

# Adaptive Channel Allocation Algorithm Suitable for WDM Networks: An Analytical Study

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**Keywords:** Performance Evaluation, Wavelength Assignment, Wavelength Division Multiplexing.

**Abstract:** In this paper, we adopt a network configuration and an efficient synchronous transmission access protocol suitable for WDM networks of passive star topology. Especially, a single control channel is assumed for the control information exchange, prior to the data packets transmission, in order to properly coordinate the data packets communication. According to the proposed WDMA scheme, a data channel is assigned at each station that attempts data packet transmission, totally avoiding the collisions over the data multi-channel system. The system performance measures are analytically derived based on a Markovian model. Numerical results are studied for diverse numbers of data channels.

## 1 INTRODUCTION

Modern trends in Wavelength Division Multiplexing (WDM) (Zheng and Mouftah, 2004) networks are dealing with the idea of exploring diverse network resources allocation methods in order to improve the performance. Many different topologies and network configurations have been proposed, not only in literature but also in implemented projects.

Especially, the use of a single control channel for the control information exchange, prior to the data packets transmission, has been extensively studied by Pountourakis (1998). The provided benefits, as compared with the case where there are no pre-transmission coordination access schemes, are based on the fact that the packets competition is restricted during the control transmission phase, while the packet loss can be totally avoided during the data transmission phase. In this way, the network is capable of reaching high performance. Also, Pountourakis et al., (2006) and Baziana (2014 and 2016) adopt the Multi-Channel Control Architecture (MCA) where there are multiple parallel control channels for the control information exchange, in order to reduce the control processing time at the station electronics (Humblet et al., 1993).

It is obvious that the WDM networks performance is restricted by the packets loss due to the concurrent transmission of more than one packets over the same channel. This phenomenon is

referred as WDM channels collisions. It causes system throughput reduction and delay increase.

In this paper, we explore the idea of adopting a wavelength assignment algorithm in order to improve the system performance. We assume a WDM passive star Local Area Network (LAN) that uses a single control channel in order to coordinate the data packets transmission and to reduce the data packets collisions over the data channels. A pre-transmission coordination scheme is considered in order for the stations to gain collisions-free access over the data channel multi-channel system. For this reason, we assume the division of the set of the data channels into two equal sets. At each station, a wavelength assignment algorithm over the two data channels sets is implied in a decentralized way. A synchronous transmission WDM Access (WDMA) protocol is proposed that takes under consideration the control channels collisions and the data packets loss due to the wavelength assignment competition. The proposed protocol provides significant performance improvement, as compared to the network case that uses a single data channel set.

The performance measures evaluation is based on the study of the closed mathematical formulas provided by a Markovian model. The proposed protocol performance is extensively studied for several, numbers of data channels.

The paper is organized as follows. Section 2 presents the network model and the assumptions. The analysis is provided in Section 3. Numerical

results are studied in Section 4. The conclusion is outlined in Section 5. The Appendix gives the proof of the closed formula for the probabilistic evaluation of the wavelength assignment access scheme over the data channels system.

## 2 NETWORK MODEL AND ASSUMPTIONS

We assume a LAN that uses a passive star coupler to interconnect a finite number  $M$  of stations, as Fig. 1 shows. The bandwidth is divided into  $(N+1)$  WDM channels, each using in a different wavelength  $\{\lambda_0, \lambda_1, \dots, \lambda_N\}$ , where  $N$  is an even integer. The channel  $\lambda_0$  is called control channel, while the channels  $\{\lambda_1, \lambda_2, \dots, \lambda_N\}$  are called data channels. The set of data channels is divided into two sets with equal number of wavelengths. Thus, the first data channel set  $A_1$  consists of the wavelengths  $\{\lambda_1, \lambda_2, \dots, \lambda_{N/2}\}$ , while the second data channel set  $A_2$  consists of the wavelengths  $\{\lambda_{1+N/2}, \lambda_{2+N/2}, \dots, \lambda_N\}$ . In this way, the data channel  $\lambda_x$ , where:  $x \in \{1, 2, \dots, N/2\}$  from the set  $A_1$  has an one to one correspondence to the data channel  $\lambda_y$ , where:  $y \in \{N/2+1, N/2+2, \dots, N\}$  from the set  $A_2$ . Each station is equipped with a tunable transmitter and a tunable receiver that can be tuned to any channel. For the tunable transceivers, we assume large tuning range.

