

# LSHADE Algorithm with a Rank-based Selective Pressure Strategy for the Circular Antenna Array Design Problem

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**Abstract:** A new algorithm called LSHADE-RSP, which is based on a modification of the Differential Evolution technique, namely the LSHADE algorithm, with a rank-based selective pressure strategy, is presented in this paper. The basic idea of the proposed approach LSHADE-RSP consists in the adaptation of its mutation strategy, DE/current-to-pbest/1, using the linear rank-based selective pressure. LSHADE-RSP is built to tackle complex high-dimensional global optimization problems, and firstly it has been successfully tested on the CEC 2018 benchmark functions. Then the LSHADE-RSP was used for solving a real-life engineering global optimization problem, more specifically, the circular antenna array design problem. The objective of the stated problem is to vary the current and phase excitations of the antenna elements and try to suppress side-lobes, minimizing beam width, and to achieve null control at the desired directions. From the obtained results, the workability and usefulness of the new approach were confirmed. In addition, it can be concluded that the proposed optimization algorithm demonstrates competitive results in comparison with most alternative algorithms, thus, LSHADE-RSP can be recommended for solving optimization problems instead of them.

## 1 INTRODUCTION

Antenna arrays are intensively used in radar (Akçakaya and Nehorai, 2011), sonar (Bellettini and Pinto, 2002) and wireless communication systems (Zaker et al., 2007) among others. Therefore, the optimum design of array patterns is an important task in order to increase the channel capacities of these systems, broadening their coverage areas and ensuring an efficient spectrum utilization (Civicioglu, 2013).

The aim of the circular antenna array design problem is to obtain its optimum parameters, thus determining the positions of array elements (Das and Suganthan, 2010). Many researchers have conducted different studies on this subject, for example (Dessouky, 2006). In this study, a new modification of the well-known LSHADE algorithm (Tanabe and Fukunaga, 2014), which, in its turn, is a modification of the Differential Evolution technique (Storn and Price, 1997), is proposed for solving the stated problem.

Although generally the original LSHADE algorithm successfully solves various difficult optimization problems, there are still difficulties in keeping the balance between exploration and exploitation when solving complex multimodal problems. In order to achieve better performance, in this study the problems of premature convergence and search diversification were solved using a modification of the LSHADE technique's mutation operator. Namely, the rank-based selective pressure strategy (Jebari and Madiafi, 2013) was used for its mutation strategy.

The developed technique was called the "LSHADE Algorithm with Rank-Based Selective Pressure Strategy" or LSHADE-RSP. Firstly, the efficiency of LSHADE-RSP was examined on test problems taken from the CEC 2018 competition on real-parameter single objective optimization (Awad et al., 2016). Experimental results demonstrated that LSHADE-RSP performs better in comparison with the alternative algorithms. Thus, LSHADE-RSP was then used for solving the circular antenna array design problem. It was established that the proposed

optimization algorithm shows competitive results in comparison to different alternative algorithms.

In this paper, firstly a brief description of the DE algorithm and consequently its modification LSHADE is given. Then the proposed LSHADE-RSP technique and its parameters settings and adaptation are presented. In the next section, the experimental results obtained by the new developed LSHADE-RSP algorithm are demonstrated and discussed. Finally, some conclusions are given in the last section

## 2 DIFFERENTIAL EVOLUTION

Differential evolution (DE) is a global optimization evolutionary meta-heuristic first introduced in 1997 for solving continuous optimization problems (Storn and Price, 1997). It is one of the most effective methods for complex high-dimensional problems, and thus, it became one of the most popular and often prize-winning optimization techniques.

The DE algorithm is simple in its implementation due its compact structure. Furthermore, it has fewer control parameters in comparison to other evolutionary algorithms.

As with biology-inspired methods, the DE is a population-based algorithm, and the population contains a number of solutions. Thus, the DE starts with a population of  $N$  candidate solutions, which may be represented as  $x_{i,j}$ , where  $i = 1, \dots, N$  denotes individual's index in the population and  $j = 1, \dots, D$  denotes a variable's index (or coordinate). The DE's work process depends on the manipulation and efficiency of three main operators: mutation, crossover and selection.

