Revisiting Population Structure and Particle Swarm Performance

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Abstract: Population structure strongly affects the dynamic behavior and performance of the particle swarm optimization (PSO) algorithm. Most of PSOs use one of two simple sociometric principles for defining the structure. One connects all the members of the swarm to one another. This strategy is often called gbest and results in a connectivity degree $k = n$, where n is the population size. The other connects the population in a ring with $k = 3$. Between these upper and lower bounds there are a vast number of strategies that can be explored for enhancing the performance and adaptability of the algorithm. This paper investigates the convergence speed, accuracy, robustness and scalability of PSOs structured by regular and random graphs with $3 \leq k \leq n$. The main conclusion is that regular and random graphs with the same averaged connectivity k may result in significantly different performance, namely when k is low.

1 INTRODUCTION

Particle Swarm Optimization (PSO) is a collective intelligence model for optimization and learning (Kennedy and Eberhart, 1995) that uses a set of position vectors (called particles) to represent candidate solutions to a specific problem. These particles move through the fitness landscape of a specified target-problem following a set of behavioral equations that define their velocity at each time step. After updating the velocity and position of each particle as well as to the global and local information about the search, the fitness of every particle is computed. The process repeats until a stop criterion is met.

Information on the current and previous state of the search flows through the graph that connects the particles, informing them on the best solutions found by their neighbors. The graph can be of any form and affects the balance between exploration and exploitation and consequently the convergence speed and accuracy of the algorithm. The reason why particles are interconnected is the core of the algorithm: particles communicate so that they acquire information on the regions explored by other particles. In fact, it has been claimed that the uniqueness of the PSO algorithm lies in the interactions of the particles (Kennedy and Mendes, 2002).

As stated, the population can be structured on any possible topology, from sparse to dense (or even fully connected) graphs), with different levels of connectivity and clustering. The classical and most used population structures are the *lbest* with ring topology (which connects the individuals to a local neighborhood) and the *gbest* (in which each particle is connected to every other individual). These topologies are well-studied and the major conclusions are that *gbest* is fast but is frequently trapped in local optima, while *lbest* is slower but converges more often to the neighborhood of the global optima.

Studies have tried to understand what makes a good structure. For instance, Kennedy and Mendes (Kennedy and Mendes, 2002) investigated several types of topologies and recommend the use of a lattice with von Neumann neighborhood (which results in a connectivity degree between that of *lbest* and *gbest*). Others, like (Parsopoulos and Vrahatis, 2005), have tried to design networks that hold the best traits given by each structure.

This paper revisits the study in (Kennedy and Mendes, 2002). Although the authors provided significant insight on the relationship between population structure and PSO performance, the study

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was mainly dedicated to random topologies and few levels of connectivity were inspected. Some aspects of the research subject that were overlooked are now worth investigating, namely the importance of graph regularity and the performance of regular and random graphs with the same level of connectivity. This paper investigates and compares the convergence speed, accuracy, robustness and scalability of PSOs structured by regular and random graphs with different connectivity. Finally, the topologies were not only tested on standard fixed-parameters PSOs, but also on a PSO with time-varying parameters.

The present work is organized as follows. Section 2 gives a background review on PSO and population structures. Section 3 describes the experiment setup and Section 4 discusses the results. Finally, Section 5 concludes the paper and outlines future lines of research.

2 BACKGROUND REVIEW

PSO is described by a simple set of equations that define the velocity and position of each particle. The position vector of the *i-th* particle is given by $\vec{X}_i = (x_{i,1}, x_{i,2}, \dots x_{1,D})$, where D is the dimension of the search space. The velocity is given by $\vec{V}_i = (v_{i,1}, v_{i,2}, \dots v_{1,D})$. The particles are evaluated with a fitness function $f(\vec{X}_i)$ and then their positions and velocities are updated by:

$$v_{i,d}(t) = v_{i,d}(t-1) + c_1 r_1 \left(p_{i,d} - x_{i,d}(t-1)\right) + c_2 r_2 \left(p_{g,d} - x_{i,d}(t-1)\right) \quad (1)$$

$$x_{i,d}(t) = x_{i,d}(t-1) + v_{i,d}(t) \quad (2)$$

were p_i is the best solution found so far by particle i and p_g is the best solution found so far by the neighborhood. Parameters $r_1$and r_2 are vectors of random values uniformly distributed in the range $[0, 1]$ and $c_1$and c_2 are acceleration coefficients.