The control packet transmission time is defined as time unit and is called mini-slot. The data packet transmission time is  $L$  time units and is called data slot. We denote as  $t_t$  and  $t_r$  the tunable transmitter and receiver tuning time respectively. We define the time interval  $T$  in time units as:  $T = \max\{t_t, t_r\}$ . The control packet consists of the source and the destination address and the data channel  $\lambda_k$ , where:  $k \in \{1, 2, \dots, N/2\}$  that has been chosen from the set  $A_1$  for the data transmission. The normalized round trip propagation delay between any pair of stations is assumed to be  $R$  time units.

All channels use the same time reference which is called cycle. The cycle is defined as the time interval that includes:  $T$  time units for the transceivers tuning, plus  $v$  time units for the control packets transmissions, plus the normalized propagation delay  $R$ , plus  $T$  time units for the transceivers tuning, plus the normalized data packet transmission time  $L$ , as Fig. 2 shows. The cycle time duration  $C$  is:

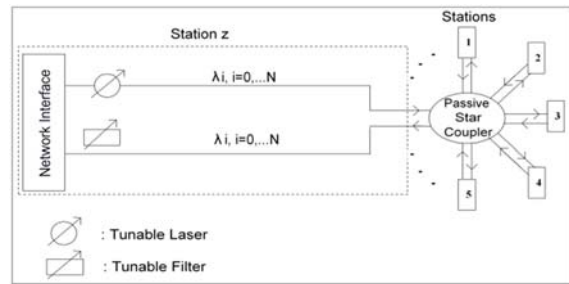


Figure 1: Network model.

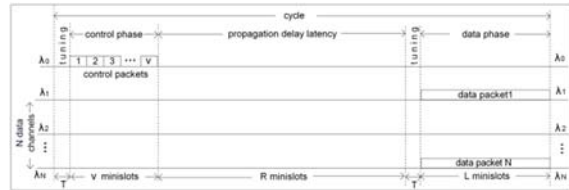


Figure 2: Cycle duration.

$$C = T + v + R + T + L \text{ time units} \quad (1)$$

Time axis is divided into contiguous cycles. Each cycle consists of the tuning phase for the control packets communication, the control phase, the propagation delay, the tuning phase for the data communication, and the data phase, as Fig. 2 shows. The control phase consists of  $v$  time units, while the data phase lasts for  $L$  time units. At the beginning of the data phase, each station is able to transmit with its tunable transmitter at a data channel  $\lambda_T$ , while simultaneously receive from a data channel  $\lambda_R$ , where  $\lambda_T, \lambda_R \in \{\lambda_1, \lambda_2, \dots, \lambda_N\}$ .

At the beginning of a cycle, each station tunes its tunable receiver to the control channel  $\lambda_0$  to monitor the control packets transmissions from all stations during the control phase. Also, if it has to send a data packet to another station, it tunes its tunable transmitter to the control channel  $\lambda_0$ . The tunable transceivers tuning is performed during the first  $T$  mini-slots of the cycle. After the end of this time period, the station chooses randomly one of the data channels from the set  $A_1$  for the data packet transmission, let's say data channel  $\lambda_i \in \{\lambda_1, \dots, \lambda_{N/2}\}$ . Also, it chooses randomly one of the control mini-slots for the control packet transmission, let's say the control mini-slot  $j \in \{1, 2, \dots, v\}$ . Then, it informs the other stations about the  $\lambda_i$  selection, by transmitting a control packet during the  $j$ -th control mini-slot with its tunable transmitter. The control packets from all stations compete according to the Slotted Aloha scheme. The station continuously monitors the control channel with its tunable receiver during the control phase and the propagation delay time

period. After the end of this period, the station is aware of the data channel claims for transmission of all stations, grace to the broadcast nature of the control channel. We can say that the successfully transmitted control packets are uniformly distributed to the  $N/2$  data channels with equal and constant probability  $2/N$ . So, if one or more other stations have selected the same  $j$ -th control mini-slot for transmission, the corresponding control packets have collided during the  $j$ -th control mini-slot and are all aborted. On the contrary if the control packet has been successfully transmitted over the  $j$ -th control mini-slot, the station has to check the data channel field of the other successfully transmitted control packets. Thus, if exactly one more station has chosen the same data channel  $\lambda_i$  for transmission and its control packet transmission was successful, then the  $i$ -th data channel from the set  $A_1$  is assigned to the first station for transmission, while the  $i$ -th data channel from the set  $A_2$  is assigned to the second station for transmission, i.e. the  $\lambda_i$  and  $\lambda_{i+N/2}$  data channels respectively. Also, if more than one stations have chosen the same data channel  $\lambda_i$  for transmission and their control packets transmissions were successful, then an arbitration rule for the data channels assignment may be considered, such as priority. In this case, only two of these stations gain access to the data channels  $\lambda_i$  and  $\lambda_{i+N/2}$  for transmission during the cycle data phase, while the other data packets transmissions are cancelled. The stations who gain the access over the data channels start tuning their tunable transmitter to the assigned channels for the transmission. The tuning period lasts for  $T$  time units. The data packet transmission will start  $R+T$  time units after the end of the control phase, as Fig. 2 shows. At the same time instant, the data packets reception will also start by the destination stations.

