State- and Uncertainty-observers-based Controller for a Class of T-S Fuzzy Models

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Abstract: A control system design based on the T-S fuzzy model with uncertainty is considered in this paper. At first, state observer is developed to estimate the state properly despite the existence of the uncertainty. Then, uncertainty observer is derived using the estimated state. Finally, a controller based on the observers is proposed in an effort to counteract the influence of the uncertainty whatever possible. In addition, the Nussbaum-type function and its relevant properties are used in the controller design to cover the observers' error and the part of estimated uncertainty that is not possibly used through the control matrices. As a result, the closed-loop control system becomes asymptotically stable.

1 INTRODUCTION

The T-S fuzzy model (Takagi and Sugeno, 1985) is widely used in the control system design. Because the consequent of fuzzy rules that compose the T-S fuzzy model is in the form of the state-space representation, usually the linear system theory can be applied straightforwardly to the T-S fuzzy model-based control system design. As a result, the control system stability will be guaranteed by certain linear matrix inequalities (LMIs), which can be solved by some existing software packages. Recently, the T-S fuzzy model has been further extended to the so-called polynomial fuzzy model (Tanaka et al., 2009; Han et al., 2017). However, whatever models may be, they are not more than a mathematical description to, usually approximately, describe the dynamics of the concerned systems. In other words, it seems almost impossible to form a model immaculately, given that nonlinear systems are usually considered, and what is more, disturbance from external/internal system, parameters' perturbation and unmodeled dynamics always exist in the real systems. Such a discrepancy between the concerned system and its model is called uncertainty in this paper.

The uncertainty may be called different names such as unknown input, disturbance in different contexts. When it comes to controller designs, H_∞ control (Wei et al., 2016), adaptive control (Khalil, 2002; Han et al., 2001; Liu et al., 2016) are thought to be very effective ways to dealing with it. Nevertheless, there is still plenty of room to improve the control performance. For example, when it comes to H_∞ control approach, the influence of the uncertainty to the state is no more than being confined to certain prescribed indexes.

Recently, uncertainty observer-based control provides a promising approach to handle the uncertainty and improve robustness. In the existing works (Han, 2016; Han and Lam, 2015), the observers involved are designed under the assumption that the uncertainty is not time varying. It is clear that such an assumption is difficult to cope with other cases such as acute time varying uncertainties. Therefore, observing the uncertainty at a more general level and involving the estimated uncertainty in the controller design to cover its influence as much as possible is the fundamental notion of this paper. However, at first, a state observer is presented for the case where the state is unavailable. Most of state observers based on T-S fuzzy models in literature (Liu and Zhang, 2003; Cao et al., 2008; Chadli and Karimi, 2013; Wei et al., 2016) are designed in the traditional sense similar to the one based on the state-space representation, in which some techniques such as H_∞ approach (Cao et al., 2008) and adaptive control (Lendek et al., 2010) are used to cover the uncertainty. Inspired by the existing works for the case of unknown input (Darouach et al., 1994; Hui and Zak, 2005), the one in this paper adopts a different structure in which some elaborately designed matrices are introduced so that the uncertainty has no influence to the the state observer at all. Then, uncertainty

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observer is derived using the estimated state. Finally, a controller based on the observers is proposed in an effort to counteract the influence of the uncertainty whatever possible. In addition, the Nussbaum-type function and its relevant properties are used in the controller design to cover the observers' error and the part of estimated uncertainty that is not possibly used through the control matrices; as a result, the closed-loop control system becomes asymptotically stable.

Throughout this paper, M^- denotes the pseudo-inverse of $M \in R^{m\times n}$, i.e., a matrix satisfying the equation $MM^-M = M$, which means M^- is either the left pseudo-inverse $M^\dagger = (M^TM)^{-1}M^T$ or right pseudo-inverse $M^\ddagger = M^T(MM^T)^{-1}$. It is clear that $M^\dagger M = I_n$, $MM^\ddagger = I_m$. In addition, $\|\cdot\|$, and $|\cdot|$ denote the p-norm with p being 2, and 1, respectively.

2 STATE AND UNCERTAINTY OBSERVERS BASED ON T-S FUZZY MODEL

Assume that a nonlinear system can be represented by the following T-S fuzzy model:

Plant Rule i :

If $\theta_1(t)$ is M_1^i and $\cdots$ and $\theta_{n_\theta}(t)$ is $M_{n_\theta}^i$, then

$$\left.\begin{array}{l} \dot{x}(t) = A_ix(t) + B_iu(t) + Dd(t) \\ y(t) = Cx(t) \end{array}\right\} \quad (1)$$

where θ_j $(j = 1, 2, \cdots, n_\theta)$ is a variable in the antecedent that is available; M_j^i $(i = 1, 2, \ldots, n_r)$, a fuzzy term corresponding to ith rule; $x(t) \in R^n$, the state vector; $u(t) \in R^m$, the input vector; $y(t) \in R^p$, the output vector; $A_i \in R^{n\times n}$, $B_i \in R^{n\times m}$, $C \in R^{p\times n}$, $D \in R^{n\times r}$, some compatible matrices; $d(t) \in R^r$, uncertainty including modeling error, external disturbance, unmodeled dynamics and parameter perturbations. In the model, it is assumed that $\text{rank}(CD) = \text{rank}(D)$.