In order to prevent particles from moving out of the limits of the search space, the positions $x_{i,d}(t)$ of the particles are limited by constants that, in general, correspond to the domain of the problem: $x_{i,d}(t) \in [-Xmax, Xmax]$. Velocity may also be limited within a range in order to prevent the explosion of the velocity vector: $v_{i,d}(t) \in [-Vmax, Vmax]$. Usually, $Xmax = Vmax$.

Although the classical PSO can be very efficient on numerical optimization, it requires a proper balance between local and global search, as it often gets trapped in local optima. In order to achieve a better balancing mechanism, (Shi and Eberhart, 1998) added the inertia weight ω for fine-tuning the local and global search abilities of the algorithm.

By adjusting ω (usually within the range [0, 1.0]) together with the constants c_1 and c_2, it is possible to balance exploration and exploitation abilities of the PSO. The modified velocity equation is:

$$v_{i,d}(t) = \omega . v_{i,d}(t-1) + c_1 r_1 \left(p_{i,d} - x_{i,d}(t-1)\right) + c_2 r_2 \left(p_{g,d} - x_{i,d}(t-1)\right) \quad (3)$$

The neighborhood of the particle defines in each time-step the value of p_g and is a key factor for the performance of the algorithm. Most of the PSOs use one of two simple principles for defining the neighborhood network. One connects all the members of the swarm to one another and is called *gbest*, where *g* stands for *global*. The degree of connectivity of *gbest* is $k = n$, where n is the number of particles. The other typical configuration, called *lbest* (*l* stands for local), creates a neighborhood that comprises the particle itself and its nearest neighbors. The most common *lbest* topology is the ring structure.

As stated above, the topology of the population affects the performance of the PSO and the configuration must be chosen according to the target-problem and the performance requirements (i.e., the acceptable compromise between convergence speed and accuracy). Since all the particles are connected to every other and information spreads easily through the network, the *gbest* topology usually converges fast but unreliably (it often converges to local optima). The *lbest* converges slower than *gbest* because average path length of the network is higher and information spreads slower, but, for the same reason, it is also less prone to converge prematurely to local optima.

In-between the ring structure with $k = 3$ and the *gbest* with $k = n$ there are several possibilities, each one with its advantages and drawbacks. Very often it is not possible to choose beforehand the optimal or near-optimal configuration: for instance, when the properties of the problem are unknown or the time requirements do not permit preliminary tests. Therefore, substantial research efforts have been dedicated to PSO's population structures.

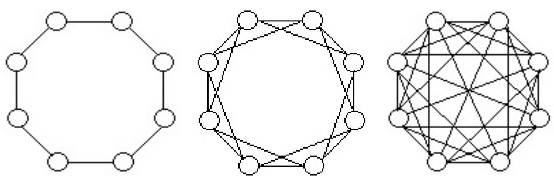
In 2002, (Kennedy and Mendes, 2002) tested several types of structures, including *lbest*, *gbest* and von Neumann configurations. They also tested populations arranged in graphs that were randomly generated and optimized to meet some criteria. The authors concluded that when the configurations were ranked by the performance at 1000 iterations the structures with $k = 5$ perform better, but when ranked according to the number of iterations needed to meet the criteria, configurations with higher degree of connectivity perform better. These results are consistent with the premise that low connectivity favors robustness, while higher connectivity favors convergence speed (at the expense of reliability).

The unified PSO (UPSO) (Parsopoulos and Vrahatis, 2005) combines *gbest* and *lbest* configurations. Equation 1 is modified in order to include a term with p_g and a term with p_i while a parameter balances the weight of each term. The authors argue that the proposed scheme exploits the good properties of *gbest* and *lbest*.

