Flexibility Definition for Smart Grid Cells in a Decentralized Energy System

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Abstract: The networking of individual cells to form a decentralised network represents a possible approach for an energy system of the future, which is up to the challenges of the energy transition, such as stochastic electricity generation through renewable energies and the participation of many small producers. In this context, the sharing of flexibility in load profiles of cells requires a uniform definition to create a communication basis. This paper presents a generic description of flexibility by defining the latter as the set of all possible and permissible load profiles, taking into account dependencies between plants, technical constraints and maintaining energy balance within networks. The resulting solution space for load optimization problems, in form of the flexibility of a cell, can be described as a partial set of the $\mathbb{R}^{p \cdot T}$ by derived constraints. The solution space is the keystone for further flexibility communication.

1 INTRODUCTION

In addition to the expansion of renewable energy resources and their integration into the existing electricity supply as well as the increase in energy efficiency, the energy turnaround comes along with a bundle of other key challenges. Besides questions concerning investment distribution, resource conservation, social acceptance and political implementation, the focus is above all on the appropriate expansion of the energy infrastructure (Mauser, 2017). A concentration on the pure expansion of grid capacity, for example through the construction of new high-voltage lines from northern to southern Germany, does not meet this challenge entirely. Alternative options such as intelligent load and application management of energy (Palensky and Dietrich, 2011) and the use of storages are indispensable. In this context, there is talk of smart grid architectures (Greer et al., 2014), energy management systems (Allerding et al., 2014), smart energy storage and demand side management (Gottwald et al., 2011). What all these concepts have in common is that they examine approaches to solutions based on intelligent use of information and data as well as their provision and exchange.

An example for the development and demonstration of a smart grid approach which is supposed to represent the blueprint for a cellularly structured energy system of the future in Southern Germany is the SINTEG project C/sells (Smart Grids-Plattform Baden-Württemberg e.V, 2017). The networking of energy units, denoted as cells, is intended to create a secure and above all robust and resilient energy infrastructure that can cope with the new requirements of a decentralised supply system with many small generators, greater complexity and stochasticity due to for the greatest part volatile renewable energies. This is to be done on the basis of the use of flexibility in the load profiles of initially individual cells and in a further step across cells. Since individual cells differ greatly in their structure, i.e. the energy-generating and consuming plants contained in them, a generic definition of flexibility is necessary as a basis for communication. Such a definition and further steps to operate the system as a whole is developed in this paper. The basic idea is to describe flexibility in its most general form as the set of all possible and permissible load profiles for a future time window. To determine this set, technical grid and plant restrictions, dependencies on energy flows over time and energy balance aspects within grids must be taken into account. In addition to its generic nature, the main criteria for the definition presented here were the suitability to

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describe a solution space for optimization problems of load profiles as well as the possibility to consider different energy sources, controllable and uncontrollable plants and storages.

2 RELATED WORK

In order to overcome the hurdles of energy system transition, it is not sufficient to consider each cell (energy unit) individually. In addition to the internal use of flexibilities in the load profile, a network-wide communication between individual cells is also necessary. The networking of the cells allows joint action, which supports and promotes the development and maintenance of a secure and robust energy infrastructure, eventually yielding both system adequacy and security. In order to create a communication basis, it is necessary in a first step to develop a generic definition of flexibility, which can be applied to a wide variety of contexts and cells. Therefore, we will now present our basic criteria (C1-C7) for such a definition:

- C1 Applicability to different cells with various controllable and non-controllable plants, as well as the possibility to integrate flexibilities of subordinate cells into higher-level cells.
- C2 Suitability as solution space for a subsequent optimization of load.
- C3 Possibility to take into account the temporal load profiles of individual plants.
- C4 Possibility of modelling loads at plant level.
- C5 Consideration of energy storage systems.
- C6 Implementability within the scope of a load optimization.
- C7 Possibility of optimization according to different criteria.

In addition to these basic criteria, we identified ten concrete requirements (R1-R10) indicated in Table 1 which utterly describe our model as presented in section 3.2.

In 1985 Gellings presented his formulation of demand side management (Gellings, 1985). His work is not the concrete concept for defining flexibility for interoperable energy systems although modern solutions are mostly based on his ideas.

In 2011 the European Commission and EFTA issued the Smart Grid Mandate M/490 to the three European Standard Organizations (ESOs), CEN, CENELEC and ETSI which requests them to develop a framework to enable a continuous standard enhancement and development in the smart grid field. In order to fulfill the requested work the ESOs, together with the relevant stakeholders, established the CEN-CENELEC-ETSI Smart Grid Coordination Group (SG-CG), being responsible for coordinating the ESOs reply to M/490. In this context, the latter association adopted the following non-technical definition of flexibility:

Table 1: Requirements for a mathematical definition of flexibilities.

