Using Goal Programming on Estimated Pareto Fronts to Solve Multiobjective Problems

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Abstract: Modern multiobjective algorithms can be computationally inefficient in producing good approximation sets for highly constrained many-objective problems. Such problems are common in real-world applications where decision-makers need to assess multiple conflicting objectives. Also, different instances of real-world problems often share similar fitness landscapes because key parts of the data are the same across these instances. We we propose a novel methodology that consists of solving one instance of a given problem scenario using computationally expensive multiobjective algorithms to obtain a good approximation set and then using Goal Programming with efficient single-objective algorithms to solve other instances of the same problem scenario. We propose three goal-based objective functions and show that on a real-world home healthcare planning problem the methodology can produce improved results in a shorter computation time.

1 INTRODUCTION

Modern multiobjective algorithms struggle to find good approximation sets on highly constrained problems or when the number of objectives is high Giagkiozis and Fleming (2012). Decision-makers often require a single solution and the Pareto front is only useful to enable them to choose the preferred solution(s). There are a few techniques that use domain knowledge to estimate Pareto fronts. Giagkiozis and Fleming (2014) propose a technique to estimate the Pareto front of continuous multiobjective problems and then use the estimated front to obtain values for the decision variables of interesting solutions. Their technique is particularly useful because it can be applied to subregions of the objective space. Another technique is Bayesian multiobjective optimisation, which consists of using a Bayesian model to learn computationally expensive objective functions and use the estimation model to quickly explore the search space Feliot et al. (2016).

In this research we tackle a workforce scheduling and routing problem (WSRP). A WSRP solution is a plan in which skilled mobile workers are scheduled to visit locations that are geographically scattered. Real-world applications of WSRP usually involve multiple objectives. However, most approaches in the literature tackle this problem using a weighted sum to combine the multiple objectives into one. Even generating single-objective solutions for the WSRP has been shown to be difficult for problem instances of considerable size Castillo-Salazar et al. (2014, 2016); Laesanklang et al. (2015a). The WSRP arises in several domains, for example when scheduling home healthcare workers, technicians, and security personnel among others. Here, we consider the problem of scheduling nurses and care workers to visit and provide care services to patients in their homes. Works in the literature tackling home healthcare planning problems usually use data from real-world scenarios Fikar and Hirsch (2017). This is also the case in the present work. We employ data from six distinct home healthcare scenarios provided by Webroster Ltd., who provides enterprise resource planning software for home healthcare companies.

The quality of solutions to the WSRP considered here can be measured using several criteria like: incurred costs, travel distance, satisfaction of staff pre-

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ferences, satisfaction of patients preferences, etc. Due to the high number of constraints and objectives in this problem, even state-of-the-art multiobjective algorithms struggle to find good approximations to the Pareto front within reasonable computation time. In problems like this where each instance is a repetition of the same situation but in different days, parts of the data repeat across multiple instances of the same problem scenario. For example, the same set of staff is available for different days with only minor changes regarding staff avilability. Also, there is a set of recurring visits that need to be scheduled in each instance of the problem scenario. This happens in other problems like vehicle routing and nurse scheduling where instances of the same problem scenario exhibit similar fitness landscapes (η-dimensional surface representing the Pareto front, where η is the number of objectives). This issue was investigated in Pinheiro et al. (2015, 2017) which proposed a technique to analyse and visualise complex objective relationships and fitness landscapes in multiobjective problems. Thus, this paper proposes a methodology that exploits this similarity between instances of a multiobjetive optimisaton problem, in order to solve instances of the problem more efficiently.

The proposed methodology works as follows. For a given dataset consisting of multiple instances of the problem scenario, a single *pilot* instance is solved using multiobjective algorithms or the best available method in order to produce an approximation to the Pareto optimal set. A decision-maker uses this approximation set to choose *target* solutions for the remaining instances of the dataset. The chosen target solutions represent the desired trade-off between the multiple objectives. Then, goal programming is applied with an efficient single-objective solving method to obtain solutions that in quality are close enough to (or better than) the chosen target solutions. Hence, the main contributions of this work are:

1. A novel methodology that employs an approximation to the Pareto optimal front of a pilot problem instance, to solve other similar instances of the same problem with goal programming and efficient single-objective algorithms.
2. The assessment of three approaches to define the goal programming objective function, one uses a Chebyshev objective function, another uses a weighted function derived from the target solution, and the third one uses an objective function that seeks to minimise the largest deviation to objective values in the target solution.

The remaining of this paper is structured as follows. Section 2 outlines the Workforce Scheduling and Routing Problems Project, which is used to illustrate the application of the proposed methodology. Section 3 presents the proposed methodology. Section 4 presents the experimental configuration while section 5 presents the results. Finally, section 7 concludes this work.

2 THE WORKFORCE SCHEDULING AND ROUTING PROBLEM

The WSRP is an optimisation problem that combines a scheduling and a routing problem. A set of m workers $\{w_1, w_2, \ldots, w_m\}$, must perform tasks on a set of n visits $\{v_1, v_2, \ldots, v_n\}$, which are distributed across distinct geographical locations. This work considers a specific case of this problem: the home healthcare planning problem, where workers are nurses, doctors and care workers and the purpose of the visits is to provide care to patients in their homes. For more information about this WSRP, please refer to Castillo-Salazar et al. (2012, 2016).