(Peram et al., 2003) proposed the fitness–distance-ratio-based PSO (FDR-PSO). The algorithm defines the neighborhood of a particle as its k closest particles in the population (measured in Euclidean distance). A selective scheme is also included: the particle selects near particles that have also visited a position of higher fitness. The authors claim that FDR-PSO performs better than the standard PSO on several test functions. However, FDR-PSO is compared only to the *gbest* configuration. Recently, (Ni et al., 2014) proposed a dynamic probabilistic PSO. The authors generate random topologies for the PSO that they use at different stages of the search.

3 EXPERIMENTAL SETUP

First, several regular graphs have been constructed using the following procedure: starting from a ring structure with $k = 3$ the degree is increased by linking each individual to its neighbors' neighbors, thus creating a set of regular graphs with $k = \{3,5,7,9,11 \ldots, n\}$, as exemplified in Figure 1 for a swarm with 8 particles (the configuration is easily generalized to other population sizes).



$k = 3$ $k = 5$ $k = n$

Figure 1: Regular graphs with population size $\boldsymbol{n} = \mathbf{8}$.

For the experiments discussed in this paper, PSOs with population size $n = 33$ have been used and regular graphs with $k = \{3,5,7,9,13,17,25,33\}$ were constructed. Please note that the regular graph with $k = 33$ corresponds to the *gbest* topology. Then, random graphs with 33, 66, 99, 132, 198, 264 and 396 bi-directional edges were also generated, corresponding to an average level of connectivity $k' = \{3,5,7,9,13,17,25,33\}$. Again, the random graph with $k' = 33$ is equivalent to the *gbest* structure.

The acceleration coefficients of the fixed-parameters PSO were set to 1.49618 and the inertia weight is 0.729844 (Rada-Vilela et al., 2013). An alternative approach to fixed parameter tuning is to let the values change during the run, according to deterministic or adaptive rules. (Shi and Eberhart, 1998) proposed a linearly time-varying inertia weight. The variation rule is given by Equation (4).

$$\omega(t) = (\omega_1 - \omega_2) \times \frac{(\max\_t - t)}{\max\_t} + \omega_2 \tag{4}$$

where t is the current iteration, max_t is the maximum number of iterations, ω_1 the inertia weigh initial value and ω_2 its final value.

Later, (Ratnaweera et al., 2004) proposed to improve Shi and Eberhart's *PSO with time-varying inertia weight* (PSO-TVIW) using a similar concept applied to the acceleration coefficients. In the *PSO with time-varying acceleration coefficients* PSO (PSO-TVAC) the parameters c_1 and c_2 change during the run according to the following equations:

$$c_1 = \left(c_{1f} - c_{1i}\right) \times \frac{t}{\max\_t} + c_{1i} \tag{5}$$

$$c_2 = \left(c_{2f} - c_{2i}\right) \times \frac{t}{\max\_t} + c_{2i} \tag{6}$$

where $c_{1i}, c_{1f}, c_{2i}, c_{2f}$ are the acceleration coefficients initial and final values. For the experiments with PSO-TVAC in the following section, parameters ω_1 and ω_2 were set to 0.9 and 0.4, the acceleration coefficient c_1 initial and final values were set to 2.5 and 0.5 and c_2 ranges from 0.5 to 2.5, as suggested in (Ratnaweera et al, 2004).

Table 1: Benchmark functions.

	mathematical representation	***search range/ initialization***	***stop criterion***
sphere f_1	$f_1(\vec{x}) = \sum_{i=1}^{D} x_i^2$	$(-100, 100)$ $(50, 100)^D$	0.000001
quadric f_2	$f_2(\vec{x}) = \sum_{i=1}^{D} \left(\sum_{j=1}^{i} x_j \right)^2$	$(-100, 100)$ $(50, 100)^D$	0.01

Table 1: Benchmark functions (cont.).