The overall T-S fuzzy model is of the following form accordingly:

$$\left.\begin{array}{l} \dot{x}(t) = \sum_{i=1}^{n_r} \alpha_i(t)\Big(A_ix(t) + B_iu(t) + Dd(t)\Big) \\ y(t) = Cx(t) \end{array}\right\} \quad (2)$$

where $\theta(t) = \big[\theta_1(t)\ \theta_2(t) \cdots \theta_{n_\theta}(t)\big]$,

$$\alpha_i(t) = \frac{\omega_i(\theta(t))}{\sum_{i=1}^{n_r} w_i(\theta(t))} \geq 0, \qquad \sum_{i=1}^{n_r} \alpha_i(t) = 1,$$

$$\omega_i(\theta(t)) = \prod_{j=1}^{n_\theta} M_j^i(\theta_j(t)).$$

Inspired by the approach of unknown input observer (UIO), the following state observer is first suggested:

State Observer Rule i :

If $\theta_1(t)$ is M_1^i and $\cdots$ and $\theta_p(t)$ is M_p^i, then

$$\left.\begin{array}{l} \dot{z}(t) = F_iz(t) + TB_iu(t) + G_iy \\ \hat{x}(t) = z(t) + Hy(t) \end{array}\right\} \quad (3)$$

where $z(t) \in R^n$, is the internal state vector of the observer; $\hat{x}(t) \in R^n$, the estimate of the state x(t); $F_i \in R^{n\times n}$, $T \in R^{n\times n}$, $G_i \in R^{n\times p}$, $H \in R^{n\times p}$, the designs parameters to be determined.

The overall observer of the following form accordingly.

$$\left.\begin{array}{l} \dot{z}(t) = \sum_{i=1}^{n_r} \alpha_i(t)\Big(F_iz + TB_iu + G_iy\Big) \\ \hat{x}(t) = z(t) + Hy(t) \end{array}\right\} \quad (4)$$

From here, unless confusion arises arguments such as t, θ will be omitted just for notational convenience.

From (4), we have

$$\begin{aligned} \dot{\hat{x}} &= \dot{z} + HC\dot{x} \\ &= \sum_{i=1}^{n_r} \alpha_i\Big(F_iz + TB_iu + G_iCx + HC(A_ix + B_iu + Dd)\Big) \\ &= \sum_{i=1}^{n_r} \alpha_i\Big(F_iz + (T + HC)B_iu \\ &\qquad + (G_iC + HCA_i)x + HCDd\Big). \end{aligned} \quad (5)$$

Defining the estimation error between x and $\hat{x}$ as:

$$e_x = \hat{x} - x \quad (6)$$

we have

$$\begin{aligned} \dot{e}_x &= \sum_{i=1}^{n_r} \alpha_i\Big(F_iz + (T + HC)B_iu + (G_iC + HCA_i)x \\ &\qquad + HCDd - A_ix - B_iu - Dd\Big) \\ &= \sum_{i=1}^{n_r} \alpha_i\Big(- (A_i - G_iC - HCA_i)x + F_iz \\ &\qquad + (T - (I - HC))B_iu + (HC - I)Dd\Big) \\ &= \sum_{i=1}^{n_r} \alpha_i\Big((A_i - G_iC - HCA_i)e_x \\ &\qquad - (A_i - G_iC - HCA_i)\hat{x} + F_iz \\ &\qquad + (T - (I - HC))B_iu + (HC - I)Dd\Big) \\ &= \sum_{i=1}^{n_r} \alpha_i\Big((A_i - G_{1i}C - HCA_i)e_x - G_{2i}Ce_x \\ &\qquad - (A_i - G_iC - HCA_i)(z + Hy) + F_iz \end{aligned}$$

$$
\begin{aligned}
&+ (T-(I-HC))B_iu+(HC-I)Dd\Big) \\
&= \sum_{i=1}^{n_r}\alpha_i\Big((A_i-G_{1i}C-HCA_i)e_x - G_{2i}C(z+Hy-x) \\
&\quad + (F_i-(A_i-G_{1i}C-HCA_i))z + G_{2i}Cz \\
&\quad - (A_i-G_{1i}C-HCA_i)Hy + G_{2i}CHy \\
&\quad + (T-(I-HC))B_iu+(HC-I)Dd\Big) \\
&= \sum_{i=1}^{n_r}\alpha_i\Big((A_i-G_{1i}C-HCA_i)e_x \\
&\quad + (F_i-(A_i-G_{1i}C-HCA_i))z \\
&\quad + (G_{2i}-(A_i-G_{1i}C-HCA_i)H)y \\
&\quad + (T-(I-HC))B_iu+(HC-I)Dd\Big).
\end{aligned} \tag{7}
$$