The WSRP studied in this work contemplates the planning of a single day. Several hard constraints are considered in the problem: visits require a set of skills and only workers possessing the required skills can be assigned; workers can only be assigned to visits if allowed by their respective contracts; every visit must commence at a specified time in which workers must already be at the visits' premises; no time clashes are allowed, i.e. a worker cannot be assigned to simultaneous visits and there must be enough time for a worker to travel between visits. Also, two soft constraints are considered: workers may have a list of time intervals in which they are available; locations are grouped by geographical areas and workers may have a list of areas in which they are available.

The objectives considered in the problem are:

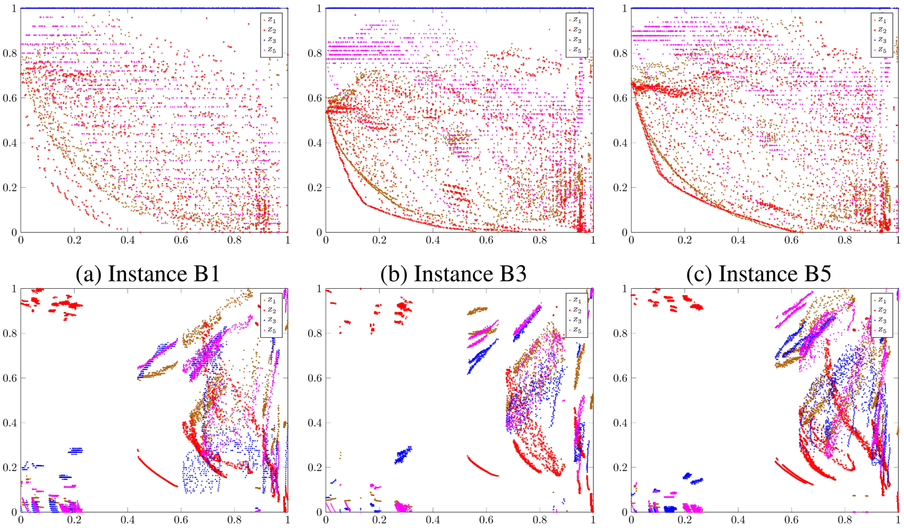
Z_1 denotes the sum of the distances travelled by the workers, including commuting from their home to the first assignment, between each assignment, and from the final assignment back to their home;

Z_2 denotes the sum of monetary assignment costs (workers wages for the assigned visits);

Z_3 denotes the sum of skill preferences in respect of assigning workers with skills that are desirable but not necessary;

Z_4 denotes the sum of patients preferences in respect of assigning preferred workers; and

Z_5 denotes the sum of workers preferences in respect of assigning workers to visits in their preferred area.



(a) Instance B1 (b) Instance B3 (c) Instance B5

(d) Instance D2 (e) Instance D4 (f) Instance D6

Figure 1: Scatter plots of three instances of datasets B and D. In each case, the x axis shows the value of objective Z_4 and the y axis shows the values of each of the other objectives in different colours.

Data from four different home healthcare companies of different sizes and characteristics was provided and six problem scenarios or dataset were generated. Each dataset (A, B, C, D, E and F) is composed of 7 instances for a total of 42 problem instances. Set A has small instances (number of visits and workers) while set F has the largest instances.

Other works in the literature have also tackled the WSRP datasets considered here:

- Laesanklang et al. (2015a) presented a computational study on the suitability of using exact methods to solve the problem. It was concluded that only the smaller instances can be solved to optimality by a mathematical solver. Later, Laesanklang et al. (2015b) proposed to decompose the problem by geographical locations in order to employ the mathematical solver on the sub-problems and then use heuristic algorithms to integrate the partial solutions.
- Algethami and Landa-Silva (2015) conducted a study of various genetic operators applied to this problem. It was observed that the high number of constraints and large size of the instances presented a considerable challenge to the standard genetic algorithm implemented. Later, Algethami et al. (2016) proposed a knowledge-based indirect representation and genetic operators to improve the efficiency of the genetic algorithm for tackling the problem.
- More recently, Pinheiro et al. (2016) presented a Variable Neighbourhood Search (VNS) metaheuristic to tackle the WSRP, incorporating two novel heuristics tailored to the problem-domain. The first heuristic restricts the search space using a priority list of candidate workers and the second heuristic seeks to reduce the violation of specific soft constraints. Also, two greedy constructive heuristics were used to give the VNS a good starting point. Since that VNS is the best-known algorithm to tackle these WSRP instances, that algorithm is employed here within the proposed goal programming framework.

As preliminary work, we applied the analysis technique proposed by Pinheiro et al. (2015, 2017) on all instances considered here. The technique consists of performing four steps: first the global pairwise relationships are analysed using the Kendall correlation method; then the ranges of the values found on the given approximation front are estimated and assessed; next these ranges are used to plot a map using Gray code, similar to Karnaugh maps, that has the ability to highlight the trade-offs between multiple objectives; and finally local relationships are identified using scatter plots. The conclusion is that in the studied datasets, instances of the same datset indeed present similar fitness landscapes. Figure 1 provides evidence of that and shows the scatter plots of instances B-01, B-03, B-05, D-02, D-04, and D-06 (arbitrarily chosen). Each datapoint corresponds to normalised objective values. For reasons of space, we omit the other datasets and the scatter plots of all instances. The resemblance between the fitness landscapes of the presented instances of datasets B and D is evident. Also, it is clear that different datasets present distinct fitness landscapes, even though they correspond to the same problem formulation. Finally, it is important to emphasise that datasets A, B and C present similar fitness landscapes with solutions well spread throughout the objective space while datasets D, E and F present distinct fitness landscapes with complex objective relationships and gaps.

3 PROPOSED METHODOLOGY

Figure 2 presents the overall concept of the proposed methodology. We use the aforementioned WSRP to illustrate the concept. Following, we explain each of the four numbered steps indicated.