	mathematical representation	*search range/ initialization*	*stop criterion*
hyper ellipsoid f_3	$f_1(\vec{x}) = \sum_{i=1}^{D} i{x_i}^2$	$(-100, 100)$ $(50,100)^D$	0. 000001
rastrigin f_4	$f_4(\vec{x}) = \sum_{i=1}^{D} ({x_i}^2 - 10\cos(2\pi x_i) + 10)$	$(-10,10)^D$ $(2.56, 5.12)^D$	100
griewank f_5	$f_5(\vec{x}) = 1 + \frac{1}{4000}\sum_{i=1}^{D} {x_i}^2 - \prod_{i=1}^{D} \cos\left(\frac{x_i}{\sqrt{i}}\right)$	$(-600, 600)$ $(300,600)^D$	0.05
weierstrass f_6	$f_7(\vec{x}) = \sum_{i=1}^{D}\left(\sum_{k=0}^{kmax}\left[a^k cos\left(2\pi b^k(x_i + 0.5)\right)\right]\right) - D\sum_{k=0}^{kmax}[a^k cos(2\pi b^k \cdot 0.5)],$ $a = 0.5, b = 3, kmax = 20$	$(-0.5,0.5)^D$ $(-0.5, 0.2)^D$	0.01
ackley f_7	$f_8(\vec{x}) = -20exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}\right) - exp\left(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i)\right) + 20 + e$	$(-32.768,3$ $(2.56,5.12)^D$	0.01
shifted quadric with noise f_8	$f_9(\vec{z}) = \sum_{i=1}^{D}\left(\sum_{j=1}^{i} z_j\right)^2 * (1 + 0.4\lvert N(0,1)\rvert),$ $\vec{z} = \vec{x} - \vec{o}$, $\vec{o} = [o_1, .. o_D]$: *shifted global optimum*	$(-100, 100)$ $(50, 100)^D$	0.01
rotated griewank f_9	$f_{10}(\vec{z}) = 1 + \frac{1}{4000}\sum_{i=1}^{D} {z_i}^2 - \prod_{i=1}^{D}\cos\left(\frac{z_i}{\sqrt{i}}\right)$, $\vec{z} = \boldsymbol{M}\vec{x}$, **M**:ortoghonal matrix	$(-600, 600)$ $(300, 600)^D$	0.05

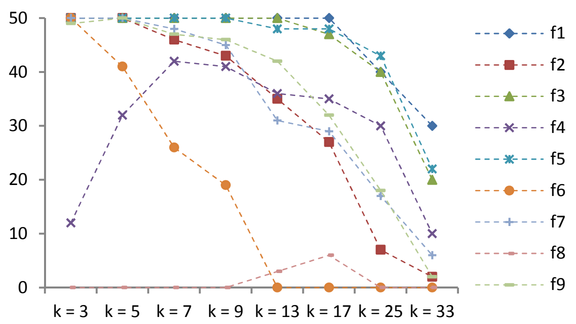


Figure 2: Success rates (50 runs). Regular graphs. Problem dimension $D = 30$. Standard PSO with fixed-parameters.

$Xmax$ is defined as usual by the domain's upper limit and $Vmax = Xmax$. A total of 50 runs for each experiment were performed. Nine benchmark problems were used. Functions f_1-f_3 are unimodal; f_4-f_7 are multimodal; f_8 is the shifted f_2 with noise and f_9 the rotated f_5 (f_8 global optimum and f_9 matrix were taken from CEC2005 benchmark). *Asymmetrical initialization* is used (initialization range for each function is given in Table 1).

Two sets of experiments were conducted. First, the algorithms were run for a specific amount of function evaluations (330000 for f_1 and f_3, 660000 for the remaining). The best solution was recorded after each run. Each algorithm has been executed 50 times in each function. Statistical measures were taken over those 50 runs. In the second set of experiments the algorithms were run for 660000 evaluations or until reaching function-specific stop criteria (given in Table 1). A success measure is defined as the number of runs in which an algorithm attains the criterion. Again, each one has been executed 50 times in each function. This setup is as in (Kennedy and Mendes, 2002)0.

The algorithms discussed in this paper are available in the OpenPSO package, which offers an efficient, modular and multicore-aware framework for experimenting with different PSO approaches. The package is implemented in C99, and transparently parallelized with OpenMP (Dagum and Menon, 1998). The library components can be interfaced with other programs and programming languages, making OpenPSO a flexible and adaptable framework for PSO research. The source code is at https://github.com/laseeb/openpso.

4 RESULTS AND DISCUSSION

The main objectives of the experiments are to examine how fixed-parameters PSOs perform with different levels of connectivity and investigate if the relative performance varies with problem dimension.