Requirement	Formulation
R1	Energy conservation principle for networks
R2	Physical limit constraints
R3	Specified power at specified times
R4	Dependencies over time
R5	Dependencies between plants
R6	Dependencies on stochastic environmental influences
R7	Number of total operation hours
R8	Specified energy at specified times
R9	Specific switch-on and switch-off times
R10	Energy conservation principle for plants

"On an individual level, flexibility is the modification of generation injection and/or consumption patterns in reaction to an external signal (price signal or activation) in order to provide a service within the energy system. The parameters used to characterize flexibility in electricity include: the amount of power modulation, the duration, the rate of change, the response time, the location etc." (Smart Grid Coordination Group, 2014)

This understanding of flexibility, as the **use** of existing leeway, is not appropriate for the context considered here, in which the ultimate goal is to identify optimal load profiles. In order to enable an optimization of the latter mentioned in a subsequent step, it is first necessary to describe flexibility as the set of all possible/permissible load profiles in an energy system. In addition, flexibility should be defined independently of external signals, since external price or activation signals do **not** change the number of possible load profiles.

In many areas, there are already very different energy management systems (EMS) in place today. These support and take over the energy, charge and load management for small units such as smart resi-

Table 2: Requirements for a mathematical definition of flexibilities.

Source	C1	C2	C3	C4	C5	C6	C7	Requirements
(Gellings, 1985)	no	no	no	no	no	no	no	-
(Weckmann et al., 2017)	no	no	no	no	no	yes	no	-
(Khoury et al., 2016)	no	yes	in part	no	yes	yes	in part	R1, R2, R3, R8, R9
(Mauser et al., 2016)	in part	yes	yes	yes	yes	yes	yes	R1-R5, R10
(Tushar et al., 2014)	no	no	no	no	in part	yes	no	-
(Liu et al., 2014)	no	yes	in part	in part	no	yes	no	R1, R2, R4
(Liu et al., 2015)	no	yes	in part	in part	no	yes	no	R2, R4

dential buildings (Khoury et al., 2016) up to companies (Weckmann et al., 2017) and entire infrastructures such as airports.

Due to the growing importance of EMS, there is also a large number of publications on this subject which have been considered in the search of a suitable flexibility definition.
(Weckmann et al., 2017) pursue the goal to ensure a stable and cost efficient energy supply in industrial energy systems where flexibilities are strongly limited by ensuring the production performance. In this respect, flexibility is understood as the possibility of cost-efficient shifting of production processes within a given scope. However, they do not consider a future time interval for which an optimal schedule is to be determined taking all available future flexibilities into account, but rather a decision on producing or not is made at any time on the basis of current flexibility key figures.
An approach that describes the determination of an optimal future load profile can be found in (Khoury et al., 2016). Having an intermittent grid electricity supply, the goal is to optimize the operation of a PV-battery backup system. Against this background, flexibility refers to two dimensions. The first is the possibility of varying a reference value for continuous working plants such as a Heating, Ventilation and Air Conditioning systems. The second relates to the possibility of shifting switch-on times, as it is practicable for a washing machine, for example. There is no generic definition of flexibility, which is why some of the requirements from Table 1 cannot be realized and power modelling at plant level is not possible.
In their article, (Mauser et al., 2016) seem to describe a similarly generic approach to the modelling of flexibility as it is the aim of this paper. However, there is no definition of flexibility in a mathematical sense, but only the two terms "Temporal Degree of Freedom" and "Energy-related Degree of Freedom" are used to define it. As part of describing the optimization of a smart residential building's load profile, it is explained that technical system details, interdependencies between plants, predefined switch-on and switch-off times, etc. can be taken into account. Nevertheless, it is not clear to what extent all possible and permissible paths for optimization are actually taken into account, i.e. whether the flexibility available in the system is fully grasped.
Further work dealing with the problem of minimising energy costs and maximising benefits through the use of flexibility is listed in Table 2. It is evident that none of the papers contains a definition of flexibility that fully meets the criteria set out in this paper.

For our purposes, in the context of C/sells, none of the studies considered provides a sufficiently generic and mathematically precise definition of flexibility (see Table 2). Thus, the great added value of this work lies in the fact that a generic, mathematically correct description of flexibility is presented. This definition can then serve as a basis (in form of the feasible solution space) for the formulation of various optimization problems, such as those of (Mauser et al., 2016).