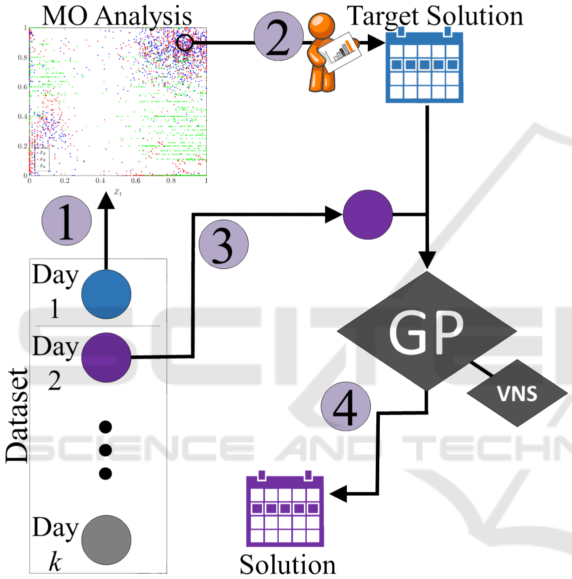


Figure 2: Overview of the proposed methodology. The numbered steps are explained below.

1. A *pilot instance* within the dataset is selected by the decision-maker and solved using multiobjective algorithms to obtain the best possible non-dominated approximation set. The pilot instance must be representative and present similar features to other instances.
2. The decision-maker chooses an appropriate solution t from the obtained non-dominated set. This chosen solution is known as the *target solution* and its objective-vector is denoted by

$$\vec{Z^t} = (Z^t_1, Z^t_2, Z^t_3, Z^t_4, Z^t_5).$$

3. Each remaining instance in the dataset may now be solved by the VNS (Pinheiro et al., 2016) using a modified objective function (goal programming variant) that attempts to reach the target objective vector in the current dataset.
4. The final solution obtained by the modified VNS is presented.

The modified objective function of step 3 has an important role in the methodology. Following, we present three approaches for determining the objective function. The first is the well known Chebyshev approach. The second aims to derive a weight-vector from the target solution and the approximation set of the pilot instance. The third approach minimises the Euclidean distances to the target objective-vector.

3.1 Chebyshev Goal Programming

Chebyshev goal programming (Flavell, 1976) is one of the most widely employed goal programming techniques that does not necessarily rely on the decision maker to set priorities between objectives (Jones and Tamiz, 2016). The technique aims to obtain a balanced solution by minimising the gap to the target of the objective that presents the highest gap. Hence, if the target goals for the objectives are similarly difficult to be reached, the Chebychev GP technique can obtain a balanced solution. However, if at least one target objective value is more difficult to achieve (i.e. the target goal is too optimistic), the quality of that objective can be a bottleneck for the remaining objectives because the search will solely focus on improving that objective. We define the Chebyshev objective function for the WSRP as follows:

$$\text{Minimise} \quad \lambda \tag{1}$$

$$\text{Subject to}$$

$$\frac{Z_1}{Z^t_1} \leq \lambda \tag{2}$$

$$\frac{Z_2}{Z^t_2} \leq \lambda \tag{3}$$

$$\frac{Z_3}{Z^t_3} \leq \lambda \tag{4}$$

$$\frac{Z_4}{Z^t_4} \leq \lambda \tag{5}$$

$$\frac{Z_5}{Z^t_5} \leq \lambda \tag{6}$$

The Chebyshev objective function given by Eq. (1) is used as the objective function for the WSRP. The main objective is now to minimise λ, thus finding a well-balanced solution regarding reaching the target values. If all targets are reached, λ can assume fractional values and a solution that shows balanced improvements on all objectives may be obtained.

3.2 Derived Weight Vector

One problem with the Chebyshev approach is that it does not guarantee Pareto efficiency. However, the optimal solution of a weighted sum (where weights are not simultaneously null) objective function is always Pareto efficient. To derive a weight vector from the target solution, we first convert the approximation set of the pilot instance into a system of linear inequalities. Considering that the approximation set is composed of n objective-vectors ($\vec{Z}^1$, $\vec{Z}^2$, ..., $\vec{Z}^n$), the linear inequalities system can be defined as follows where the aim is to determine the values of $\vec{w} = (w_1, w_2, w_3, w_4, w_5)$:

$$\begin{cases} \vec{w}\vec{Z}^t \leq \vec{w}\vec{Z}^1 \\ \vec{w}\vec{Z}^t \leq \vec{w}\vec{Z}^2 \\ \quad\vdots \\ \vec{w}\vec{Z}^t \leq \vec{w}\vec{Z}^n \end{cases} \tag{7}$$

There is no guarantee that the system of linear inequalities has a solution because the fitness landscape is non-convex, meaning that no set of weights can be set to achieve some points in the front. Therefore, instead of finding a solution for the system, we aim to find a weight vector $\vec{w}$ that satisfies the largest number of inequalities. Hence, we define the problem of finding the best weight vector as the following MIP (mixed-integer programming) minimisation problem.

$$\text{Minimise} \sum_{j=1}^{n} \bar{x}_j \tag{8}$$

subject to

$$\vec{w}\vec{Z}^t - \vec{w}\vec{Z}^j \leq M\bar{x}_j \qquad , j = 1, \ldots, n \tag{9}$$

$$w_i \in (0,1], \bar{x}_j \quad \text{binary} \qquad , \begin{cases} i = 1, \ldots, 5 \\ j = 1, \ldots, n \end{cases} \tag{10}$$

Note that M is a very large constant.