Packets are generated independently at each station following a geometric distribution, i.e. a packet is generated at each cycle with birth probability  $p$ . A backlogged station retransmits the unsuccessfully transmitted packet following a geometric distribution with probability  $p_1$ . We assume that each station is equipped with a transmitter buffer with capacity of one data packet. If the buffer is empty the station is said to be free, otherwise it is backlogged. If a station is backlogged and generates a new packet, the packet is lost. Free stations that unsuccessfully transmit on the control channel or in case of loss at the channel assignment competition during a cycle, are getting backlogged in the next cycle. A backlogged station is getting free at the next cycle, if it manages to retransmit

without collision over a control channel and its data packet retransmission is not aborted due to the channel assignment competition.

### 3 ANALYSIS

The examined system performance can be described by a discrete time Markov chain. We denote the state of the system by  $X_t$ ,  $t=1,2,\dots$  where  $X_t=0,1,\dots,M$  is the number of backlogged stations at the beginning of a cycle. Let:

$H_c$  = The number of new control packets arrivals at the beginning of a cycle,  $c=0,1,2,\dots$

$A_c$  = The number of successfully transmitted data packets over the  $N$  data channels at the end of a cycle,  $c=0,1,2,\dots$

$S(x)$  = The number of successfully transmitted control packets during the  $v$  control mini-slots, conditional that  $x$  (re)transmissions occurred during a cycle,  $c=0,1,2,\dots$

The probability of  $y$  successes over the  $v$  control mini-slots from  $x$  (re)transmissions during a cycle is given by Szpankowski (1983):

$$\Pr[S(x) = y] = \frac{(-1)^y v! x!}{v^x y!} \sum_{j=y}^{\min(v,x)} \frac{(-1)^j (v-j)^{x-j}}{(j-y)!(v-j)!(x-j)!} \quad (2)$$

and  $0 \leq y \leq \min(v,x)$  and  $x-y \neq 1$

Also, let:

$A(y)$  = The number of successfully transmitted data packets over the  $N$  data channels, conditional that  $y$  successful (re)transmissions occurred during the  $v$  control mini-slots, during a cycle.

The probability  $\Pr[A(y) = z]$  of  $z$  successfully transmitted data packets over the  $N$  data channels, conditional that  $y$  successful (re)transmissions occurred during the  $v$  control mini-slots, during a cycle, is given by:

$$\Pr[A(y) = z] = \begin{cases} 0, & \text{if } (y < z) \text{ OR } (y > z \text{ AND } z < 2) \\ 1, & \text{if } (y = z \text{ AND } y \leq 2) \\ \text{prob}(y, z), & \text{if } (y \geq z \text{ AND } y > 2) \end{cases} \quad (3)$$

where:

$$\text{prob}(y, z) = \left(\frac{2}{N}\right)^y \times \sum_{i=u}^{\lfloor \frac{z}{2} \rfloor} \binom{\frac{N}{2}}{z-i} \binom{y}{z-2i} (z-2i)! \binom{z-i}{i} i! S_2(y-z+2i, i) \quad (4)$$

and:

$$u = \begin{cases} 1, & \text{if } y > z \\ 0, & \text{if } y = z \end{cases} \quad (5)$$

The proof of (4) is given in the Appendix.

We define the function  $\Phi_v(x,y,z)$  as the product of the probability of  $y$  successes from  $x$  (re)transmissions during the  $v$  control mini-slots, times the probability of  $z$  successfully transmitted data packets over the  $N$  data channels during a cycle, i.e.:

$$\Phi_v(x, y, z) = \Pr[S(x) = y] \Pr[A(y) = z] \quad (6)$$

The Markov chain  $X_t \ t=1,2..$  is homogeneous, aperiodic and irreducible. The one step transition probabilities,  $0 \leq i, j \leq M$ , are defined as:

$$P_{ij} = \Pr(X_{t+1} = j | X_t = i) \quad (7)$$

and they are given by Pountourakis and Baziana (2005).