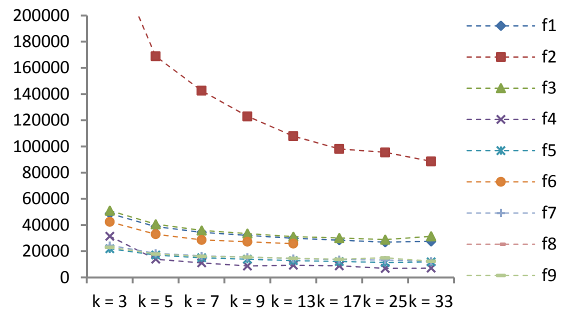


Figure 3: Evaluations required to meet criteria: median values (50 runs). Regular graphs. Problem dimension $D = 30$. Fixed-parameters PSO.

Then, study the differences between the performance of PSOs with regular and random graphs. Finally, confirm if the same general conclusions apply to time-varying strategies for parameter setting.

4.1 Regular Graphs

The first experiment compares the success rates, convergence speed and accuracy (best solutions) of fixed-parameters PSO on regular graphs. Problem dimension is $D = 30$. Figure 2 shows the success rates of the algorithm on each function with each regular graph. In general, better success rates are attained with lower connectivity, but there are two exceptions: functions f_4 and f_8. However, these results are in general terms in accordance with those in (Kennedy and Mendes, 2002): configurations with lower connectivity attain better success rates.

Figure 3 represents the median values of the evaluations required to meet the stop criteria. Clearly, the convergence speed increases with connectivity degree k. These findings are in different from those in (Kennedy and Mendes, 2002), where it is reported that the configurations with $k = 5$ (from a set with $k = 3$, $k = 5$ and $k = 10$ graphs) required less evaluations to meet the stop criteria. However, those experiments were conducted under different conditions, like population size and, namely, graph types: here, we are testing PSO on regular graphs with varying size.

Table 2 shows the median values of the best fitness attained in each of the 50 runs, for each function and each graph. The best graphs according

Table 2: Best fitness. Median values. Regular graphs. Problem dimension D = 30. Fixed-parameters PSO.

	$k = 3$	$k = 5$	$k = 7$	$k = 9$	$k = 13$	$k = 17$	$k = 25$	$k = 33$
f_1	1.96e-89	7.85e-90	3.93e-90	1.96e-90	1.96e-90	**0.00e00**	**0.00e00**	3.93e-90
f_2	7.59e-13	1.04e-20	2.49E-25	4.41e-29	3.03e-34	**6.04e-37**	1.00e+04	2.00e+04
f_3	1.67e-88	3.34e-89	5.89e-90	1.96e-90	0.00e00	**0.00e00**	**0.00e00**	4.50e+04
f_4	1.18e+02	8.71e+01	**8.31e+01**	7.26e+01	8.31e+01	8.66e+01	8.71e+01	1.28e+02
f_5	**0.00e+00**	**0.00e+00**	**0.00e+00**	**0.00e+00**	1.11e-02	7.40e-03	9.86e-03	6.85e-02
f_6	**0.00e+00**	**0.00e+00**	6.17e-03	6.78e-02	1.02e+00	2.03e+00	4.33e+00	6.03e+00
f_7	**7.55e-15**	**7.55e-15**	**7.55e-15**	**7.55e-15**	**7.55e-15**	**7.55e-15**	1.25e+00	1.90e+00
f_8	2.02e+02	1.32e+01	9.23e-01	**3.43e-01**	4.98e+03	9.30e+03	2.86e+04	4.74e+04
f_9	**0.00e00**	**0.00e00**	**0.00e00**	8.63e-03	1.23e-02	1.72e-02	5.09e-01	4.25e+01

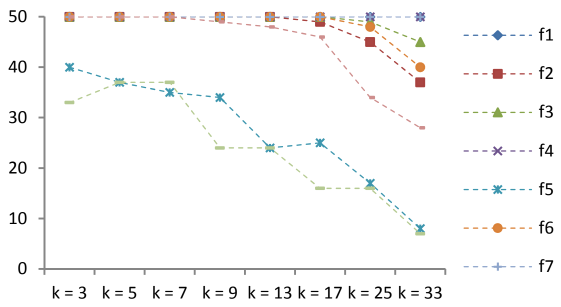


Figure 4: Success rates (50 runs). Regular graphs. Problem dimension $D = 10$. Standard PSO with fixed-parameters.

to the accuracy criteria depend on the type of function. For unimodal functions (f_1, f_2, f_3 and f_8) best results are attained with highly connected graphs, while multimodal functions require graphs with lower connectivity.

