implementing the strategy in solving problems, as explained by (Anthony & Walshaw, 2009) when students have the strategic knowledge needed to correct existing problems but, applying them ineffectively, will fail to use the right strategy, the same thing is explained by (Sulak, 2010) that students who are able to develop sound strategies will succeed in solving problems.

Furthermore, the interconnection process that occurs at the implementation stage of the plan to the evaluation stage is also influenced by the stage of understanding carried out by students. This can be seen from the evaluation process carried out by AM students. In order to be sure of the correctness of the answers obtained, AM students re-match the values of each variable obtained from the implementation stage, with mathematical models arranged based on the elements known in the problem. The process is illustrated by the following work results of AM students:

![Figure 2: Work Results II of AM Student.](image)

Based on the results of the AM student’s work and the interview process, it is known that AM students connect evaluation. Evaluation connections are the interconnections shown through the relationship between representation and justification in the problem-solving process. Students must have the ability to use different methods in the evaluation process to have the ability to solve problems (Esen & Belgin, 2017). (Eli et al., 2013) explains that there is a connection between representation and justification, namely the ability of students to find connections between the final results obtained with representations based on the data obtained at the understanding stage will lead students to obtain the appropriate problem solutions.
5 CONCLUSIONS

The Intraconnection process that occurs in the Polya problem-solving stage begins from the understanding stage. Students demonstrate the ability to identify elements that are known and asked in the matter, these conditions are identified as understanding connections. Furthermore, in the planning stage students demonstrate the ability to build a hierarchical relationship between two concepts or one of the concepts that are components of another concept, this condition is identified as a hierarchical connection. In the implementation stage of the student plan shows the ability to identify a concept that has a logical dependency on another concept, this condition is identified as a connection if then. In addition, at the planning stage also identified procedural connection occurred. Procedural connections are demonstrated by students' ability to use a concept when working with a particular method or procedure. Subsequent connections that occur at the stage of the implementation of the plan is equivalent representational connections, this connection is indicated by the ability of students to represent a concept with a variety of relevant representations.

The interconnection process that occurs between the Polya problem-solving steps is started from the coherence built between the understanding stage and the problem-solving planning stage. These connections are identified as planning connections, these connections are demonstrated through a relationship of the understanding level to the maturity of the completion strategy to be developed. Further connections that occur between the stages of planning and stages. The implementation of the plan i.e. connection syntax. Syntax connection shows the realization of the concept or procedure from the planning stage to the implementation stage of the planning stage. Further connections that arise between the evaluation stage and the understanding stage of the evaluation connection. The evaluation connection shows the relationship of justification by checking the conformity of the solution obtained with the representation of mathematical models arranged based on known elements at the understanding stage.

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