The steady state probabilities  $\pi_i, 0 \leq i \leq M$ , are given by Pountourakis and Baziana (2005).

*Performance Measures.* The conditional throughput  $Thr(i)$  is defined as the expected value of the successful data packet transmissions over the  $N$  data channels during a cycle, given that the number of the backlogged stations at the beginning of the cycle is  $i$ , i.e.:

$$Thr(i) = \begin{cases} \sum_{k=1}^v \sum_{m=0}^{\min(\frac{N-i}{2}, v)} Q_{m,i} \sum_{n=0}^{\min(v-m,i)} q_{n,i} \sum_{s=0}^{v-k} \Phi_v(n+m, k+s, k) + \\ + \sum_{k=1}^{v-1} \sum_{m=0}^{\min(\frac{N-i}{2}, v)} Q_{m,i} \sum_{n=v+1}^i q_{n,i} \sum_{s=0}^{v-k-1} \Phi_v(n+m, k+s, k) + \\ + \sum_{k=1}^{v-1} \sum_{m=N+1}^{\frac{N-i}{2}} Q_{m,i} \sum_{n=0}^i q_{n,i} \sum_{s=0}^{v-k-1} \Phi_v(n+m, k+s, k) \end{cases} \quad (8)$$

where:  $q_{i,n}$  gives the conditional probability that  $i$  out of  $n$  backlogged stations attempt to retransmit with probability  $p_i$ , while  $Q_{i,n}$  gives the conditional probability that  $i$  out of  $(M-n)$  free stations attempt to transmit with probability  $p$  during the cycle.  $Q_{i,n}$  and  $q_{i,n}$  are given by Pountourakis and Baziana (2005).

The steady state average throughput  $Thr$  is given by:

$$Thr = \frac{L}{C} E[Thr(i)] = \frac{L}{C} \sum_{i=0}^M Thr(i) \pi_i \quad (9)$$

The steady state average number  $B$  of backlogged stations is given by:

$$B = E[i] = \sum_{i=0}^M i \pi_i \quad (10)$$

The conditional input rate  $Thr_{in}(i)$  is the expected number of new packet arrivals during a cycle given that the backlogged stations at the beginning of the cycle are  $i$ :

$$Thr_{in}(i) = E[H_t | X_t = i] = (M - i)p \quad (11)$$

The steady state average input rate  $Thr_{in}$  is given by:

$$Thr_{in} = \sum_{i=0}^M (M - i)p \pi_i \quad (12)$$

The average input rate  $Thr_{in}$  should equal to the average throughput  $Thr$ , i.e. it is:

$$Thr = (M - B)p \quad (13)$$

The average delay  $D$  is defined as the average number of cycles that a packet has to wait until its successful transmission. According to the Little's formula, it is:

$$D = (T + v + R + T + L) \left( 1 + \frac{B}{Thr} \right) \quad (14)$$

## 4 PERFORMANCE EVALUATION

For the numerical solutions, we consider that:  $L=10$  time units,  $R=50$  time units,  $T=2$  time units and  $p_1=0.3$ . Also, for computational reasons, we assume that  $N=2 \times M$ .

Fig. 3 shows the average throughput  $Thr$  versus the birth probability  $p$ , for  $v=50$  control mini-slots,  $N=100, 140, 200$  data channels. It is remarkable that the  $Thr$  is an increasing function of  $N$ . For example for  $p=0.9$ , the  $Thr$  is: 1.59 data packets/cycle for  $N=200$ , 1.5 data packets/cycle for  $N=140$ , and 1.35 data packets/cycle for  $N=100$ . This is because as the number  $N$  increases, the sets  $A_1$  and  $A_2$  of data channels over which the number of stations with successfully transmitted control packets distribute their data packets, are getting larger. As a sequence, the probability that more than two stations whose control packets have been successfully transmitted to have selected the same data channel for

transmission is getting lower. Thus, as the number  $N$  increases, the number of correctly transmitted data packets over the data channels system increases too, giving rise to the Thr values.