3 DEFINITION OF FLEXIBILITIES

In this chapter, the idea underlying the definition of flexibilities is first motivated in Section 3.1, followed by the formal definition in Section 3.2.

3.1 Idea

The C/sells project which aims to design a cellulary structured energy system in southern Germany and to model the individual cells in a network, raises the question of an appropriate definition of flexibility. In the first step, it must provide a suitable starting point for an optimization of power generation and consumption (load profiles or paths) in a single cell and, moreover, it must be generic enough to permit meaningful communication between the individual cells of the compound. A cell is understood as a combination of all

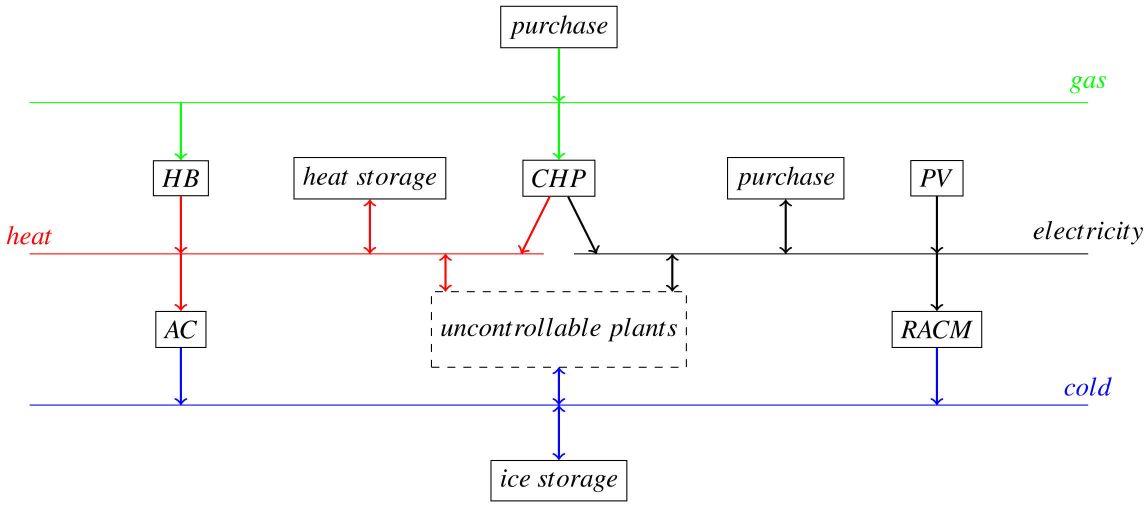

Figure 1: Example of a cell with four different grids (gas, heat, cold, electricity), ten different controllable and uncontrollable plants and 17 connections between them.The abbreviations mean: HB = heating boiler, CHP = combined heat and power unit, PV = photovoltaic system, AC = absorption chiller, RACM = refrigeration and air conditioning machine.

plants, all networks and the existing links between them in a energy system. An important aspect is that the flexibility definition remains the same even if a cell is integrated into a higher-level cell. To achieve this, flexibility is first of all considered at the lowest level, i.e. at the plant level. If the flexibility of an entire cell is defined as the aggregated flexibilities of the individual plants contained in it, then if it is embedded in a higher-level cell, only an extension of the number of all considered plants is carried out. For a single plant, on the one hand flexibility can describe the possibility to vary a load (in form of purchased or provided power) at a specific point of time or over a time interval. A second type of flexibility is offered when there is the possibility of load shifting over time. That means a predefined load is deferrable or even interruptible. Overall there can be flexibilities in energy-related or temporal terms. Furthermore, only such plants can offer flexibilities which can be controlled in the described sense. Therefore the following distinction between controllable and uncontrollable plants is made in the discussed model:

(1) A plant can be controlled (called *controllable*) if its power consumption or output can be controlled/defined within a range permitted for the plant. Limits of the permissible range can depend on stochastic influencing variables. Such plants fulfill the requirements introduced in section 3.2.1 to 3.2.10.

(2) All plants that cannot be controlled are called *uncontrollable*. The power flows for these plants are random variables for all time intervals.

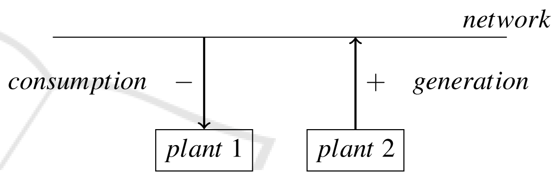

Figure 2: Visualisation of two plants in a cell with different flow direction and corresponding signs (+ indicates generation; - indicates consumption).