where

$$G_i = G_{1i} + G_{2i}. \tag{8}$$

Let

$$(HC-I)D = 0 \tag{9}$$
$$F_i = A_i - G_{1i}C - HCA_i \tag{10}$$
$$G_{2i} = F_iH \tag{11}$$
$$T = I - HC \tag{12}$$

then we have

$$\dot{e}_x = \sum_{i=1}^{n_r}\alpha_iF_ie_x. \tag{13}$$

Therefore, e_x converges to zero if F_i is a Hurwitz matrix as long as the relations (8)-(12) are held. Now, let us elaborate on it. From (9), we have $HCD = D$. When $\text{rank}(CD) = \text{rank}(D)$ holds true, CD is a full column rank matrix, a solution H of the equation is:

$$H = E\,(CD)^{\dagger}. \tag{14}$$

Once H is determined, G_{1i} in (10) can be obtained by

$$G_{1i} = Q^{-1}M_{1i} \tag{15}$$

where $Q = Q^T \in R^{n\times n}$ and $M_{1i} \in R^{n\times p}$ are decision parameters in the following LMIs that make F_i a Hurwitz matrix:

$$Q > 0 \tag{16}$$
$$
\begin{aligned}
&A_iQ + QHCA_i - M_{1i}C \\
&+ (A_iQ + QHCA_i - M_{1i}C)^T < 0 \\
&\qquad \text{for } i = 1,2,\cdots,n_r.
\end{aligned} \tag{17}
$$

The above LMIs follow from the Lyapunov's stability theorem by defining a Lyapunov function $V = e_xQe_x$. Finally, having H and F_i in mind, G_{2i}, and T can be straightforwardly obtained from (11), and (12), respectively. □

After establishing the state observer, we move on to consider how to observe the uncertainty d. As shown later, there is no necessity to pinpoint d itself, but the whole term of Dd from the viewpoint of controller design. Letting

$$D_d = Dd \tag{18}$$

an observer of D_d follows from (2):

$$\hat{D}_d = \sum_{i=1}^{n_r}\alpha_i\Big(A_i\hat{x} + B_iu\Big) - \dot{\hat{x}}. \tag{19}$$

Defining the estimation error between D_d and $\hat{D}_d$ as:

$$e_d = \hat{D}_d - D_d \tag{20}$$

it is clear that $e_d \to 0$ as long as $e_x \to 0$.

In the following, our interest is how to make most of $\hat{D}_d$ in controller design. Considering a special case of the existence of B_i^{-1}, it would be able to design a controller that is based on a regular control input u with an extra element such as $u - B_i^{-1}\hat{D}_d$ to counteract the influence of D_d completely. However, such a special case is highly unlikely to happen in the real system due to the fact that $m \le n$ which is the dimensions of the control matrix B_i usually. This implies that $\hat{D}_d$ cannot be used in the controller design at this stage. Nevertheless, we have to diminish the influence of M_d whatever possible. For this purpose, let us recall the following lemma.

Lemma 1. *(Baksalary and Kala, 1979) Let $B \in R^{m\times k}, M \in R^{l\times n}$, and $N \in R^{m\times n}$. The equation*

$$BX - MY = N \tag{21}$$

has a solution $X \in R^{k\times n}, Y \in R^{m\times l}$, if and only if

$$\big(I - BB^-\big)N\big(I - M^-M\big) = 0. \tag{22}$$

If this is the case, the general solution of (21) has the form

$$X = B^-N + B^-ZM + \big(I - B^-B\big)W \tag{23}$$
$$Y = -\big(I - B^-B\big)NM^- + Z - \big(I - B^-B\big)ZMM^- \tag{24}$$

where $W \in R^{k\times n}, Z \in R^{m\times l}$ being arbitrary. □

It is evident that the condition (22) is held if $M = -I$. In this case, (21) admits $X = B^-N$ and $Y = -(I = BB^-)NM^-$ as a solution. Therefore, according to Lemma 1, $\hat{D}_d$ can be divided into the following form:

$$\hat{D}_d = B_i\psi_i + \Delta E_{di} \tag{25}$$

where $\psi_i = B_i^\dagger \hat{D}_d$ and $\Delta E_{di} = \hat{D}_d - B_i\psi_i$ are a solution of the equation $B_iX + Y = \hat{D}_d$. By doing so, ψ_i can be used in the controller design partially to counteract the influence of D_d. However, as a solution of the equation, in fact there is no guarantee that the magnitude of ΔE_{di}, for example $||\Delta E_{di}||$, is less than that of $\hat{D}_d$. In other words, in the case of $||\Delta E_{di}|| > ||\hat{D}_d||$, on the contrary, ψ_i used in the controller design will worsen the influence of the uncertainty. Therefore, in order to make sure the effect of the counteraction of the uncertainty, ψ_i in (25) is obtained by

$$\psi_i = \begin{cases} B_i^\dagger \hat{D}_d & ||\hat{D}_d - B_i\psi_i|| \le ||\hat{D}_d|| \\ 0 & \text{otherwise.} \end{cases} \tag{26}$$

From (25) we observe that ψ_i shares the same control matrix B_i with the control input u; therefore, it is the maximum amount to counteract the uncertainty D_d from the estimated $\hat{D}_d$. It is clear that $\psi_i = 0$ means that there is no way to use the extracted information form $\hat{D}_d$ to diminish the influence of D_d at this stage. The smaller $||\Delta E_{di}||$ is, the better control performance can be expected.

In what follows, let us consider the boundedness with regard to the estimation errors and solutions of equation (25). First, define

$$\Xi_{ij} = (F_i - A_i)\, e_x - e_d + B_i(\psi_i - \psi_j) + \Delta E_{di}. \tag{27}$$

In view of the convergence of e_x and e_d along with the relation in (25), it is clear that Ξ_{ij} is bounded.

Let $q_{ij(k)}$ $(k = 1,2,\cdots,n)$ be the k-th entry of the vector $\Xi_{ij} \in R^n$, i.e.,

$$\Xi_{ij} = \left[q_{ij(1)}\ q_{ij(2)}\ \cdots\ q_{ij(n)}\right]^T, \tag{28}$$

and $b_{i(k)} \in R^{1\times m}$ be the k-th row of B_i, i.e.,

$$B_i = \left[b_{i(1)}^T\ b_{i(b)}^T\ \cdots\ b_{i(n)}^T\right]^T. \tag{29}$$

Bearing in mind that the boundedness of Ξ_i in (27), it is reasonable to assume that there exists a scalar $\kappa > 0$ such that

$$\max_{j=1\sim n_r} \left|q_{ij(k)}\right| \le \kappa \cdot |b_{i(k)}|. \tag{30}$$

In this case, it is easy to check that the following inequality holds:

$$\Gamma\Xi_{ij} \le \kappa|\Gamma B_i| \tag{31}$$

where $\Gamma \in R^{1\times n}$ is an arbitrary vector.

Before starting the controller design, we introduce the Nussbaum-type function (Ye and Jiang, 1998) that is adopted to design a part of controller in compensation for the influence of Ξ_{ij}.

Any continuous function $N(\zeta) : R \to R$ is a function of Nussbaum-type function if it has the following properties (Nussbaum, 1983):

$$\limsup_{z\to+\infty} \frac{1}{z}\int_0^z N(\zeta)d\zeta = +\infty \tag{32}$$

$$\liminf_{z\to+\infty} \frac{1}{z}\int_0^z N(\zeta)d\zeta = -\infty \tag{33}$$

For example, continuous functions $\zeta^2\cos(\zeta)$, $\zeta^2\sin(\zeta)$ and $\exp(\zeta^2)\cos(\frac{\pi}{2}\zeta)$ are commonly used as Nussbaum-type functions. Regarding Nussbaum-type functions, there is the following lemma.

Lemma 2. *(Ye and Jiang, 1998) Let $V(\cdot)$ and $\zeta(\cdot)$ be smooth functions defined on $[0,t_f)$ with $V(t) \ge 0$, $\forall t \in [0,t_f)$, $N(\cdot)$ be even smooth Nussbaum-type function. If the following inequality holds:*

$$V(t) \le c_0 + \int_0^t \Big(gN(\zeta(\tau)) + 1\Big)\dot{\zeta}(\tau)d\tau, \quad \forall t \in [0,t_f) \tag{34}$$

where g is a nonzero constant and c_0 represents some suitable constant, then $V(t)$, $\zeta(t)$ and $\int_0^t \left(gN(\zeta(\tau)) + 1\right)\dot{\zeta}(\tau)d\tau$ must be bounded on $[0,t_f)$.

Also, the following lemma, which is an alternative to the Barbalat Lemma (Khalil, 2002), will be used in the system stability analysis later.

Lemma 3. *(Tao, 1997) A function $f:\ R^n \to R$ is square integrable, i.e.,*

$$\int_0^\infty f^T(t)Pf(t)dt < \infty, \tag{35}$$

where $P = P^T > 0$, and has a bounded derivative, then

$$f(t) \to 0, \quad as \quad t \to \infty. \tag{36}$$

3 CONTROLLER DESIGN

Based on the state and uncertainty observers, the following controller is proposed.