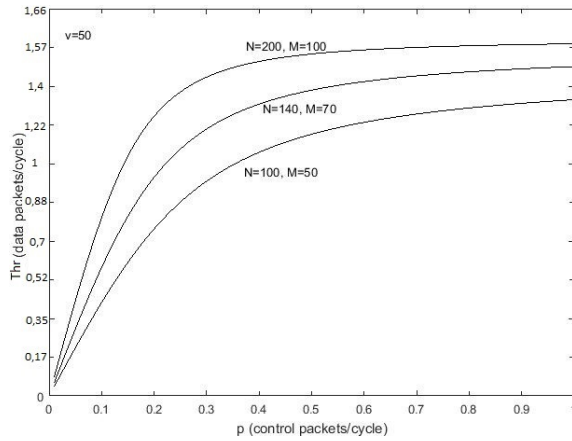


Figure 3: Throughput Thr versus birth probability  $p$ , for  $v=50$ ,  $N=100, 140, 200$ .

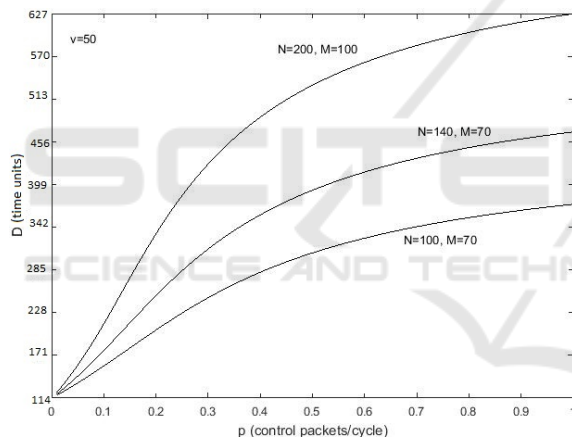


Figure 4: Delay  $D$  versus birth probability  $p$ , for  $v=50$ ,  $N=100, 140, 200$ .

Fig. 4 shows the average delay  $D$  versus the birth probability  $p$ , for  $v=50$  control mini-slots,  $N=100, 140, 200$  data channels. As it is shown, the proposed protocol behaviour when the number  $N$  increases conforms to the above discussion. In other words, as the number  $N$  increases and consequently the number  $M$  increases, the total delay  $D$  is getting higher. This is because as the number of stations increases, the offered load to the system increases too. In this case, the probability of control packets collisions over the  $v$  control mini-slots rises. This fact gives rise to the probability of a data packet rejection when requesting access over the data channels due to the applied wavelength assignment algorithm. This is because, as the number  $M$

increases the probability that more than two stations whose control packets have been successfully transmitted to have selected the same data channel for transmission is getting higher. This is the reason why, the total delay  $D$  reaches higher values.

## 5 CONCLUSIONS

In this paper, we explore an effective wavelength assignment algorithm suitable for passive star WDM LANs that use as single control channel. Our objective is to improve the system performance by dividing the set of multiple data channels into two groups and by applying an efficient access scheme that avoids the collisions over the WDM data channels. An analytical model for the probabilistic evaluation of the wavelength assignment is adopted, while the performance measures of average throughput and delay are derived by a Markovian model study.

## REFERENCES

- Jun Zheng, and H. T. Mouftah, *Optical WDM Networks: Concepts and Design Principles*. J. Willey & Sons Inc. Publication - IEEE Press, 2004, pp. 3-6.
- I.E.Pountourakis, "Performance Evaluation with Receiver Collisions Analysis in Very High-Speed Optical Fiber Local Area Networks Using Passive Star Topology", *IEEE Journal of Lightwave Technology*, Vol. 16, No. 12, pp. 2303-2310, Dec. 1998.
- I.E.Pountourakis, P.A.Baziana, G.Panagiotopoulos, "Propagation Delay and Receiver Collision Analysis in WDMA Protocols", in *Proc. 5th International Symposium on Communication Systems Networks and Digital Signal Processing (CSNDSP 2006)*, Patra, Greece, 2006, pp. 120-124.
- P.A.Baziana: "An Approximate Protocol Analysis with Performance Optimization for WDM Networks", *Optical Fiber Technology*, Vol. 20, Issue 4, pp. 414-421, 2014.
- P.A.Baziana: "Performance Analysis and Transmission Strategies Comparison for Synchronous WDM Passive Star LANs", *Springer Photonic Network Communications Journal*, Vol. 31, Issue 3, pp 457-465, June 2016.
- P.A. Humblet, R. Ramaswami, K.N. Sivarajan, An Efficient Communication Protocol for High-Speed Packet Switched Multichannel Networks, *IEEE Journal on Selected Areas Communications*, Vol. 11, pp. 568-578, 1993.
- W. Szpankowski, "Packet switching in multiple radio channels: analysis and stability of a random access system," *Comput. Netw.*, vol. 7, no. 1, pp. 17-26, 1983.

