CRVM: Circular Random Variable-based Matcher
A Novel Hashing Method for Fast NN Search in High-dimensional Spaces

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Abstract: Nearest Neighbour (NN) search is an essential and important problem in many areas, including multimedia databases, data mining and computer vision. For low-dimensional spaces a variety of tree-based NN search algorithms efficiently cope with finding the NN, for high-dimensional spaces, however, these methods are inefficient. Even for Locality Sensitive Hashing (LSH) methods which solve the task approximately by grouping sample points that are nearby in the search space into buckets, it is difficult to find the right parameters. In this paper, we propose a novel hashing method that ensures a high probability of NNs being located in the same hash buckets and a balanced distribution of data across all the buckets. The proposed method is based on computing a selected number of pairwise uncorrelated and uniformly-distributed Circular Random Variables (CRVs) from the sample points. The method has been tested on a large dataset of SIFT features and was compared to LSH and the Fast Library for Approximated NN search (FLANN) matcher with linear search as the base line. The experimental results show that our method significantly reduces the search query time while preserving the search quality, in particular for dynamic databases and small databases whose size does not exceed 200k points.

1 INTRODUCTION

The Nearest Neighbour (NN) search is one of the most basic and time-consuming processes in many applications such as machine learning, multimedia databases, data mining, computer vision and image processing. In many of these applications, the data are typically represented as high-dimensional vectors. For example, a local feature extraction operation on an image will result in hundreds of dimensions. Consider a database of points $P \subset \mathcal{R}^d$ are given as $P = \{p_1, p_2, p_3, \ldots, p_n\}$. Now, the problem of NN search is to find the closest point in P to a given query point q using some similarity measure (e.g., the Hamming or Euclidean distance).

$$\begin{aligned}\mathrm{NN}(P,q) = \{p \in P \mid \forall p_i \in P \wedge p_i \neq p \\ : dist(p,q) \leqslant dist(p_i,q)\}\end{aligned}$$

The simplest solution for this problem is scanning the whole database, computing the distances from the query point to every point in the database sequentially yielding the point with the smallest distance. However, the running time of this operation is proportional to the number and dimensionality of data points. Therefore, this solution becomes dramatically inefficient for high-dimensional and large-scale databases (Har-Peled et al., 2012).

Over the last three decades, a lot of research has addressed the problem of finding efficient solutions for the NN search problem in high-dimensional spaces. The proposed solutions have in common that they organise the database contents in complex data structures (trees, graphs or hash tables) in such a way that querying a NN can be found without exploring the whole database. Unfortunately, the operation of storing new data into data structure as well as querying the data structure keeps growing exponentially in the number of dimensions. Recently, many papers have been published to overcome this problem by approximating the search for NN; instead of reporting the closest point to the query, it is allowed to report any point within a distance less than $(1+\varepsilon)$ times the distance between the query and its exact NN. The Approximated NN search (ANN) is sufficient for some applications and search can be performed in polynomial time w.r.t. the data dimensionality.

In this paper, a novel hashing method for the NN search in high-dimensional space is proposed. Our method is based on extracting several uncorrelated and uniformly-distributed Circular Random Variables (CRVs) from the data to be searched. These CRVs

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are then used to cluster the nearby data in the search space into sub-spaces. Seeking the NN of some query then means that only the corresponding sub-space has to be explored instead of the whole search space. This leads to a significant lower query time.

The remainder of the paper is organised as follows. In the next section, we review the related work that has addressed the problem of NN search in high-dimensional spaces. In Section 3, we introduce our method based on CRVs and show its application for SIFT features in Section 4. In Section 5, we evaluate our method on a number of datasets of SIFT features (with different sizes) and compare it with the LSH and FLANN matcher. Finally we conclude the paper in Section 6.

2 RELATED WORK

The simplest way to find the NN is a brute-force search which includes computing all the metric distances from a query point to every point in the dataset for finding the point with the smallest distance. This algorithm finds the exact NN, but it is inefficient for large datasets and high-dimensional spaces. The NN search can be accelerated by relaxing the problem and searching for the Approximated NN (ANN) instead of the exact one. In general, ANN search algorithms can be divided into three major categories: graph-based, tree-based, and hash-based algorithms (Muja and Lowe, 2009).