Controller Rule i :

If θ_1 is M_1^i and $\cdots$ and θ_p is M_p^i, then

$$u = k_i\hat{x} - \psi_i + \phi_i \tag{37}$$

where $k_i \in R^{m\times n}$ is the feedback control gain that is obtained by $k_i = X_iQ^{-1}$ with $Q = Q^T > 0$ and $X_i \in R^{m\times n}$ being the decision variables of the following LMIs:

$$A_iQ + B_iX_j + QA_i^T + X_j^TB_i^T < 0 \tag{38}$$

for $i,j = 1,2,\cdots,n_r$;

ψ_i is extracted from the estimated uncertainty $\hat{D}_d$ in (26) in an effort to counteract the influence of the uncertainty Dd whatever possible; and $\phi_i \in R^m$ is designed in compensation for the influence of Ξ_{ij}:

$$\phi_i = B_i^T P\hat{x} + N(\zeta) \cdot \left(\sum_{j=1}^{n_r} \alpha_j B_j^T P\hat{x} + \kappa \cdot \mathrm{sgn}\left(\sum_{j=1}^{n_r} \alpha_j B_j^T P\hat{x} \right) \right) \quad (39)$$

where sgn denotes the sign function, κ is given in (30), $P = Q^{-1}$, $N(\zeta) \in R$ is a Nussbaum-type function (Nussbaum, 1983), and

$$\dot{\zeta} = \left\| \sum_{i=1}^{n_r} \alpha_i B_i^T P\hat{x} \right\|^2 + \kappa \left| \sum_{i=1}^{n_r} \alpha_i B_i^T P\hat{x} \right|. \quad (40)$$

The overall fuzzy controller (37) is of the following form accordingly.

$$u = \sum_{i=1}^{n_r} \alpha_i \Big(k_i\hat{x} - \psi_i + \phi_i \Big). \quad (41)$$

Substituting (6), (20), (25) and (41) into (2), the closed-loop control system becomes as follows.

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{n_r} \alpha_i \Big(A_i(\hat{x} - e_x) + B_i u + \hat{D}d - e_d \Big) \\ &= \sum_{i=1}^{n_r}\sum_{j=1}^{n_r} \alpha_i\alpha_j \Big((A_i + B_i k_j)\hat{x} - A_i e_x + B_i(\psi_i - \psi_j) \\ &\qquad + B_i\phi_j + \Delta E_{di} - e_d \Big). \end{aligned} \quad (42)$$

Substituting (13) and (43) into $\dot{\hat{x}} = \dot{x} + \dot{e}_x$ which follows from (6), we have

$$\begin{aligned} \dot{\hat{x}} &= \sum_{i=1}^{n_r}\sum_{j=1}^{n_r} \alpha_i\alpha_j \Big((A_i + B_i k_j)\hat{x} - A_i e_x + B_i(\psi_i - \psi_j) \\ &\qquad + B_i\phi_j + \Delta E_{di} - e_d + F_i e_x \Big) \\ &= \sum_{i=1}^{n_r}\sum_{j=1}^{n_r} \alpha_i\alpha_j \Big((A_i + B_i k_j)\hat{x} + B_i\phi_j + \Xi_{ij} \Big) \end{aligned} \quad (43)$$

where Ξ_{ij} is defined in (27).

Bearing in mind that the relation in (13) where F_i is made to be a Hurwitz matrix, an asymptotically stable system in (43) does lead to another asymptotically stable system in (42). The stability of the system in (43) is investigated based on the Lyapunov stability theory. Let us consider the following quadratic Lyapunov function candidate:

$$V = \hat{x}^T P\hat{x}. \quad (44)$$

Taking the time derivative of V, we have

$$\begin{aligned} \dot{V} =& \dot{\hat{x}}^T P\hat{x} + \hat{x}^T P\dot{\hat{x}} \\ =& \sum_{i=1}^{n_r}\sum_{j=1}^{n_r} \alpha_i\alpha_j \hat{x}^T \left((A_i + B_i k_j)^T P + P(A_i + B_i k_j) \right) \hat{x} \\ &+ 2\sum_{i=1}^{n_r}\sum_{j=1}^{n_r} \alpha_i\alpha_j \hat{x}^T P B_i B_j^T P\hat{x} \\ &+ 2\sum_{i=1}^{n_r} \alpha_i \hat{x}^T P B_i N(\zeta) \\ &\quad \cdot \left(\sum_{j=1}^{n_r} \alpha_j B_j^T P\hat{x} + \kappa \cdot \mathrm{sgn}\left(\sum_{j=1}^{n_r} \alpha_j B_j^T P\hat{x} \right) \right) \\ &+ 2\sum_{i=1}^{n_r}\sum_{j=1}^{n_r} \alpha_i\alpha_j \hat{x}^T P\Xi_{ij} \\ =& -\hat{x}^T H\hat{x} + 2\sum_{i=1}^{n_r} \alpha_i \hat{x}^T P B_i \\ &\quad \cdot \left(\sum_{j=1}^{n_r} \alpha_j B_j^T P\hat{x} + \kappa \cdot \mathrm{sgn}\left(\sum_{j=1}^{n_r} \alpha_j B_j^T P\hat{x} \right) \right) \\ &- 2\sum_{i=1}^{n_r} \alpha_i \hat{x}^T P B_i \cdot \kappa \cdot \mathrm{sgn}\left(\sum_{j=1}^{n_r} \alpha_j B_j^T P\hat{x} \right) \\ &+ 2N(\zeta)\dot{\zeta} + 2\sum_{i=1}^{n_r}\sum_{j=1}^{n_r} \alpha_i\alpha_j \hat{x}^T P\Xi_{ij} \\ =& -\hat{x}^T H\hat{x} + 2\left(1 + N(\zeta)\right)\dot{\zeta} \\ &- 2\kappa \left| \sum_{i=1}^{n_r} \alpha_i \hat{x}^T P B_i \right| + 2\sum_{i=1}^{n_r}\sum_{j=1}^{n_r} \alpha_i\alpha_j \hat{x}^T P\Xi_{ij} \end{aligned} \quad (45)$$

where

$$H = -\sum_{i=1}^{n_r}\sum_{j=1}^{n_r} \alpha_i\alpha_j \left((A_i + B_i k_j)^T P + P(A_i + B_i k_j) \right). \quad (46)$$

It is not difficult to see that, using (38), $H > 0$.

Taking the relations (30) and (31) into consideration, let us pay attention to the last block in (45).

$$\begin{aligned} 2\sum_{i=1}^{n_r}\sum_{j=1}^{n_r} \alpha_i\alpha_j \hat{x}^T P\Xi_{ij} \le& 2\sum_{i=1}^{n_r}\sum_{j=1}^{n_r} \alpha_i\alpha_j \kappa \left| \hat{x}^T P B_i \right| \\ =& 2\kappa \left| \sum_{i=1}^{n_r} \alpha_i \hat{x}^T P B_i \right| \end{aligned} \quad (47)$$

where the fact that $\alpha_i \ge 0$ is used.
Substituting (47) into (45), it follows

$$\dot{V} \le -\hat{x}^T H\hat{x} + \Big(1 + N(\zeta)\Big)\dot{\zeta} \quad (48)$$

which also implies

$$\dot{V} \le \Big(1 + N(\zeta)\Big)\dot{\zeta} \quad (49)$$

where the fact that $H > 0$ is used. Integrating (49), it follows to we get

$$V(t) \leq V(0) + \int_0^t \Big(1 + N(\zeta)\Big)\dot{\zeta}(\tau)d(\tau). \quad (50)$$

Applying Lemma 2 to (50), we conclude that $V(t)$, $\zeta(t)$, and $\int_0^t \Big(1 + N(\zeta)\Big)\dot{\zeta}(\tau)d(\tau)$ must be bounded. Again, integrating (48), we have

$$\begin{aligned} V(t) \leq & V(0) - \int_0^t x^T(\tau)Hx(\tau)d\tau \\ & + \int_0^t \Big(1 + N(\zeta)\Big)\dot{\zeta}(\tau)d(\tau). \end{aligned} \quad (51)$$

Because $V(t)$ (as well as $V(0)$) and $\int_0^t \Big(1 + N(\zeta)\Big)\dot{\zeta}(\tau)d(\tau)$ are all bounded, it is evident that $\int_0^t \hat{x}^T(\tau)H\hat{x}(\tau)d\tau$ must be bounded. Therefore, we conclude that $\hat{x}(t)$ is square integrable, and that $\lim_{t\to\infty} \hat{x}(t) \to 0$ according to Lemma 3. $\square$

It is worth noting that when ψ_i and ϕ_i in (37) are removed:

$$u = \sum_{i=1}^{n_r} \alpha_i k_i x \quad (52)$$

reduces to the regular parallel distributed compensation (PDC) controller (Tanaka and Wang, 2001), where LMIs in (38) are the stability conditions for which the uncertainty Dd in (2) is not considered.