One of the main features of a DE is the mutation scheme, which was shown to automatically adapt to the scale of the optimized function, improving the performance. Therefore, the key idea of differential evolution is in constructing a mutant vector using the difference between two other vectors from the current population.

The LSHADE algorithm (Tanabe and Fukunaga, 2014) is an extension of the SHADE algorithm (Tanabe and Fukunaga, 2013), based on one of the adaptive DE modifications JADE (Zhang and Sanderson, 2009). LSHADE was first presented at CEC 2014, and ranked as the winner-algorithm for bound-constrained continuous optimization.

The original LSHADE algorithm uses the *DE/current-to-pbest/1* mutation scheme, shown below:

$$v_j = x_{i,j} + F(x_{b,j} - x_{i,j}) + F(x_{r1,j} - x_{r2,j}) \quad (1)$$

Here  $x_{i,j}$  is the  $j$ -th coordinate of the  $i$ -th individual  $x_i$ ,  $r1$  and  $r2$  are mutually random numbers representing indexes of the individuals,  $v_j$  is the so-called mutant vector, which will be used in crossover operation, and  $x_b$  is randomly chosen as one of the top  $100p\%$  individuals of the current population with  $p$  from the range  $(0, 1]$ . The scaling factor  $F$  is the parameter, usually in range  $[0, 1]$ . The random index  $r2$  is uniformly selected from the joint set of the population and the external archive. The external archive keeps parent individuals which were replaced by new solutions.

The next step is the crossover, which is performed for each individual  $x_i$  as a calculation of the trial vector  $t$  with the crossover rate  $Cr$ . The  $j$ -th variable of the trial vector  $t$  is the same as the  $j$ -th variable of the mutant vector  $d$  if a randomly generated number in the range  $(0, 1)$  is smaller than the crossover rate  $Cr$  or if  $j$  is equal to  $jrand$ , where  $jrand$  is a randomly chosen index from 1 to  $D$ , otherwise it is the same as the corresponding variable of the individual  $x_i$ .

$$t_j = \begin{cases} d_j, rand(0,1) < Cr \vee j = jrand \\ x_{i,j}, rand \geq Cr \end{cases} \quad (2)$$

In the last formula,  $Cr$  is the control parameter of the algorithm in the range  $[0, 1]$ .  $Cr = 1$  means that there is no crossover, and the trial vector is equal to the mutant vector. The  $jrand$  index ensures that at least one variable is taken from the newly generated vector.

The selection step is performed after calculating the fitness value of the trial vector. If the trial vector is better than the  $i$ -th individual in the population, than it is replaced by the trial vector.

In addition, it should be noted that the LSHADE algorithm uses the Linear Population Size Reduction (LPSR) scheme (Tanabe and Fukunaga, 2014), which significantly boosts its performance. This scheme decreases the number of individuals in the population by deleting the least fit ones at every generation.

## 3 PROPOSED APPROACH

In this section, the proposed algorithm LSHADE-RSP is introduced. Firstly, a description of the LSHADE-RSP is given and then the parameter settings of the new algorithm are presented.

### 3.1 LSHADE-RSP

In this section, the modification of the LSHADE algorithm with a Rank-based Selective Pressure mutation (LSAHDE-RSP) is described. The rank-based mutation scheme, which was called *current-to-pbest/r*, modifies the *current-to-pbest/1* strategy so that the second part containing two random vectors receives selective pressure. More precisely,  $r_1$  and  $r_2$  are selected according to the rank selection typically used in genetic algorithms (Jebari and Madiafi, 2013), with the ranks assigned as follows:

$$Rank_i = k(N - i) + 1 \quad (3)$$

The largest rank is assigned to the individual with the largest fitness, and the smallest rank to the least fit, i.e. here  $i$  taken from the range  $[1, N]$  is the index in a sorted fitness array. We are considering a minimization problem, so larger fitness means a smaller goal function value. In (3)  $k$  is the scaling factor responsible for the greediness of the rank selection. Thus, the probability that the individual  $i$  will be selected is calculated as follows:

$$pr_i = \frac{Rank_i}{\sum_{j=1}^N Rank_j} \quad (4)$$

The new mutation strategy, *current-to-pbest/r*, tends to select individuals with larger fitness values more often, although even the worst individual still has a possibility of being selected for the mutation operation. The motivation behind this is that rank-based selection should boost the exploitation capabilities of the mutation strategy without significantly affecting the exploration. The resulting mutation strategy uses the modification proposed for the jSO algorithm (Brest et al., 2017), i.e. different scaling factors for the first ( $F_w$ ) and second part ( $F$ ) of the equation. Additionally, the scaling factor  $F_w$  depends on the scaling factor  $F$ . At the beginning of the search, while  $NFE < NFE_{max} * 0.2$ ,  $F_w$  is set to be equal to  $0.7F$ . Next, while  $NFE < NFE_{max} * 0.4$ ,  $F_w = 0.8F$ , and  $F_w = 1.2F$  for the rest of the search, where  $NFE$  is the current number of function evaluations and  $NFE_{max}$  is the total available number of goal function evaluations.

### 3.2 Parameter Settings

The two parameters adapted in LSHADE-RSP, are the scaling factor  $F$  and crossover rate  $Cr$ . The adaptation uses the same scheme as the original LSHADE algorithm, but the initial values and some constraints are taken from (Brest et al., 2017). The

scaling factor  $F$  for every mutation operation is computed using a Cauchy distribution with location parameter  $\mu F_r$  and scale parameter 0.1, while  $Cr$  is computed using a normal distribution with mean  $\mu Cr_r$  and variance 0.1.

Moreover,  $\mu F_r$  and  $\mu Cr_r$  are randomly chosen from the memory  $M$  of successful parameter settings (where the memory size is defined as  $H$ ), and  $r$  is a uniformly chosen random index. Initially all  $\mu F_r$  are set to 0.3, and  $\mu Cr_r$  are set to 0.8, and in addition to this, one memory cell always keeps  $\mu F_r$  and  $\mu Cr_r$ , which are equal to 0.9. The values in  $\mu F_r$  and  $\mu Cr_r$  in one memory cell are updated at the end of each generation using the Lehmer mean, which takes into consideration the fitness improvement.

While the memory is being updated, the new values are calculated as the mean of the old  $F$  or  $Cr$  value and the newly generated value.

For the first  $0.6NFE_{max}$  evaluations, (here  $NFE_{max}$  denotes the maximum number of function evaluations) the  $F$  value is constrained to be not larger than 0.7 and not larger than 1.0 during the remaining computation resource. The  $p$  value for the *current-to-pbest/r* strategy, responsible for the greediness of the search, is computed by the following formula:

$$p = 0.085 + 0.085 \frac{NFE}{NFE_{max}} \quad (5)$$

The idea behind increasing the number of best individuals is to prevent premature convergence by gradually decreasing the selective pressure as the algorithm runs.

## 4 EXPERIMENTAL RESULTS

The efficiency of the new LSHADE-RSP was investigated firstly on a set of benchmark problems taken from the CEC 2018 competition on real-parameter single objective optimization (Awad et al., 2016). Then, a real-world engineering problem, namely the circular antenna array design problem, was solved by the proposed technique. Subsequently, the experiments as well as the obtained results are described.

### 4.1 CEC 2018 Benchmark Problems

Firstly, the algorithm performance was evaluated on the CEC 2018 Competition on Single Objective Real Parameter Numerical Optimization (Awad et al., 2016). Therefore, the workability of the new

algorithm was tested on 30 benchmark functions, which were shifted and rotated. The functions in the competition were tested for the corresponding numbers of variables: 10 (10D), 30 (30D), 50 (50D) and 100 (100D). The computational resource for all algorithms and all dimensions was equal and was calculated as 10000D.

In the performed experiments, the parameter  $k$  for rank-based selection varied from 1 to 9, to be more specific  $k$  was equal to 1, 2, 3, 5, 7 and 9 in this study. However, only the results obtained with  $k = 3$  and 10 variables are presented here in Table 1.

Table 1: Algorithm results for 10D.