Another aspect that has to be taken into account defining flexibility, is the interaction of different networks that supply power to the plants or into which power is fed. Various energy vectors, such as electricity, gas, heat and cold, play a particularly important role here. In this respect, the sign of the power should indicate the direction in which it flows. Figure 2 illustrates the flow directions with the corresponding signs. If a plant consumes power from the grid, the sign is negative and vice versa. An example of a cell with different networks and plants is given in Figure 1.

3.2 Mathematical Model

The time interval for which all energy flows of a cell are considered is indicated with $[0,T]$ for $T \in \mathbb{N}$ where T is the number of time steps. The variable t describes a specific point in time within the interval $[0,T]$. The time steps are discrete and in 15-minutes intervals to draw a link to the scenario of peak demand management. For other use cases the length of the time steps can easily be modified. Some of our later constraints have to be changed to these modifications.

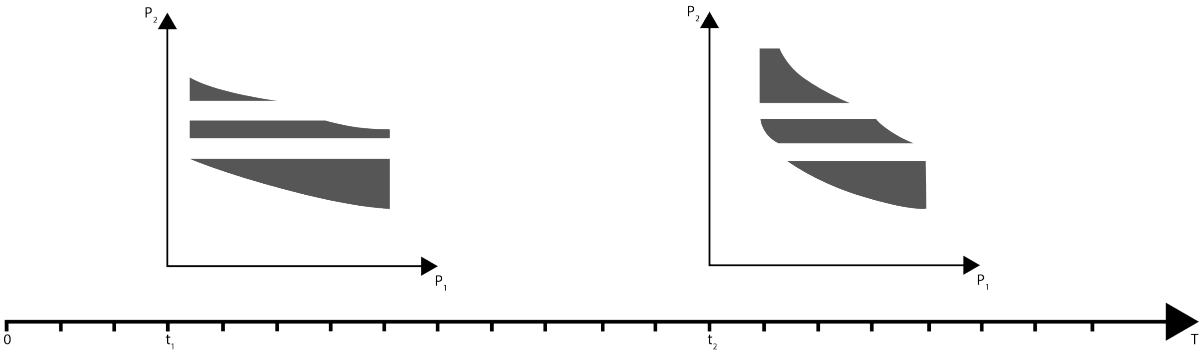

Figure 3: Schematic illustration of the flexibility of two plants A_1 and A_2 with possible power flows P_1 and P_2. Dependencies to other plants of the cell were neglected for the sake of representability. The combinations of P_1 and P_2 in the gray area are possible loads and the whole area reflects the total flexibility of the two plants at the time points t_1 and t_2.

The number of plants in the cell under consideration is given by Z and the plants are induced by $i \in \{1,\ldots,Z\}$
The differentiation of plants in controllable and non-controllable is made using the parameter $s \in \{c,nc\}$.
The parameter n specifies the network from which a plant draws power or into which a plant feeds power. This implies that energy flows take place exclusively between plants and networks i.e. any energy loss in networks is neglected. The set of considered networks is given by $N = \{E_1,\ldots,E_k,H_1,\ldots,H_l,C_1,\ldots,C_m,G_1,\ldots,G_o\}$, where electricity grids are marked by E_{j_1}, heat grids by H_{j_2}, cold grids by C_{j_3} and gas grids by G_{j_4} for $j_1 \in \{1,\ldots,k\}$, $j_2 \in \{1,\ldots,l\}$, $j_3 \in \{1,\ldots,m\}$ and $j_4 \in \{1,\ldots,o\}$.
The variable $P_{t,i,s,n}$ describes the power consumed or provided by plant i at time t. The network in which the power transfer takes place is specified by the parameter n and the controllability of the plant by parameter s. Every $P_{t,i,s,n}$ is understood as the average load within the time step $[t,t+1)$. Because not all combinations of i,s and n exist just the real existing links between networks and plants and the corresponding indices are taken into account. The set of all links between plants and networks can be described as $\mathcal{P} = \{(i,n)|$ the link between i and n exists$\}$. The number of these links is denoted by $p = |\mathcal{P}|$.
For plants that cannot be controlled, their power flows $P_{t,i,nc,n}$ represent stochastic quantities at all points t in time. For the following theory it is assumed that forecasts on the time interval $[0,T]$ in the form of defined values from $\mathbb{R}$ exist for all these uncontrollable power flows.