4 SIMULATION

Consider the following altered Van der Pol oscillator:

$$\left.\begin{aligned} \ddot{z} &= -z + \left(1 - z^2\right)\dot{z} + u + f \\ y &= 2z + \dot{z} \end{aligned}\right\} \quad (53)$$

where f denotes uncertainty in the system. In this simulation, f is assumed to be:

$$f(t) = \begin{cases} 0, & \text{if } 0 \leq t < 2 \\ 5.3, & \text{if } 2 \leq t < 4 \\ 0, & \text{if } 4 \leq t < 6 \\ -5 + \sin(t-6) + 2\,\text{rand}(), & \text{if } 6 \leq t < 15 \\ 3, & \text{otherwise} \end{cases} \quad (54)$$

where rand() is a function that returns a single uniformly random number in the interval (0, 1). The f is depicted in Fig. 1. In the absence of u and f, the phase plane $z - \dot{z}$ is shown in Fig. 2, where the circle denotes the initial point, the square the end point. It is clear that the system is unstable without control.

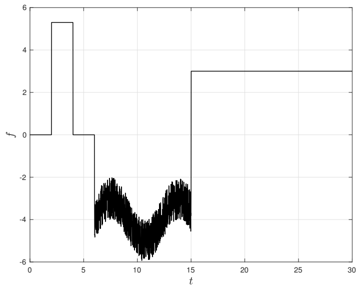

Figure 1: Uncertainty f considered in the system (53).

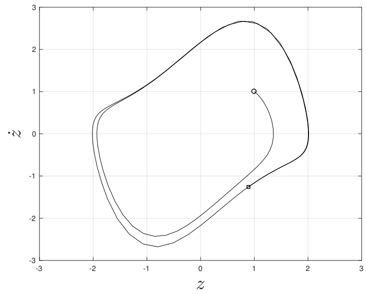

Figure 2: $z - \dot{z}$ phase plane with $u = 0$ and $f = 0$.

The above nonlinear system can be represented by the following two-rule T-S fuzzy model:

$$\text{Rule } i : \text{If } z \text{ is } M^i(z), \text{ then} \quad \left.\begin{aligned} \dot{x} &= A_i x + B_i u + Dd \\ y &= Cx \end{aligned}\right\} \quad (55)$$

where $x^T = [x_1\ x_2] = [z\ \dot{z}]$, $i = 1, 2$,

$$M^1(z) = \begin{cases} \frac{9-z^2}{9}, & -3 \leq z \leq 3 \\ 0, & \text{otherwise} \end{cases}, \quad M^2(z) = 1 - M^1(z)$$

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ -1 & -8 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$B_2 = B_1, \quad C = [2 \quad 1], \quad D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

and Dd denotes the uncertainty corresponding to f in (53). In this paper, the state x is supposed to be unavailable; therefore, instead of $z = x_1$ in the membership functions, $z = \hat{x}_1$ is used in this simulation.

By solving LMIs in (38) with the help of some software package, k_i and $P = Q^{-1}$ are set to be

$$\left.\begin{aligned} k_1 &= \begin{bmatrix} -0.4286 & -2.0714 \end{bmatrix} \\ k_2 &= \begin{bmatrix} -0.4286 & 6.9286 \end{bmatrix} \end{aligned}\right\} \quad (56)$$

$$P = \begin{bmatrix} 0.0145 & 0.0058 \\ 0.0058 & 0.0145 \end{bmatrix}. \quad (57)$$

A Nussbaum-type function, $N(\zeta) = \zeta^2\cos(\zeta)$ is used in this simulation. In addition, κ in (39) is set to be $\kappa = 0.5 + ||\hat{x}||^2$.

For comparison purposes, the plant (53) in the absence of the uncertainty f is first controlled by the regular PDC controller (52), the control result is shown in Fig. 3. It is clear that the regular PDC controller is very effective in this case. However, when the uncertainty in Fig. 1 is applied to the system, as the control result shown in Fig. 4, the regular controller is no longer able to control the system properly.

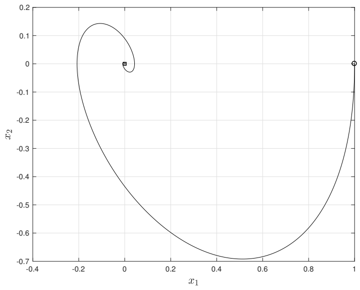

Figure 3: State driven by the regular PDC controller (52) without the uncertainty.

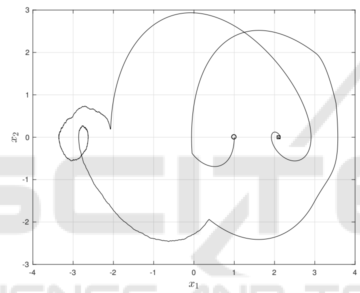

Figure 4: State driven by the regular PDC controller (52) with the uncertainty in (54).