$N_f$	Worst	Best	Mean	Std. Dev
$f_1$	0.000e+00	0.000e+00	0.000e+00	0.000e+00
$f_2$	0.000e+00	0.000e+00	0.000e+00	0.000e+00
$f_3$	0.000e+00	0.000e+00	0.000e+00	0.000e+00
$f_4$	0.000e+00	0.000e+00	0.000e+00	0.000e+00
$f_5$	2.985e+00	0.000e+00	1.405e+00	7.155e-01
$f_6$	0.000e+00	0.000e+00	0.000e+00	0.000e+00
$f_7$	1.281e+01	1.067e+01	1.180e+01	5.087e-01
$f_8$	2.999e+00	0.000e+00	1.446e+00	6.647e-01
$f_9$	0.000e+00	0.000e+00	0.000e+00	0.000e+00
$f_{10}$	2.240e+02	2.339e-01	1.717e+01	4.244e+01
$f_{11}$	0.000e+00	0.000e+00	0.000e+00	0.000e+00
$f_{12}$	6.244e-01	0.000e+00	3.632e-01	2.054e-01
$f_{13}$	5.584e+00	0.000e+00	3.464e+00	2.303e+00
$f_{14}$	0.000e+00	0.000e+00	0.000e+00	0.000e+00
$f_{15}$	5.000e-01	8.096e-06	1.742e-01	2.067e-01
$f_{16}$	9.357e-01	4.094e-02	5.644e-01	2.172e-01
$f_{17}$	1.625e+00	7.174e-02	6.615e-01	4.071e-01
$f_{18}$	5.000e-01	3.935e-05	1.685e-01	2.003e-01
$f_{19}$	1.973e-02	0.000e+00	8.150e-03	9.504e-03
$f_{20}$	6.243e-01	3.122e-01	4.224e-01	1.492e-01
$f_{21}$	2.039e+02	1.000e+02	1.221e+02	4.210e+01
$f_{22}$	1.003e+02	1.000e+02	1.000e+02	6.716e-02
$f_{23}$	3.046e+02	3.000e+02	3.012e+02	1.533e+00
$f_{24}$	3.340e+02	1.000e+02	2.531e+02	1.082e+02
$f_{25}$	4.433e+02	3.977e+02	4.024e+02	1.350e+01
$f_{26}$	3.000e+02	3.000e+02	3.000e+02	0.000e+00
$f_{27}$	3.895e+02	3.890e+02	3.894e+02	1.762e-01
$f_{28}$	6.118e+02	3.000e+02	3.061e+02	4.323e+01
$f_{29}$	2.450e+02	2.267e+02	2.343e+02	3.406e+00
$f_{30}$	8.176e+05	3.945e+02	1.642e+04	1.133e+05

During the algorithm run, the error was calculated as the difference between the current best solution  $f(x)$  and the global optimum  $f(x^*)$ . If this difference was less than  $10^{-8}$ , then it was considered to be small enough and taken as zero. Table 1 contains the worst, best, mean and standard deviation values for every function calculated over 51 program runs for 30 variables.

The performance of the LSHADE-RSP algorithm was compared to the other methods participating in the CEC 2017 competition on single objective bound constrained optimization, including the original LSHADE algorithm. This was possible

due to the fact that the test functions were the same for the CEC 2017 and the CEC 2018 competitions. Therefore, all methods had the same amount of computational resources and runs. To compare different methods, the Wilcoxon's rank sum test with  $p = 0.05$  was used. For comparison, the jSO (Brest et al., 2017), the EBOwithCMAR (Kumar et al., 2017), and the LSHADE-SPACMA (Mohamed et al., 2017) algorithms were chosen.

Table 2: Comparison with other methods using statistical tests.

$D$	EBOwith CMAR	jSO	LSHADE-SPACMA	LSHADE-RSP ( $k = 0$ )
10	8+/12=/10- 2-	2+/26=/2- 0	12+/14=/4- 8+	2+/25=/3- 1-
30	10+/8=/12- 2-	7+/19=/4- 3+	12+/11=/7- 5+	8+/21=/1- 7+
50	13+/7=/10- 3+	13+/13=/4- 9+	13+/12=/5- 8+	10+/19=/1- 9+
100	13+/8=/9- 4+	16+/9=/5- 11+	8+/6=/16- 8-	15+/15=/0- 15

The numbers in the table represent the number of wins (+), losses (-) and equal results (=) when comparing LSHADE-RSP ( $k = 3$ ) with other methods. The obtained results demonstrate that the proposed approach outperformed most of the alternative optimization techniques, including jSO, and was outperformed only by the winners of the CEC 2017 competition for some dimensions. Furthermore, LSHADE-RSP is more successful in comparison to other optimization techniques when the number of variables increases. Thus, it can be concluded that the selective pressure gives an improvement and the workability and usefulness of the new LSHADE-RSP algorithm were established.