The objective function given by Eq. (8) aims to find a weight vector $\vec{w}$ that minimises the number of linear inequalities in the system (7) which do not fulfill the condition $\vec{w}\vec{Z}^t \leq \vec{w}\vec{Z}^i$ expressed by constraint (9). Note that constraint (10) guarantees that zero cannot be chosen as a weight-value as we do not want criteria to be removed.

Finally, the weight vector $\vec{w}$ obtained from the MIP model is used in the objective function for the WSRP as given by Eq. (11).

$$\text{Minimise} \qquad \sum_{i=1}^{5} w_i Z_i \tag{11}$$

3.3 Euclidean Distances

We propose an alternative based on the Euclidean distances to the target vector. In essence, this is a method that considers all objectives as equally important. Hence, minimising the Euclidean distances alone does not guarantee Pareto efficiency. In order to reduce this issue, the proposed method consists of minimising the distances to the target vector for the objectives that are worse than the target. If the current distance for the objectives that are worse than the target vector is small ($< \varepsilon$), then the aim is to maximise the distances of the objectives that are better than the target vector.

Henceforth, the objective function in Eq. (12) becomes the objective function for the WSRP.

$$\text{Minimize} \begin{cases} z & ,\text{if} \quad z > \varepsilon \\ -z' & ,\text{otherwise} \end{cases} \tag{12}$$

where

$$z = \sqrt{\sum_{i=1}^{5} z_i} \tag{13}$$

$$z' = \sqrt{\sum_{i=1}^{5} z'_i} \tag{14}$$

$$z_i = \begin{cases} (Z_i - Z_i^t)^2 & ,\text{if} \quad Z_i > Z_i^t \\ 0 & ,\text{otherwise} \end{cases} \tag{15}$$

$$z'_i = \begin{cases} (Z_i - Z_i^t)^2 & ,\text{if} \quad Z_i \leq Z_i^t \\ 0 & ,\text{otherwise} \end{cases} \tag{16}$$

In summary, when the Euclidean distances of the objectives that are worse than the target vector are larger than the given parameter ε, the objective function consists of minimising the Euclidean distances (z). Otherwise, when $z \leq \varepsilon$, the objective consists of maximising the distances for the objectives that are better than the target solution (z'). Thus, if the solution has not reached the target, the objective function attempts to close the gap to the target. If the solution is close or better than the target, the objective function attempts to further improve it.

4 EXPERIMENTAL CONFIGURATION

We applied the proposed methodology to the WSRP datasets mentioned previously. The first instance of each dataset (A-01, B-01, C-01, D-01, E-01 and F-01) was selected as the pilot instance of its respective dataset. We used the approximation sets obtained as described next. Fifteen target vectors were randomly selected (uniformly distributed) from each approximation set and the same target vectors were used for the Derived Weight Vector (WV) objective function, the Euclidean Distances (ED) objective function, and the Chebyshev (CV) objective function.

For each remaining instance in each dataset, the Full-VNS algorithm proposed by Pinheiro et al. (2016) was applied to the fifteen selected target vectors from the pilot instance of its respective dataset using all three objective functions. For each target vector of each instance, we ran the Full-VNS algorithm eight times for one minute each. The results shown comprise the aggregate data.

Multiobjective algorithms struggle to find good approximation sets in combinatorial problems with many objectives (more than three) (Giagkiozis and Fleming, 2012). Hence, we resort to a tailored procedure to obtain an improved approximation set. Giagkiozis and Fleming (2014) state that most multiobjective algorithms can be classified as either Pareto-based or decomposition-based. This study utilises NSGA-II (Deb et al., 2002) as the Pareto-based algorithm and MOEA/D (Zhang and Li, 2007) as the decomposition-based. Thus, **for each problem instance** the approximation set was obtained as follows:

1. run both the NSGA-II and MOEA/D for one million objective evaluations on each possible bi-objective vector (Z_1, Z_2), (Z_1, Z_3), $\ldots$ (Z_4, Z_5);
2. run both the NSGA-II and MOEA/D for one million objective evaluations on each possible three-objective vector (Z_1, Z_2, Z_3), (Z_1, Z_2, Z_4), $\ldots$ (Z_3, Z_4, Z_5);
3. run both the NSGA-II and MOEA/D for one million objective evaluations on each possible four-objective vector (Z_1, Z_2, Z_3, Z_4), (Z_1, Z_2, Z_3, Z_5), $\ldots$ (Z_2, Z_3, Z_4, Z_5);
4. create an archive composed of the non-dominated solutions found in the previous three steps;
5. generate a population of individuals where half of the elements are randomly generated and the other half are randomly drawn from the archive built in the previous step;
6. run both the NSGA-II and MOEA/D four times each, for two million objective evaluations, using the initial population generated in the previous step and the five-objective vector; and
7. compile an approximation set with all non-dominated solutions found in all steps.

The number of solution vectors obtained for each pilot instance was as follows: 1162 for A-01, 1302 for B-01, 1830 for C-01, 2470 for D-01, 3689 for E-01 and 1500 for F-01.

5 EXPERIMENTAL RESULTS

First, we show the effectiveness of the derived weight vector obtained from the MIP model presented in Eq. (8)–(10). The effectiveness of a weight vector $\vec{w}$ is given by the percentage of solutions (in the pilot instance of the approximation set) in which $\vec{w}\vec{Z}^t \leq \vec{w}\vec{Z}^i$, $i = 1, \ldots, 5$. Hence, if the effectiveness is 100%, it means that the MIP model found a solution for the inequalities system in Eq. (7).