The graph-based algorithms construct a graph of nodes and edges where the nodes represent the data points and the edges connect each two neighbouring points; edge weights reflect how close two points are in the dataset. In the literature, many strategies to explore graphs have been published. In (Hajebi et al., 2011), the starting points are chosen randomly and a hill-climbing strategy is used. Another approach is shown in (Sebastian and Kimia, 2002). There, a few well-separated nodes are used as starting points and the graph is then explored in a best-first order. However, the graph-based algorithms suffer from the high computational cost for the construction of the NN graph structure. In spite of this, tree-based algorithms are founded on the recursive partitioning of the search space into sub-spaces. The most widely used algorithm in this class is the *k*-d tree (Bentley, 1975; Friedman et al., 1977). The *k*-d tree is a *k*-dimensional binary tree which partitions the space hierarchically resulting in a tree-based data structure. To search for the NN, the *k* split coordinates of the query point are used to determine the NN leaf node. Then, the linear search is performed to determine the closest point within the NN leaf. The k-d tree operates successfully in low-dimensional search space, but the performance degrades exponentially with an increasing number of dimensions. To overcome this restriction, many variations of the *k*-d tree approach were introduced by relaxing the NN search (Beis and Lowe, 1997; Arya et al., 1998). Silpa-Anan et al. (Silpa-Anan and Hartley, 2008) proposed the use of multiple randomised *k*-d trees, which are created by choosing the split dimensions randomly from among a few dimensions for which the data have a high variance. A constant number of trees are built using independent random choices. While searching for a query point, a single priority queue is maintained across all the randomised trees so that the search results can be ordered by the increasing distance to each bin boundary. Another example of a tree-based algorithms is the hierarchical *k*-means tree (Fukunaga and Narendra, 1975). The hierarchical *k*-means tree partitions the space hierarchically by using the *k*-means clustering algorithm instead of using hyper-planes as in the case of the *k*-d trees and its variants. The *k*-means clustering algorithm determines a set of *k* points randomly (called centroids). Then, *k* initial clusters are created by associating every data point with the nearest centroid. The centroids are updated at each iteration by setting them to the cluster means. This procedure is repeated until convergence is achieved.

The most popular hash-based method for ANN search is the Locality-Sensitive Hashing(LSH). The LSH scheme was proposed by Indyk et al. (Indyk and Motwani, 1998) for use in binary Hamming space and was later extended by Datar et al. (Datar et al., 2004) for the use in Euclidean space. The primary choice of constructing an LSH function for the Euclidean space is to project data points (represented as vectors in $\mathcal{R}^d$) along a randomly chosen line (identified by a random vector) which is segmented into intervals of the same width. Then, data points projected to the same interval are viewed as colliding in the hashing scheme and each interval is considered as a bucket. In (Andoni and Indyk, 2008), Andoni and Indyk used a set of hash functions to cluster data points that are nearby in the search space into the same bucket with high probabilities. The performance of hash-based algorithms highly relies on the quality of the hash functions used and the tuning of algorithmic parameters. Therefore, many papers addressing these issues have been published (e.g. (Kulis and Grauman, 2009; Wang et al., 2010; Bawa et al., 2005)). In (Muja and Lowe, 2009), Muja and Lowe compared many different algorithms for ANN search using datasets with a wide range of dimensionality; they found that two algorithms obtained the best performance, depending on the dataset and the desired precision. These algorithms used either the hierarchical k-means tree or the multiple randomised k-d trees. They developed a *Fast Library for ANN search* (FLANN) in high dimensional spaces. FLANN contains a collection of the best algorithms for NN search and an automatic mechanism for choosing the best algorithm and optimal parameters depending on the dataset.

In this paper, we will introduce a novel hashing method for NN search for high dimensional spaces. The idea is to extract a set of CRVs from the dataset, so that they are pairwise uncorrelated and tend to be uniformly-distributed. The CRVs are then used to hash the dataset points into one or few hash-trees. For each set of pairwise uncorrelated and uniformly-distributed CRVs, a hash tree can be constructed. Despite the simplicity of our method, it outperforms all the state-of-the art methods for ANN search in high-dimensional spaces specially for dynamic databases; it does not require a complex data structure, training phase, or parameter tuning operation.