The control results by the proposed controller are shown in Figs. 5∼9. Figs. 5, and 6 depict the responses of x_1 (dotted line), its estimate $\hat{x}_1$ (solid line), and x_2 (dotted line), its estimate $\hat{x}_2$ (solid line), respectively. We observe that the proposed control is able to make the states of x_1 and x_2 asymptotically stable, while the state observer (4) is effective to estimate the real state. Figs. 7, and 8 depict the real uncertainty $D_d(1) = 0$ (dotted line), its estimate $\widehat{D}_d(1)$ (solid line), and $D_d(2) = f$ (dotted line), its estimate $\widehat{D}_d(2)$ (solid line), respectively. It is evident from them that $\widehat{D}_d$ follows D_d properly. The control input is shown in Fig. 9. In comparison with Fig. 8, control input u going in the opposite direction to the uncertainty f accordingly is thought to be mainly contributed by ψ_i that is filtered from $\widehat{D}_d$ by (25) and (26). Finally, the behaviors of the other parameters, $N(\zeta)$, ζ, are depicted in Figs. 10, and 11, respectively. It can be seen that all the parameters involved in the system are bounded.

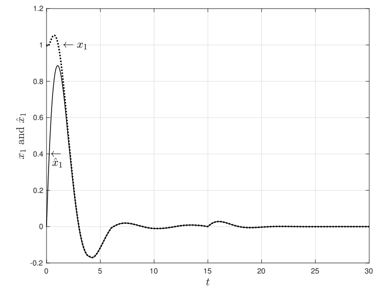

Figure 5: State x_1 and its estimate $\hat{x}_1$ driven by the proposed controller (41) along with the observer (4) where the uncertainty in (54) is applied.

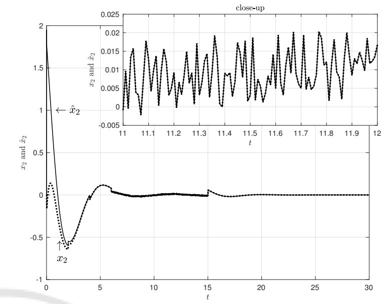

Figure 6: State x_2 and its estimate $\hat{x}_2$ driven by the proposed controller (41) along with the observer (4) where the uncertainty in (54).

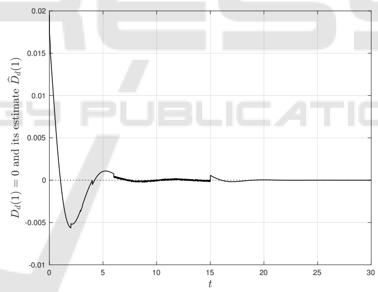

Figure 7: Uncertainty $D_d(1) = 0$ (dotted line) and its estimate $\widehat{D}_d(1)$ (solid line).

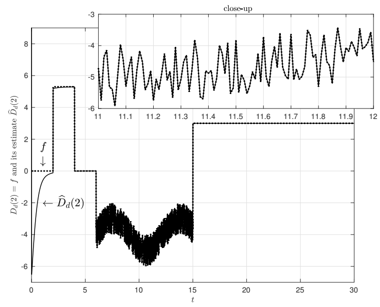

Figure 8: Uncertainty $D_d(2) = f$ (dotted line) as shown in Fig. 1 and its estimate $\widehat{D}_d(2)$ (solid line).

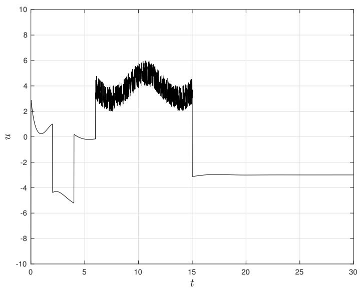

Figure 9: Control input obtained by (41).

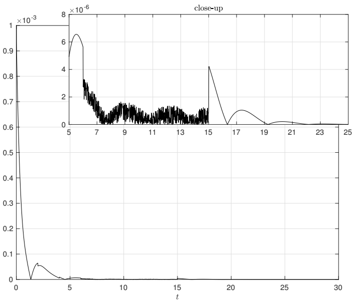

Figure 10: The behavior of ζ.

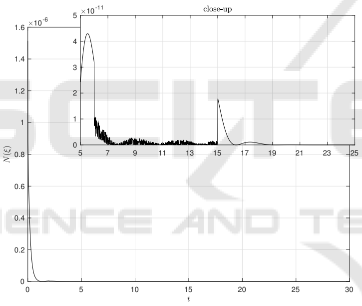

Figure 11: The behavior of $N(\zeta)$.

5 CONCLUSIONS

A design of control system based on a class of T-S fuzzy models with uncertainty was considered in this paper. For the case that the state is unavailable, an state observer was firstly designed, and then an uncertainty observer was derived using the state estimate. While it is almost impossible to use the whole estimated uncertainty directly in control design, the paper made an effort trying to use it as much as possible to counteract the influence of the uncertainty. On the basis of the observers, a controller was proposed, in which the Nussbaum-type function and its relevant properties were used to make the closed-loop system asymptotically stable.

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