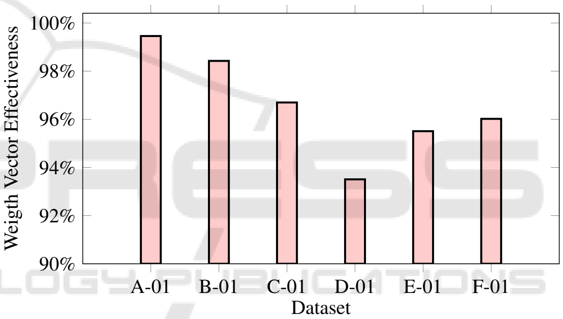


Figure 3: Average percentage of the solutions in the approximation set of each target instance such that $\vec{w}\vec{Z}^t \leq \vec{w}\vec{Z}^i$.

Figure 3 presents the results of the effectiveness analysis of the weight vectors obtained. In all pilot instances, the effectiveness surpassed 90% on average. Pilot instance D-01 presented the worst overall effectiveness, with an average of 93.5%, while pilot instance A-01 presented the most effective weight vectors averaging 99.5%. In all cases, the MIP model provided good weight vectors and these were applied to the WV objective function in Eq. (8).

Next, we show the results for each dataset in three charts. The *target achievement* chart displays the percentage of solutions, in the given dataset, that achieved the target value in each objective. The *gap to target* chart contains the average gap to the target solutions for the solutions that did not reach the target. Finally, the *overall comparison* chart displays the average quality of solutions where positive values indicate that, on average, the solutions found are better than the target solution and negative values indicate that the solutions are worse than the target solution.

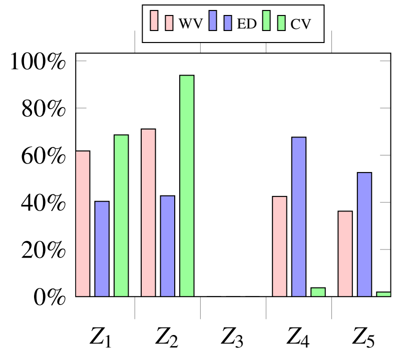


Figure 4: Dataset A – target achievement.

Figures 4–6 present the results of applying the VNS algorithm with the WV, ED and CV objective functions to the remaining instances of dataset A. Results comprise the average values of eight runs for each target vector of each problem instance for each objective function.

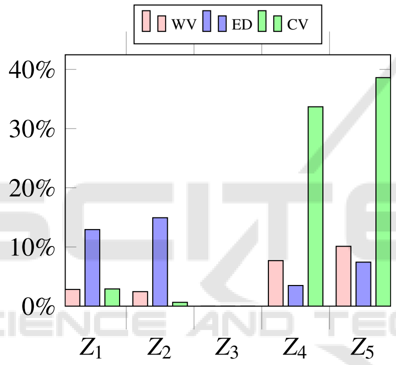


Figure 5: Dataset A – gap to the target.

In Figure 4, the percentage of achievement of individual target objective is shown. Note that there are no values for Z_3 because dataset A does not present preferred skills information. It is clear that the CV objective function presented better overall target achievement on Z_1 and Z_2, followed by WV, while the ED objective function is better on Z_4 and Z_5, whilst CV clearly underperforms.

Figure 5 shows that the average gap of the solutions that did not reach the target is always smaller than 16% for WV and ED, but high for CV. Also, in conformity with the previous chart, the CV objective function presented better values for the objectives Z_1 and Z_2 while the ED objective function obtained better values for Z_4 and Z_5. The CV objective function presented large gaps on the two latter objectives. Finally, in Figure 6, it is clear that on average solutions are at most 10% off the target for WV and ED, but the CV approach clearly presents a stronger bias towards Z_1 and Z_2 causing the quality of Z_4 and Z_5 to deteriorate.

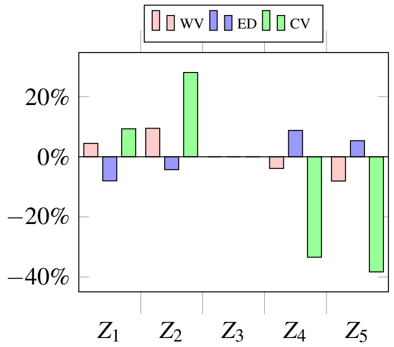


Figure 6: Dataset A – overall comparison.

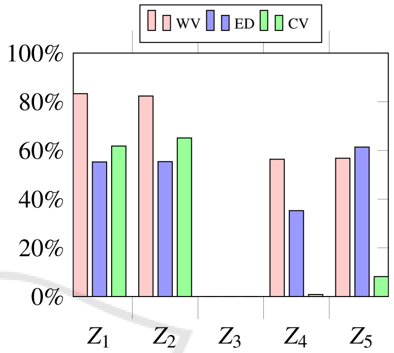


Figure 7: Dataset B – target achievement.

The results for dataset B are presented in Figures 7–9. In Figure 7, it is clear that the WV objective function obtains better target achievement than the ED on Z_1, Z_2 and Z_3. On Z_4 the ED has a small advantage. Also, when compared to dataset A, the average overall achievement of the targets is higher for WV and ED, but not for CV. The CV objective function shows competitive results on Z_1 and Z_2, but presents extremely low achievement rates on Z_4 and Z_5.