3 CRV-BASED MATCHING

In this section, we outline the CRV-based matching algorithm. We start with defining the Circular Random Variables (CRVs).

Definition 1. *Let $\boldsymbol{x} = (x_0 \ \ldots \ x_{n-1})^T$ be a n-dimensional descriptor vector. We divide $\boldsymbol{x}$ into k segments $\boldsymbol{s}_0, \ldots, \boldsymbol{s}_{k-1}$, each of length l with $k = \lceil \frac{n}{l} \rceil$. Each segment $\boldsymbol{s}_i$ contains the following descriptor components $\boldsymbol{s}_i = \{x_{i \cdot l}, x_{i \cdot l+1}, \ldots, x_{i \cdot l+(l-1)}\}$. We associate to each segment the index of the maximum value as a random variable v_i with $v_i = \{j \in [0, l-1] \mid x_{i \cdot l+j}$ is the maximum value in $\boldsymbol{s}_i\}$.*

Once the random variables v_i are calculated for all segments, a hash tree with l^k leaves can be constructed, where the hash keys are determined by a polynomial of order $(k-1)$ according to

$$h(\boldsymbol{x}) = I = \sum_{i=0}^{k-1} l^i \cdot v_i. \tag{1}$$

l denotes the CRV period and k the number of the uncorrelated and uniformly-distributed CRVs that can be extracted from the descriptor vector.

For an example illustrated in Fig. 1, the data points are segmented with a period $l = 5$, each segment $\boldsymbol{s}_i$ gets a value v_i equal to the index of peak value in the respective segment.

Becaue neibouring points have similar descriptors, the CRVs computed from these descriptors tend to be identical, which lead to hash neibouring points into the same hash-tree leaf with high probability.

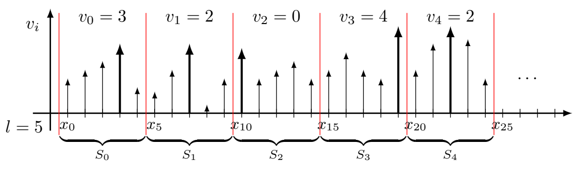


Figure 1: The extraction of CRVs from a data point represented as d-dimensional vector. In this example, the CRV period is 5.

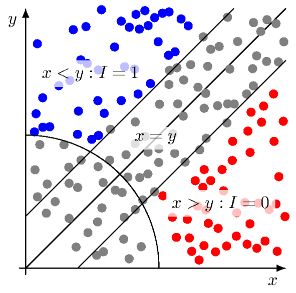


Figure 2: 2D classification results using one CRV v.

For the sake of simplicity, our idea is illustrated in Fig. 2 for two-dimensional search space. For this case, the whole vector is considered as one segment. Therefore the period of the random variables is $l = 2$ and only one CRV v can be extracted. If the abscissa of a feature is larger than the ordinate ($x > y$) we get $v = 0$ (red features in Fig. 2, otherwise $y > x$ yields $v = 1$ (blue dots in the figure). For boundary points, that at least one of whose segments has no dominate maximum (grey dots in Fig. 2), we get a boundary problem. In this case, boundary problem can be avoided by adding boundary points to both hash leaves.

In the general case ($l > 2$ and $k > 1$), the boundary problem is solved by considering not only the maximum indices that define the CRVs, but also the second maximum indices during determining the hash keys of boundary points, when the second to first maximum ratio greater than a certain threshold T. This ratio threshold can be used to make trade-off between search speed and accuracy. This consideration can be taken into account while storing or/and querying stages. To explain this, we assume that we have a dataset of 6D points, so we can segment each point vector into two segments of length 3. The dataset points will be stored in a hash tree of 9 leaves as shown in Fig. 3.