In order to describe the flexibility of each plant at a time t, conditions are placed on $P_{t,i,s,n}$. The set of all permissible values for $P_{t,i,s,n}$ forms the flexibility of plant i at time t and is a subset of $\mathbb{R}$. It must be born in mind that the flexibility of a single plant cannot be considered separately from the behaviour/performance of other plants, since there are dependencies between the consumed/disposed powers of all plants. In addition, dependencies between plants not only refer to a fixed point in time, but also have to be taken into account across time periods. The flexibility $\mathcal{F}$, i.e. the set of all permissible load profiles for an energy system (also called cell), is therefore defined as follows:

Definition (Flexibility) *Let E be an energy system that includes Z plants and N networks and is considered on the discrete time interval $[0,T]$. In addition, the number of connections between plants and networks is indicated by p, where $p \leq Z * N$. Then the flexibility $\mathcal{F}_E$ of the energy system is given by the following subset of $\mathbb{R}^{p*T}$:*

$$\mathcal{F}_E = \left\{x \in \mathbb{R}^{p*T} \,|\, \text{3.2.1 to 3.2.10 applied}\right\} \quad (1)$$

The entries of vector x represent the power flows between plants and networks at all times $t \in [0,T]$ and therefore define a complete permissible load profile of the following form:

$$x = (\ldots,P_{0,i,s,n},\ldots,P_{T-1,i,s,n},\ldots) \quad \forall (i,n) \in \mathcal{P}$$

An active formulation of the conditions 3.2.1 to 3.2.10 only takes place for controllable plants. Power flows of uncontrollable plants in the form of fixed, predicted values are taken into account indirectly (considering dependencies or conservation principles) if necessary.

The flexibility concept defined in this way is visualized in Figure 3 using the example of two (controllable) plants A_1 and A_2 with one network connection each (P_1 and P_2). In sections 3.2.1 to 3.2.10, the conditions for $x \in \mathbb{R}^{p*T}$ are now discussed.

3.2.1 Energy Conservation Principle for Networks

In each network belonging to the cell, the sum of consumption (negative sign) and generation/feed-in (positiv sign) must result in zero. Mathematically, this can be described by the following equation:

$$\forall n \in N \quad \forall t \in [0,T] \quad \sum_{i \in Z} P_{t,i,s,n} = 0. \tag{2}$$

The sum also includes the power flows of uncontrollable plants in form of fixed, previously forecast values.

3.2.2 Physical Limit Constraints

The power consumption or supply of each plant i is subject to plant- and grid-inherent physical restrictions in form of lower and upper bounds (LB and UB). This is formulated as follows:

$$\forall i,n \quad \exists LB_1, \ldots, LB_p, UB_1, \ldots, UB_p$$

with $p \in \mathbb{N}$, such that

$$LB_j(i,n) \leq P_{t,i,c,n} \leq UB_j(i,n) \tag{3}$$

with

$$j \in \{1, \ldots, p\}.$$

Power plants have often the physical limit constraints $LB_j(i,n) = 0$ and consuming plants $UB_j(i,n) = 0$.

3.2.3 Specified Power at Specified Times

It is often the case that power consumption or supply of a plant i is prescribed for certain time intervals $[t, t+1)$. Therefore, at these points conditions are provided to $P_{t,i,c,n}$ in the following form:

$$P_{t,i,c,n} = C(t,i,n). \tag{4}$$

3.2.4 Dependencies Over Time

For some plants the provided or purchased power at time t depends on the power of earlier points in time $\tilde{t}$ with $\tilde{t} \in [0,t)$. This can be modelled by setting upper and lower limits to the power that do not only depend on i and n (physical limit constraints) but also on $P_{\tilde{t},i,c,n}$. If the power of plant i at time t to network n depends on $\tilde{t}_1, \ldots, \tilde{t}_j$, the following condition is obtained:

$$LB = LB(i,n,P_{\tilde{t}_1,i,c,n}, \ldots, P_{\tilde{t}_j,i,c,n})$$

and

$$UB = UB(i,n,P_{\tilde{t}_1,i,c,n}, \ldots, P_{\tilde{t}_j,i,c,n})$$

with

$$LB \leq P_{t,i,c,n} \leq UB. \tag{5}$$

3.2.5 Dependencies Between Plants

Just as the power of a plant i at time t can depend on previous performances of the same plant, dependencies between different plants are also possible. For example, if the provided or purchased power of plant i at time t to network n depends on the power of plant $h \in \{1, \ldots, Z\}$ at times $\tilde{t}_1, \ldots, \tilde{t}_j \in [0,t]$, this can be formulated as follows using upper or lower limit constraints:

$$LB = LB(i,n,P_{\tilde{t}_1,h,s,n}, \ldots, P_{\tilde{t}_j,h,s,n})$$

and

$$UB = UB(i,n,P_{\tilde{t}_1,h,s,n}, \ldots, P_{\tilde{t}_j,h,s,n})$$

with

$$LB \leq P_{t,i,c,n} \leq UB. \tag{6}$$

3.2.6 Dependencies on Stochastic Environmental Influences

In some cases, the upper and lower limits of the permissible power spectrum of a plant i at time t to network n may also depend on environmental influences EI_t such as the global horizontal irradiance GHI_t (in the case of PV plants). Assuming that forecasts for the time interval $[0,T]$ are available for such stochastic influencing variables, the following dependencies and restrictions on the power at time t result:

$$LB = LB(i,n,EI_t)$$

and

$$UB = UB(i,n,EI_t)$$

with

$$LB \leq P_{t,i,c,n} \leq UB. \tag{7}$$

3.2.7 Number of Total Operation Hours

For some plants the operation hours are limited to a certain number $\#S$. This condition is modelled using the indicator function $\mathbb{1}$:

$$\left(\sum_{t \in [0,T]} \mathbb{1}_{\{|P_{t,i,c,n}|>0\}} \right) : 4 \leq \#S \tag{8}$$

The number of time intervals having positive or negative power is divided by 4 in order to take into account that the time steps are set at quarter-hourly basis.

3.2.8 Specified Energy at Specified Times

Often, such as in the case of batteries, a storage device must hold a certain amount of energy C. If the condition should be fulfilled at time t_1, this can be guaranteed by the following inequality:

$$\sum_{t\in[0,t_1]} P_{t,i,c,n}\cdot\frac{1}{4}h \geq C(0,t_1,i) \tag{9}$$

3.2.9 Specific Switch-on and Switch-off Times

There are plants that can only be switched on or off for a limited number of consecutive time intervals. A classic example is a ventilation system that cannot be switched off for longer than a certain period of time. On the other hand, there are systems that must be switched on/off for a minimum number of consecutive time intervals. Another variant is a plant that hast to comply with a predefined switch-on/switch-off pattern. Such a pattern could be the condition that after switching off the ventilation for two time units, it must be switched on for the same period of time. Some of the most frequently required conditions are formulated below.
For this purpose, the binary variable $\delta_{t,i}$ is defined by

$$\delta_{t,i} = \begin{cases} 0 & \text{, if plant } i \text{ is switched off at time } t \\ 1 & \text{, if plant } i \text{ is switched on at time } t \end{cases}$$

1. *The plant is allowed to be switched on for a maximum of k consecutive time steps.* To ensure this condition, the inequality

   $$\sum_{\tilde{t}=t+1}^{t+k} \delta_{\tilde{t},i} - (k-1) \leq 1 - \delta_{t,i}$$

   must be fulfilled for all $t \in \{0,\ldots,T-k\}$.

2. *The plant must be switched on for at least k consecutive time steps.* This condition is met if

   $$\sum_{\tilde{t}=t+2}^{t+k} \delta_{\tilde{t},i} - (k-1) \geq (1-k)(1-\delta_{t+1,i}) + (1-k)\delta_{t,i}$$

   and applies for all $t \in \{0,\ldots,T-k\}$.

3. *The plant may be switched off for a maximum of k consecutive time steps.* That means the inequality

   $$\sum_{\tilde{t}=t+1}^{t+k} \delta_{\tilde{t},i} - 1 \geq -\delta_{t,i}$$

   has to be valid for all $t \in \{0,\ldots,T-k\}$.

4. *The plant must be switched off for at least k consecutive time steps.* This condition holds if

   $$\sum_{\tilde{t}=t+2}^{t+k} \delta_{\tilde{t},i} \leq (k-1)\delta_{t+1,i} + (k-1)(1-\delta_{t,i})$$

   for all $t \in \{0,\ldots,T-k\}$.

5. *If the plant has been switched of for k consecutive time steps, then it must be switched on for l consecutive time steps.* To ensure this condition, the inequality

   $$\sum_{\tilde{t}=t+k}^{t+k+l-1} \delta_{\tilde{t},i} - l \geq -l\cdot\sum_{\tilde{t}=t}^{t+k-1} \delta_{\tilde{t},i}$$

   must be fulfilled for all $t \in \{0,\ldots,T-k-l+1\}$.