In (Kennedy and Mendes, 2002), configurations with $k = 5$ yielded the best fitness values and required less evaluations to meet the criteria, while $k = 3$ had the best success rates. The results in this paper, although they do not necessarily contradict the experiments in (Kennedy and Mendes, 2002) (which were conducted under different conditions), provide some more insight on the performance of PSO populations with different connectivity levels.

4.2 Problem Dimension

The next test investigates the behavior of the algorithm with different problem dimension. For that purpose, D was set to $D = 10$ and $D = 50$. The algorithms were tested as in the previous experiment.

Figure 4 and Figure 5 show the success rates for $D = 10$ and $D = 50$ respectively. Changing the problem dimension does affect the general behavior of the PSO on regular graphs with different connectivity levels: lower k graphs yield better success rates for $D = 10$ and $D = 50$ (the most notorious exception is f_5 when $D = 50$). Some functions behave differently, namely f_5 and f_8 (Griewank and rotated Griewank), for which the success rates tend to increase with D. However, the overall performance scales as expected, as seen in Figure 6, which depicts the percentage of successful runs of each type of graph for each D averaged over the whole set of functions.

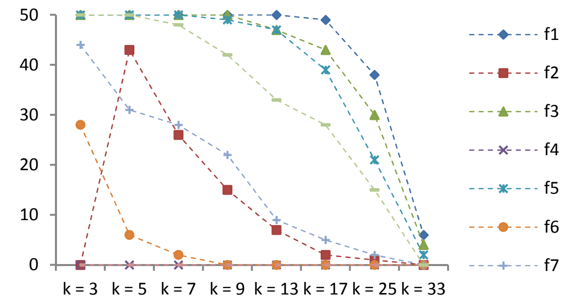


Figure 5: Success rates (50 runs). Regular graphs. Problem dimension $D = 50$. Standard PSO with fixed-parameters.

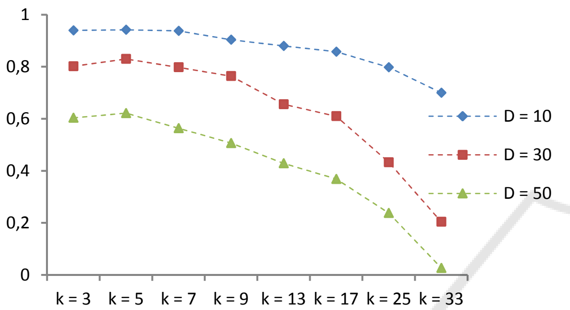


Figure 6: Percentage of successful runs averaged over the set of functions. Regular graphs. Fixed-parameters PSO.

As for the convergence speed and accuracy, the results lead to the same conclusions as in Section 4.1 for $D = 30$: convergence speed for $D = 10$ and $D = 50$ increases with k and accuracy depends on the type of function: unimodal are better tackled with highly connected graphs while multimodal problems require graphs with lower connectivity.

4.3 Time-varying Parameters

A final experiment implemented and tested the PSO-TVAC on the set of regular graphs. Success rates are shown in

Figure 7. PSO-TVAC is able to meet the criteria in every function (except f_8) with $k = 3$ and $k = 7$. It also improves the performance of standard PSO on several functions for higher k values. On the other hand, it is significantly slower than the standard PSO on every function and every k.

Mann-Whitney tests were performed to compare the distributions of the number of evaluations to meet criteria of each graph in each function confirming that the PSO is significantly faster than PSO-TVAC in every function and k. Comparing Figure 8 and Figure 3 gives an overall idea on the magnitude of the differences in convergence speed.

Table 3: Best fitness. Median values. Random graphs. Problem dimension D = 30. Fixed-parameters PSO.