## 4.2 Circular Antenna Array Design Problem

As was mentioned before, the circular shaped antenna arrays find various applications in sonar, radar, mobile and other communication systems. Let us consider  $N$  antenna elements spaced on a circle of radius  $r$  in the  $x$ - $y$  plane (Das and Suganthan, 2010). The antenna elements are said to constitute a circular antenna array. The array factor for the circular array is written as follows:

$$AF(\phi) = \sum_{n=1}^N I_n \exp[kr(\cos \alpha_1 - \cos \alpha_2) + \beta_n] \quad (6)$$

where  $\alpha_1 = \phi - \phi_{ang}^n$ ,  $\alpha_2 = \phi_0 - \phi_{ang}^n$ ,  $kr = Nd$ . In this formula, the following denotations were used:

- $\phi_{ang}^n = 2\pi(n-1)$  is the angular position of the  $n$ -th element on the  $x$ - $y$  plane;
- $k$  is the wave number;
- $d$  is the angular spacing between elements;
- $r$  is the radius of the circle defined by the antenna array;
- $\phi_0$  is the direction of maximum radiation;
- $\phi$  is the angle of incidence of the plane wave;
- $I_n$  is the current excitation;
- $\beta_n$  is the phase excitation of the  $n$ -th element.

The current and phase excitations of the antenna elements should be varied in order to suppress side-lobes, minimize beam width and achieve null control at desired directions. In addition, a symmetrical excitation of the circular antenna array was considered due to (Das and Suganthan, 2010). Thus, the objective function is taken as follows:

$$OF = \frac{|AR(\phi_{sil}, \vec{I}, \vec{\beta}, \phi_0)|}{|AR(\phi_{max}, \vec{I}, \vec{\beta}, \phi_0)|} + \frac{1}{DIR(\phi_0, \vec{I}, \vec{\beta})} + \sum_{k=1}^{num} |AR(\phi_k, \vec{I}, \vec{\beta}, \phi_0)| \quad (7)$$

The first component attempts to suppress the side-lobes.  $\phi_{sil}$  is the angle at which a maximum side-lobe level is attained. The second component attempts to maximize the directivity of the array pattern. Nowadays, directivity has become a very useful figure of merit for comparing array patterns. The third component strives to drive the maxima of the array pattern close to the desired maxima  $\phi_{des}$ . The fourth component penalizes the objective function if sufficient null control is not achieved.  $num$  is the number of null control directions and  $\phi_k$  specifies the  $k$ -th null control direction.

The following parameters were also used for this study:

- the number of elements in circular array was equal to 12;
- the input string can be any string within the bounds;
- $null = [50, 120]$  in radians (no null control);
- $d$  is the angular spacing between elements;
- $\phi_{des} = 180^\circ$ ;
- the distance was equal to 0.5.

The first six optimized variables for this problem are in the range [0.2, 1], while the second six are in the range [-180, 180], and the problem has only bound constraints. For our experiments, we have set the maximum number of function evaluations equal to 150000, as stated in (Das and Suganthan, 2010), allowing the results to be compared to those achieved by other researchers.

To show the advantage of using selective pressure in the LSHADE-RSP algorithm, we have performed several tests, first for the algorithm without selective pressure ( $k=0$ ), and next for different coefficients  $k$ . Table 3 contains the best, average and standard deviation of the results. There were 25 runs performed for each algorithm configuration.

Table 3: Results of LSHADE-RSP for the Circular Antenna Array Design Problem.