3.2.10 Energy Conservation Principle for Plants

When a plant interacts with different networks like gas, electricity, heat and cold grids, it must be ensured that the energy conservation principle is fulfilled. A co-generation plant, which draws power from the gas grid and supplies power to the electricity and heat grid, is a good example. Let $\{n_{i_0},\ldots,n_{i_k}\}$ designate the different networks from which a plant i draws power at time t and $\{n_{i_{k+1}},\ldots,n_{i_l}\}$ the different networks fed by plant i. When η describes the efficiency of plant i, then the following equation must apply for all $t\in[0,T]$:

$$-\eta\sum_{j=n_{i_0}}^{n_{i_k}} P_{t,i,s,j} = \sum_{j=n_{i_{k+1}}}^{n_{i_l}} P_{t,i,s,j} \tag{10}$$

The powers provided to different networks are in a certain ratio to each other, which is why

$$-\eta\sum_{j=n_{i_0}}^{n_{i_k}} P_{t,i,s,j} = \frac{1}{x_j}P_{t,i,s,n_j} \tag{11}$$

must also be fulfilled for all $j\in\{k+1,\ldots,l\}$ and $\sum_j x_j = 1$. The x_j result from the partial efficiencies $\eta_{k+1},\ldots,\eta_l$ through

$$x_j = \frac{\eta_j}{\eta} \quad \forall j \in \{k+1,\ldots,l\}.$$

4 EVALUATION

Since the definition given in this paper is first of all theoretical in nature and will be incorporated into optimization problems for load profiles of cells in a next step, an evaluation at this point in time will also take place on a theoretical level.

One aspect to be considered is the possibility that conditions 3.2.1 through 3.2.10 in their current formulation may approximate reality too simplistic in some cases. This is explained by the fact that a compromise must be found between a complete representation of reality and an excessively high complexity of the solution space (in the form of flexibility). The extent to which the degree of abstraction chosen in this paper proves suitable for the application remains to be verified. However, since a generic approach to describe flexibility was the main objective of this work, there only has been a deliberate simplification for condition 3.2.10. The complexity was reduced by assuming the efficiency η to be independent of the power flows. Actually, the following should apply, with the notations from 3.2.10:

$$\eta = \eta(P_{t,i,s,n_{i_0}}, \dots, P_{t,i,s,n_{i_k}}) \tag{12}$$

The same applies to the proportion of power provided to power consumed by the plant, which also changes depending on the latter:

$$x_j = x_j(P_{t,i,s,n_{i_0}}, \dots, P_{t,i,s,n_{i_k}}) \tag{13}$$

for all $j \in \{k+1, \dots, l\}$ and $\sum_j x_j = 1$.

The example of a cell from Figure 1 will be used to evaluate, whether the conditions formulated in 3.2.1 to 3.2.10 are suitable to describe the flexibility present in this cell.

First of all, it is determined that the observation period should be one day which equals 96 time steps. The number of plants Z is given by 10 and the number of networks by 4 (gas, heat, electricity and cold), with a total of $p = 17$ connections between plants and networks. The flexibility can then be formulated according to definition (1) as follows:

$$\mathcal{F}_E = \left\{ x \in \mathbb{R}^{17*96} | \text{ 3.2.1 to 3.2.10 apply} \right\}$$

In practice, the set of all possible load profiles is first of all limited by the fact that the power balance within a network must always be zero. In the case of Figure 1, the equation in 3.2.1 must therefore apply specifically to the four grids gas, heat, electricity and cold (described as the set $N = \{G, H, E, C\}$):

$$\forall t \in [0, 96-1] \quad \sum_{i=1}^{10} P_{t,i,s,n} = 0.$$

Secondly, technical/physical restrictions of plants and also networks for determining cell flexibility must be taken into account. If, for example, the CHP, the HB, or the AC is considered, plausible physical limit constraints could be the following:

CHP: Combustion heat output 0 kW or between 500 kW and 2 MW. Electrical power 0 kW or between 500 and 2 kW.

HB: Combustion heat output 0 kW or between 500 kW and $4,9$ MW.

AC: Cooling capacity 0 kW or between 50 kW and 200 kW.

If these restrictions are combined, exactly the upper and lower bounds defined in 3.2.2 are obtained.