- I.E.Pountourakis, P.A.Baziana: “Multi-channel Multi-access Protocols with Receiver Collision Markovian Analysis”, *WSEAS Transactions on Communications*, Is. 8, Vol. 4, pp. 564-569, Aug. 2005.
- P. Baziana, G. Fragkouli, and E. Sykas: “Analytical Receiver Collisions Performance Modeling of a Multi-channel Network”, in *Proc. of the 2017 IEEE Conference of Russian Young Researchers in Electrical and Electronic Engineering (ElConRus 2017)*, Paper No. 008, Feb. 01-03, 2017, St. Petersburg, Russia.
- L. Comtet, *Advanced Combinatorics: The Art of Finite and Infinite Expansions*, D. Reidel Publishing Company, 1974, pp. 221-222.

## APPENDIX

We explore the probability  $\Pr[A(y)=z]$  of  $z$  successfully transmitted data packets over the  $N$  data channels, conditional that  $y$  successful (re)transmissions occurred during the  $v$  control mini-slots, during a cycle, in case that ( $y \geq z$  AND  $y > 2$ ). This problem is a special case of the problem studied Baziana et al., (2017). Let:

$E_d$  = The number of data channels from the set  $A_1$  where each of them has been selected by exactly  $d$  stations whose control packets transmission was successful, by the end of a cycle, where:  $d \in [0, \min(v, M)]$ ,  $E_d \in [0, N/2]$ .

$M_d$  = The number of data channels from the set  $A_1$  where each of them has been selected by more than  $d$  stations whose control packets transmission was successful, by the end of a cycle, where:  $d \in [0, \min(v, M)]$ ,  $E_d \in [0, N/2]$ .

Let be:  $M_2 = i$ .

$J$  = The number of ways to choose the data channels from the set  $A_1$  in order at least one station whose control packet transmission was successful to have selected each of them. It is:

$$J = \binom{N}{z-i} \quad (15)$$

$K$  = The number of ways in which the stations whose control packet transmission was successful can be chosen in order each of them to have selected a data channel from the set  $A_1$  with no other station to have selected it. It is

$$K = \binom{y}{z-2i} \quad (16)$$

$L$  = The number of possible replacements of stations whose control packet transmission was successful and that have selected a data channel from the set  $A_1$  with no other station to have selected it. It is:

$$L = (z-2i)! \quad (17)$$

$P$  = The number of ways in which the data channels from the set  $A_1$  can be chosen in order at least two stations whose control packet transmission was successful to have selected each of them. It is:

$$P = \binom{z-i}{i} \quad (18)$$

$Q$  = The number of ways in which the data packets from the stations whose control packet transmission was successful can be distributed to the data channels from the set  $A_1$  in order at least two stations to have selected each of them. It is:

$$Q = i! S_2(y-z+2i, i) \quad (19)$$

where the function  $S_2(n, k)$  denotes the 2-associated Stirling number of the second kind and provides the number of ways  $n$  data packets from the stations whose control packet transmission was successful are distributed to  $k$  data channels from the set  $A_1$ , in order at least two packets to have selected each of them.  $S_2(n, k)$  is given by the retroactive equation (Comtet, 1974):

$$S_2(n, k) = k S_2(n-1, k) + \binom{n-1}{1} S_2(n-2, k-1) \quad (20)$$

where:

Thus, in case that: ( $y \geq z$  AND  $y > 2$ ), it is:

$$\Pr[A(y)=z] = \text{prob}(y, z) = \left(\frac{2}{N}\right)^y \times \sum_{i=u}^{\lfloor \frac{z}{2} \rfloor} \binom{N}{2} \binom{y}{z-2i} (z-2i)! \binom{z-i}{i} i! S_2(y-z+2i, i) \quad (21)$$

where:

$$u = \begin{cases} 1, & \text{if } : y > z \\ 0, & \text{if } : y = z \end{cases} \quad (22)$$

Finally, we denote as  $\lfloor \frac{a}{b} \rfloor$  the integer part of the division  $\frac{a}{b}$ .