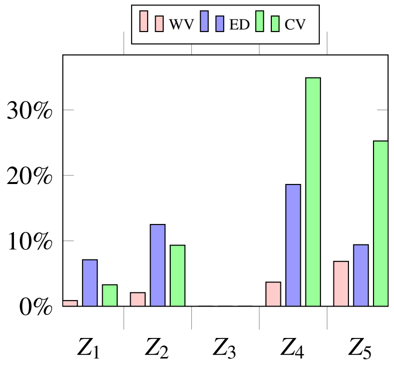


Figure 8: Dataset B – gap to the target.

Figure 8 shows that on all objectives, the WV objective function obtained smaller average gaps for the solutions that have not met the target. Conversely, the

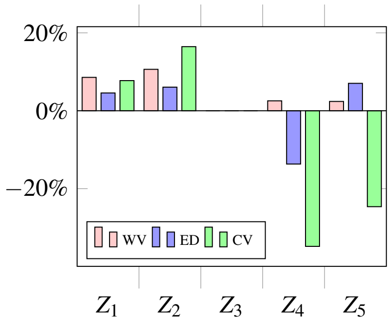


Figure 9: Dataset B – overall comparison.

CV objective function presents large gaps on Z_4 and Z_5. Figure 9 displays that on average, the objective function WV can provide improved objective-values on all objectives. However, the objective function ED presents improved results on Z_1, Z_2 and Z_5 only, failing to deliver good results on Z_4, while CV presents a similar behaviour as on dataset A: good performance on Z_1 and Z_2 but bad performance on Z_4 and Z_5. Thus, in this dataset, the WV objective function presents a better alternative to solve the problem.

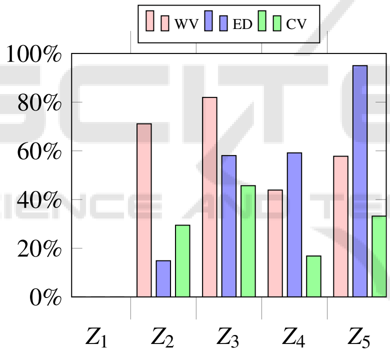


Figure 10: Dataset C – target achievement.

Figures 10–12 present the results for dataset C. Note that since that dataset does not have distances information, Z_1 is always zero. In Figure 10, it is clear both the objective functions WV and ED are balanced, with WV presenting better results for Z_2 and Z_3 while the ED has the edge on Z_4 and Z_5. However, the ED objective function presents deteriorated results on Z_2 and exceptional results on Z_5. The CV objective function underperforms compared to the other methods.

On Figure 11, it is clear that the ED objective function not only struggles to reach the target for Z_2, but it also misses it by 40% on average. This information is also reflected in Figure 12. Overall, the WV objective function presents a more consistent choice for this dataset as only on Z_4 the overall results are negative and the average overall performance is superior. Again, the CV objective function presents the worst results as it can be seen that for every objective there is a deficit in the overall quality obtained.

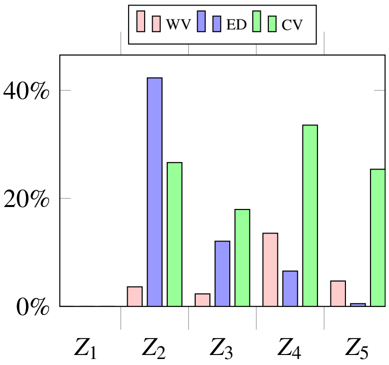


Figure 11: Dataset C – gap to the target.

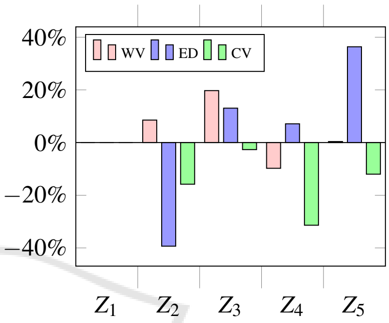


Figure 12: Dataset C – overall comparison.

Also, it is important to notice that this dataset presents a high variance in the sizes of the instances. Therefore, because C-01 is a large instance, the target solution simply cannot be achieved on the smaller instances of that dataset (C-02, C-04, C-05 and C-07).

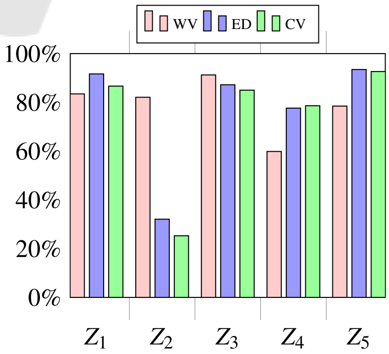


Figure 13: Dataset D – target achievement.

The results for dataset D are presented by Figures 13–15. In Figure 13 we see an overall increase on the target achievement when compared to the previous datasets (A, B and C). As with dataset C, the ED objective function displayed poor performance for Z_2

but had better performance on three other objectives (Z_1, Z_4 and Z_5). The CV objective function presented competitive results on this dataset.

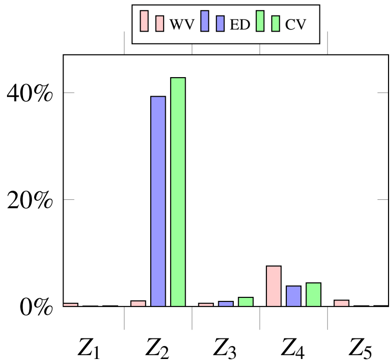


Figure 14: Dataset D – gap to the target.