RSP	Mean	Best	Std. Dev	Reliability
$k = 0$	-21.6376	-21.6445	<b>0.03214</b>	0.00
$k = 1$	-21.6675	-21.8425	0.08624	0.16
$k = 2$	-21.6519	-21.8424	0.03889	0.04
$k = 3$	-21.6600	-21.8425	0.05380	0.08
$k = 5$	-21.6773	-21.8425	0.08855	0.20
$k = 7$	-21.6599	-21.8425	0.05383	0.08
$k = 9$	<b>-21.6996</b>	<b>-21.8426</b>	0.08908	<b>0.28</b>

The reliability of the algorithm is according to the value of the best known solution, found by LSHADE-RSP, so if at the end of the search the found value was lower than -21.8, the run was considered as successful, and the ratio of the number of successful runs to the total number of runs was a reliability estimation. The algorithm with the highest selective pressure coefficient,  $k = 9$ , achieved the best results in terms of the mean value and best found value. However, the mutation strategy without selective pressure had a lower standard deviation. It should be mentioned that applying even small selective pressure with  $k = 1$  allows the algorithm to find very good solutions, whose goal function is close to -21.8425, while the algorithm without selective pressure could not achieve this goal function value.

In Figure 1, the graphs of the average goal function values achieved at different stages of the search process, namely, 0.01, 0.1, 0.2, ..., 1.0 are presented. The goal function values are shifted up by 21.9, and the graphs are built in logarithmic scale so that the difference can be seen more easily.

From the graphs, it can be seen that most of the variants of the algorithm achieve the best solution at around 0.5 of the available computational resource,

i.e. 75000 function calculations. However, algorithms with higher selective pressure tend to converge faster at the beginning of the search. Moreover, as the selective pressure grows, LSHADE-RSP is capable of finding better solutions, i.e. it increases its search capabilities.

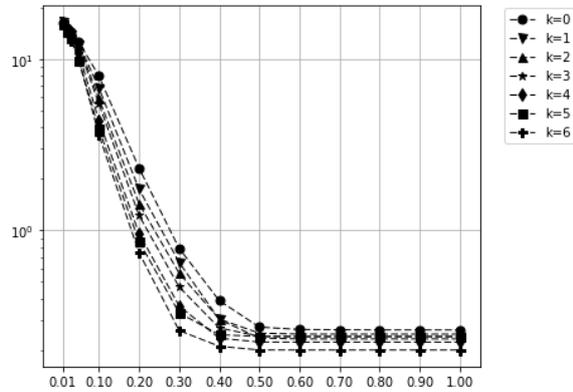


Figure 1: Comparison of the effect of different selective pressures on the convergence speed.

The comparison to other algorithms used to solve the same problem is presented in Table 4. The amount of computational resource was the same for all methods.

Table 4: Comparison of LSHADE-RSP with other methods for the Circular Antenna Array Design Problem.

Algorithm	Mean	Best
LSHADE-RSP	-21.6996	-21.8426
OXCoDE (Li and Yin, 2011)	-21.591	<b>-21.865</b>
WI-DE (Haider et al, 2011)	-21,70	-21.80
GA-MPC (Elsayed et al., 2011a)	<b>-21.702</b>	-21.8425
ED-DE (Wang et al., 2011)	-21.421	-21.832
Adap.DE171 (Asafuddoula et al., 2011)	-20.958	-21.808
EA-DE-MA (Singh et al., 2011)	-21.2554	-21.7956
SAMODE (Elsayed et al., 2011b)	-21.6589	-21.8216

The comparison shows that although LSHADE-RSP is outperformed in terms of the best value by one method, and in terms of the mean value by another, the difference in the performance is insignificant. Considering both mean and best values, the closest method is GA-MPC, which has almost the same best value found, and similar average performance.

## 5 CONCLUSIONS

In this paper, the LSHADE-RSP algorithm was presented, which is a modification of Linear population size reduction Success History based Adaptive Differential Evolution with Rank-based Selective Pressure. This algorithm implements a number of various parameter adaptations, but most importantly, it uses the modified mutation strategy, *current-to-pbest-w/r*, which allows the convergence speed of the algorithm to be improved.

The problem of designing the Circular Antenna Array was solved by LSHADE-RSP with different selective pressure parameters, and it was observed that higher selective pressure results in better results in terms of both mean and best values. The achieved results are comparable to the best known up-to-date results for this problem.

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