Further constraints arise as a result of temporal dependencies in the power flows, as in this example for the two storages (heat and ice storage). The general formulation in 3.2.4 results into

$$0 \leq \sum_{\tilde{t}=0}^{t} (P_{\tilde{t},HS,c,H} \cdot \frac{1}{4} h) \leq S_{max} \quad \forall t \in [0, 96-1]$$

$$\Leftrightarrow -\sum_{\tilde{t}=0}^{t-1} P_{\tilde{t},HS,c,H} \leq P_{t,HS,s,H} \leq S_{max} \cdot \frac{4}{h} - \sum_{\tilde{t}=0}^{t-1} P_{\tilde{t},HS,c,H}$$

where the two terms on the left and right side represent the lower and upper bounds respectively and S_{max} is the maximal storage capacity of the heat storage. In the case of the ice storage, LB and UB can be formulated in the same way.

For the PV system of the cell, the upper power limit also depends on the stochastic influencing variable of the global horizontal irradiance. As from 3.2.6

$$UB = UB(PV, E, GHI_t) = f(GHI_t)$$

is another condition for the set of all permissible load profiles, where the function f determines the relationship between the predicted value of GHI_t and the maximum possible power generation.

Next, the maximum switch-off time prescribed for the RACM must be taken into account. If the air quality is not to deteriorate too much, a plausible specification could be half an hour. Using the binary variable $\delta_{t,i}$, defined in 3.2.9, and $k = 2$ ($2 \cdot 15\text{min} = 30\text{min}$), the following constraint is obtained for all $t \in \{0, \dots, (96-1)-2\}$:

$$\sum_{\tilde{t}=t+1}^{t+2} \delta_{\tilde{t},RACM} - 1 \geq -\delta_{t,RACM}$$

Finally, in order to describe the flexibility in the exemplary energy system shown in Figure 1, it should be noted that the energy balance for individual plants connected to more than one network will be maintained. In this context it concerns the four plants HB, AC, CHP and RACM, whereby in the following the condition from 3.2.10 is formulated as an example for the CHP. It is assumed that the CHP has an electrical efficiency of $43,5\%$ and a thermal efficiency of $41,5\%$, resulting in an overall efficiency of 85%. The following equations must therefore be fulfilled for all $t \in [0, 96-1]$:

$$-0.85 \cdot P_{t,CHP,c,G} = P_{t,CHP,c,H} + P_{t,CHP,c,E}$$

and

$$-0.85 \cdot P_{t,CHP,c,G} = \frac{0.85}{0.435} \cdot P_{t,CHP,c,E}$$

and

$$-0.85 \cdot P_{t,CHP,c,G} = \frac{0.85}{0.415} \cdot P_{t,CHP,c,H}$$

This example shows that the generic description of flexibility made in this paper is suitable for applying it to a wide range of cells.

5 CONCLUSION AND OUTLOOK

In order to create a common communication basis for cells of different types enabling the exchange of information regarding existing and thus usable flexibility, a generic definition of flexibility is developed in this paper. This is particularly important against the background of the energy transition, since the use of flexibility within a cellularly structured energy system is one of the most widely pursued approaches to deal with the challenges of an increasingly decentralised energy system.

The basic idea is to define flexibility within cells containing any number of plants and networks as the set of all possible and permissible load profiles. This can be described as a subset of $\mathbb{R}^{p \cdot T}$, where p describes the number of all existing connections between plants and networks and T the number of considered time steps. Each point defines a complete load profile for all networks and all plants for a pre-defined, future time interval. In order to determine the subset, conditions have been introduced which include interdependencies between plants and laws for energy conservation within networks and individual plants. Further criteria for the definition of flexibility are that it creates a suitable solution space for load optimization problems within cells, enables easy integration into higher-level cells and supports the modelling of power flows at plant level.
Individual sub aspects in the plant behaviour, such as start-up ramps, energy loss in networks or power-dependent efficiency ratios, are currently still being neglected, as these would not change the basic idea of describing the system, but would create additional complexity. However, such aspects can be added to the given definition at any time.

Based on the now generally defined concept of flexibility, it is to be investigated how it can be embedded in different methods for the optimization of load profiles within but also across cells. This also raises the question of the visualization ability of flexibility, which is currently still implicitly described by boundary conditions to the permissible subset of $\mathbb{R}^{p \cdot T}$. Therefore, this aspect is also closely related to the investigation of possibilities for an explicit representation of flexibility. It may be necessary to make further simplifying assumptions on the energy systems under consideration in order to facilitate the application of the definition within the context of load optimization in real cells.
In addition, another application objective of the flexibility definition given in this paper is to be seen in the context of fleet management of electric vehicles. The energy management system of a shared electric vehicle fleet described in (Ostermann and Koetter, 2016) implements the flexibility idea presented in this paper for a special case in practice. This offers a possible application case in which a transfer of the theoretical definition into practice can be the subject of future research work.

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