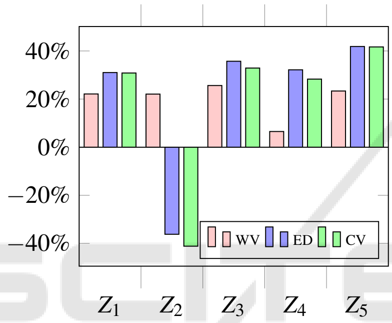


Figure 15: Dataset D – overall comparison.

Figure 14 shows that, except for the ED and CV objective functions on Z_2, the gap for the solutions that have not met the target is small. Also, except for that objective, the overall average results (Figure 15) show that all three objective functions present good results, with the ED surpassing the WV on all objectives except on Z_2 and the CV presenting results slightly worse than the ED. Still, the WV objective function manages to present a higher consistency than the other objective functions, and it also manages to present good results for all objectives.

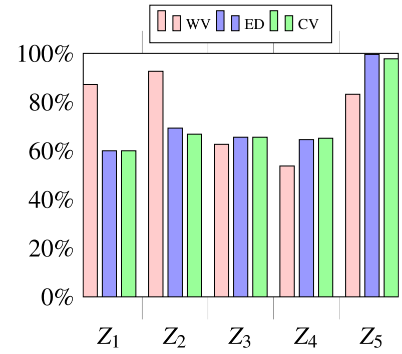


Figure 16: Dataset E – target achievement.

Dataset E results are presented in Figures 16–18. Like with dataset D, a high overall target achievement rate is shown (Figure 16). The WV objective function present exceptional results on Z_1 and Z_2, but fail to beat the other objective functions on the remaining objectives.

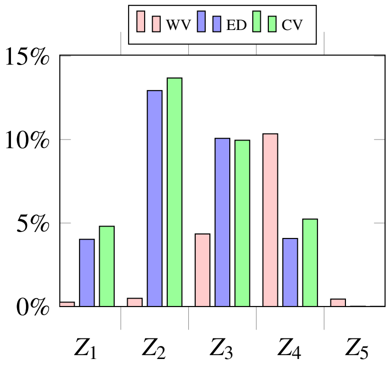


Figure 17: Dataset E – gap to the target.

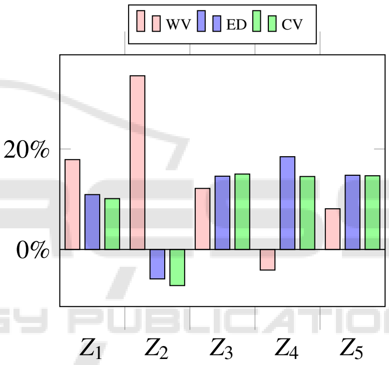


Figure 18: Dataset E – overall comparison.

It is clear from Figure 17 that the ED and CV objective functions present higher gaps to the target solutions. Figure 18 shows that all objective functions perform well on this dataset. The WV objective function presents better performance on Z_1 and Z_2, while the ED objective function presents better results on Z_4. The CV objective function presents the best results on Z_3.

Lastly, the results for dataset F are presented in Figures 19–21. The target achievement rates (Figure 19) show that all three objective functions perform well on this dataset. However, on Z_2 the WV objective function is noticeably better.

Figure 20 reinforces that claim as the gap for objective Z_2 is much larger for the ED and CV objective functions. Finally, Figure 21 shows that on average, the solutions provided by the WV objective function are better than the target solutions for all objectives, and, while the ED and CV objective functions present better results than the WV on Z_1, Z_3, Z_4 and Z_5, on Z_2 the average results are worse than the target values.

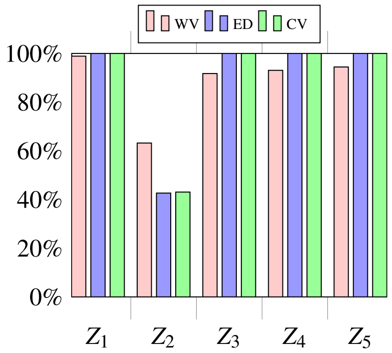


Figure 19: Dataset F – target achievement.

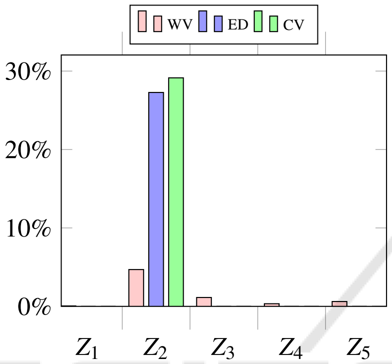


Figure 20: Dataset F – gap to the target.

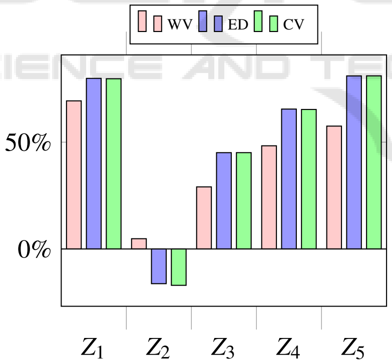


Figure 21: Dataset F – overall comparison.