	$k' = 3$	$k' = 5$	$k' = 7$	$k' = 9$	$k' = 13$	$k' = 17$	$k' = 25$
f_1	1.57e-89	5.89e-90	3.93e-90	1.96e-90	1.96e-90	**0.00e+00**	**0.00e+00**
f_2	5.00e+03	6.39e-26	3.02e-29	1.13e-31	6.85e-35	**6.65e-38**	1.00e+04
f_3	1.21e-88	1.08e-89	5.89e-90	**0.00e+00**	**0.00e+00**	**0.00e+00**	**0.00e+00**
f_4	1.13e+02	7.56e+01	**6.91e+01**	7.91e+01	8.51e+01	8.31e+01	9.60e+01
f_5	1.23e-02	9.86e-03	**3.70e-03**	7.40e-03	7.40e-03	1.23e-02	9.86e-03
f_6	2.52e+00	**1.51e-01**	2.00e-01	7.52e-01	1.13e+00	1.92e+00	3.64e+00
f_7	1.78e+00	**7.55e-15**	**7.55e-15**	**7.55e-15**	**7.55e-15**	1.11e-14	1.16e+00
f_8	2.26e+04	3.16e+02	**1.77e-01**	3.45e+01	2.58e+02	1.33e+04	2.20e+04
f_9	3.31e-02	9.86e-03	9.86e-03	**7.40e-03**	2.22e-02	1.23e-02	5.20e-01

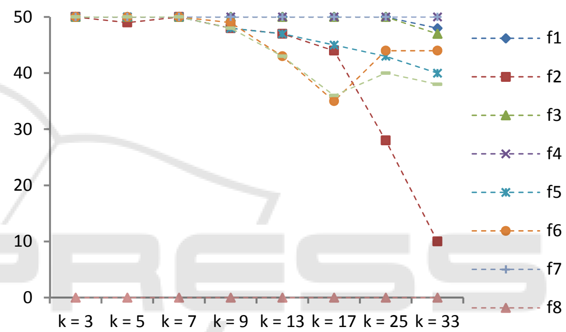


Figure 7: Success rates (50 runs). Regular graphs. Problem dimension D = 30. PSO-TVAC.

5 CONCLUSIONS

This paper investigates the performance of PSOs with regular and random structures. A set of regular and random graphs with different levels of connectivity were constructed and used as network topologies for the algorithms. Success rates, convergence speed, accuracy and scalability have been investigated. Results show that the probability of meeting the stop criteria (success rates) is higher when the degree of connectivity k is lower. However, convergence speed increases with k. As for the accuracy, the experiments showed that best results on unimodal functions are attained with highly connected graphs while lower connectivity is more suited for multimodal functions. The general behavior maintains when varying the search space dimension. Also, PSOs with fixed and time-varying parameters behave similarly throughout the range of regular graphs.

One of the most interesting results concerns the comparison between regular and random graphs. The experiments demonstrated that switching from a regular to a random graph with the same level of connectivity degrades PSO success rates and accuracy when k is low, while for higher k the results are similar. This is probably due to the high variance of the average k in graphs with low connectivity but further investigation is required to confirm this hypothesis.

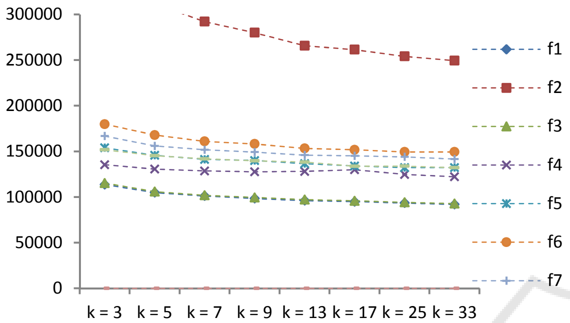


Figure 8: Evaluations required to meet criteria: median values (50 runs). Regular graphs. Problem dimension $D = 30$. PSO-TVAC.

The analysis in this paper has been mainly qualitative and supported by graphical depiction of the results. In the future, the data will organized and normalized in order to perform exhaustive statistical tests that will hopefully give more insight on the relationship between performance and population structure and provide support to the conclusions and hypothesis raised by this study. In addition, more random graphs will be generated and tested, with different standard deviation of k and clustering degree. Finally, the effect of dynamic structures in the performance of PSO will be investigated.

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