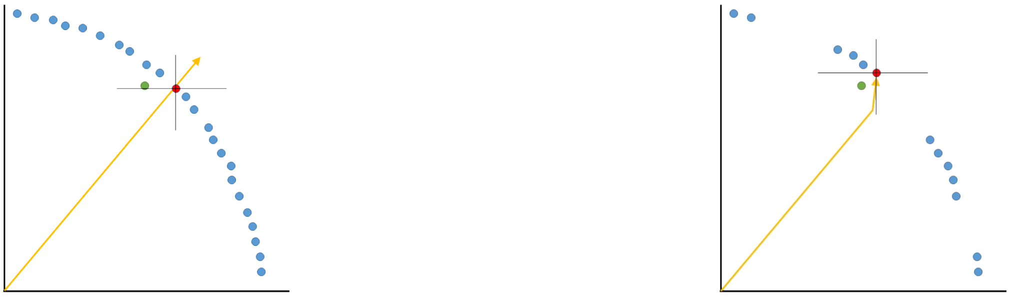
6 DISCUSSION

It was shown that both the WV (derived weighted vector) and ED (Euclidean distances) objective functions provide good results on the majority of the scenarios, with the WV objective function being slightly better than the ED objective function on average. Also, for the first three scenarios, the CV objective function clearly presented inferior results. It is noticeable that the results were better on the largest datasets D, E and F. In Section 2 we explained that the smallest datasets are also the ones with solutions well spread throughout the objective space. Also, it was mentioned that datasets D, E and F present unique fitness landscapes with several regions containing no solutions and several local relationships. Therefore, in the smallest datasets, if the objective function is not accurate enough (regarding reaching the target solution), the search can deviate from the region of interest more easily because the objective space is smoother. On the largest datasets, because the objective space contains gaps and local relationships, it may be enough for the search (driven by the objective function) to reach the same region as the target solution.

Take for instance the example in Figure 22. The blue dots represent the Pareto front, the green dot is the target solution, the red dot is the solution found by the search and the yellow arrow is the search direction given by the objective function. Note that the search direction is not accurate, hence it does not point towards the target solution (this is a valid assumption given that we are either estimating the weight vector or approximating the Euclidean distances). In Figure 22a, we have a well spread Pareto front and because there exists a solution aligned with the search vector (red dot), that solution is taken as the best solution. In the given example, the chosen solution is better than the target regarding the objective presented in the x axis, but it is worse than the target regarding the objective represented by the y axis.

In Figure 22b, we have a Pareto front with gaps and local relationships. The search direction is the same as before but in that direction, there are no solutions, hence the search seeks a closer solution that, in this case, is one that dominates the target solution (surpasses the target in all objectives).

A reason for the ED objective function to underperform when compared to the WV is the different ranges of objective-values. Distances and costs can range from about a few thousand units while the preferences are measured in dozens of units. This would lead us to believe that the ED objective function would provide better results on objectives Z_1 and Z_2 because those are the objectives with high ranges, but that does not happen, and the algorithm frequently provides better results on Z_3, Z_4 and Z_5. An explanation for this phenomenon lies in Eq. (15). When the search reaches local optima, it becomes very difficult for the algorithm to escape it because of the condition in that equation. If the search reaches local optima and a few (but not all) objectives fulfil the condition $Z_i > Z_i^t$, then, slightly disturbing the solution to escape local optima might result in other objectives fulfilling the condition and making the new solution much worse. In summary, the VNS used was tailored for the



(a) Well-spread Pareto front.

(b) Pareto front with gaps.

Figure 22: Example of an inaccurate objective-function on a Pareto front that is well-spread (a) and on a Pareto front with gaps (b).

WSRP using a weighted function. The ED objective function drastically changes the objective space and the algorithm loses performance. The phenomenon is even worse for the CV objective function.

Additionally, the overall target achievement rate for all objective functions was higher on the larger datasets. This happened because the size of those datasets was probably too large for the multiobjective algorithms to find near-optimal solutions while the VNS was able to provide improved solutions.

Nonetheless, it is clear that estimating the Pareto front for problem instances that have similar fitness landscape to the pilot instance in the same dataset, is an effective way to tackle the problem. While the multiobjective algorithms required up to four hours to obtain the approximation set for hepilot instance of a dataset, the VNS managed to find competitive solutions in five minutes. For the majority of the experiments, targets were achieved and the overall quality of results was high.

7 CONCLUSION

In this work we proposed a solving methodology to use efficient single-objective algorithms to solve a multiobjective WSRP problem. The methodology consists of solving an instance of the WSRP using multiobjective techniques (which are typically computationally expensive) to obtain an approximation set and then having the decision-maker to choose adequate target compromise solutions. Then, employ goal programming to solve other instances of the same dataset using the selected solution as target. Additionally, we used three different objective functions to guide the algorithms to reach the target. The first one is the well-known Chebychev approach. The second one attempts to obtain a weight vector from the target solution. The third on uses Euclidean distances to the target solution.

The proposed methodology can potentially be applied to other multiobjective problems where instances present similar fitness landscapes. In other real-world problems, instances of the same scenario have the same partial data as each instance is a repetition of the same situation for a different time frame. For example, instances of a give vehicle routing problem may have the same fleet every day and recurring deliveries. Similarly, instances of a given educational timetabling scenario may have the same facilities and teachers for different terms. Hence, the technique proposed i this paper can be of practical use in that type of problems. In addition, the multiobjective analysis technique proposed by (Pinheiro et al., 2015, 2017) offers an effective tool to evaluate whether using goal programming to solve the problem is applicable or not.

We showed that the methodology proposed here is capable of finding good trade-off solutions in a fraction of the time required by multiobjective algorithms to process the approximation set. Also, because of the strong performance by the chosen single-objective algorithm in this case, it was common for solutions to present better objective values than the target solution.

The fact that instances of the same dataset present similar fitness landscapes in the WSRP tackled here is not surprising because many features are recurring in these home healthcare planning scenarios. The same or very similar pool of workers is used for different days and many visits are recurring each day (patients need daily help to get out of bed, take baths, take medications, etc).

Proposed future work could investigate adaptive objective functions that can change the search direction according to the current state of the search and possibly reach improved results.

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