While querying, if both segments of the query point have dominate maxima (as the case of point A in table 1), then only one of the 9 leaves have to be searched. If one of the segments has no dominate maxima (as the case of points B and C), then we have to consider the maximum and the second maximum and hence two leaves have to be searched. In

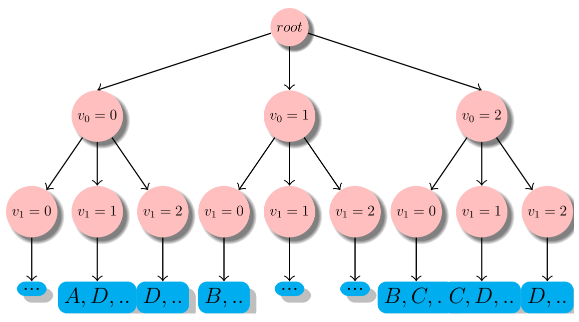


Figure 3: Hash tree for 6D dataset with two CRVs of period=3.

the worst case that both segments have no dominate maxima (as in the case of point D), then we need to explore 4 leaves out of 9. For better explanation, table 1 shows the maximum, second maximum and their indices of two segments for 4 example points, and Fig. 3 shows how they are stored(or queried)in the hash tree, when the ratio threshold T is set to 0.5.

One property of the above hash function is that features are mapped uniformly over the integer indices of Eq. 1 if (1) the used CRVs are uniformly-distributed; and (2) the CRVs are pairwise uncorrelated.

To examine whether the CRVs meet the uniformly-distributedness condition, their probability density functions (PDF) are estimated from a large dataset of points. The PDFs are computed by building histograms of the CRV values ranging between $[0, l-1]$. Once the PDFs are constructed, the χ^2 test is used to quantitatively evaluate the goodness fit to the uniform distribution.

To examine, whether the CRVs meet the pairwise uncorrelatedness condition, it is necessary to measure the dependence between each pair of them. The most familiar measure of dependence between two quantities is the Pearson product-moment correlation coefficient. For CRVs, the Circular Correlation Coefficient (CCC) is used to measure the association between two circular variables.

Once the CCC and the test statistic values are computed for all the extracted CRVs, a subset of them is chosen so that CCC and x values are as small as possible. An example with CCCs computed for SIFT features is shown in Figure 4. Based on the CCC, only uncorrelated CRVs are chosen to construct a hash tree and to hash dataset points into it. When querying a query point, its hash key is determined and the NN search is restricted only to points having the same hash key.

The above conditions ensure that the data are evenly distributed over all the buckets. This leads to the following observation.

Table 1: Maximum, second maximum and corresponding indices of 4 6D points.

point	v_1		v_2	
	1max-Id	2max-Id	1max-Id	2max-Id
A	100- 0	40-1	120-1	50-2
B	130-1	110-2	100-0	10-2
C	60-2	5-1	20-1	18-0
D	80- 0	79-2	60-2	59-1

Corollary 1. *For a database of size S, and a number of buckets $b = l^k$, we can calculate the size of every bucket as $B = \frac{S}{b}$. Therefore, compared to linear search, the query process can theoretically be sped up by a factor ranging from $f = (\frac{l}{2})^k$ (if all segments do not have dominate maxima) to $f = \frac{S}{B} = l^k$ (if all segments have dominate maxima).*

Generally, our method consists of three different stages. In the first stage as shown in Algorithm 1, a sample database is studied to figure out the statistical properties of the descriptor, such as the mean value of each dimension and the dependency between them. These properties are descriptor-dependent and do not depend on the database size. So this stage is needed only once when the kind of descriptor is changed. At this stage, each descriptor is divided into a certain number of segments. For each segment, the relative positions of the peaks over the whole sample database are saved and considered as a discrete CRV V. After that, their probability density functions and the correlation coefficients between them are estimated.

Algorithm 1: Computing pairwise uncorrelated and uniformly-distributed CRVs.

```
input : Descriptor vectors $D_i$, $l$ the period of
        a CRV
output: Uncorrelated and uniformly
        distributed CRVs $u_1, \dots, u_m$

foreach descriptor $D_i$ in DB do
    $[s_0 s_1, \dots, s_{k-1}] \leftarrow$ splitSegments($D_i, l$)
    foreach $s_j$ do
        $v_j \leftarrow$computeCRV($s_j$)
    end
end
foreach $v_j$ do
    PDF($j$) $\leftarrow$computePDF($v_j$);
end
foreach $v_j$, $v_i$ do
    $CCC(j,i) \leftarrow$ computeCCC($v_j, v_i$)
end
$[u_1, \dots, u_m] \leftarrow$ filter($[v_1, \dots, v_n]$, PDF($j$),
  $CCC(v_j, v_i)$)
Save positions of $S$ that corresponded to $u$
```

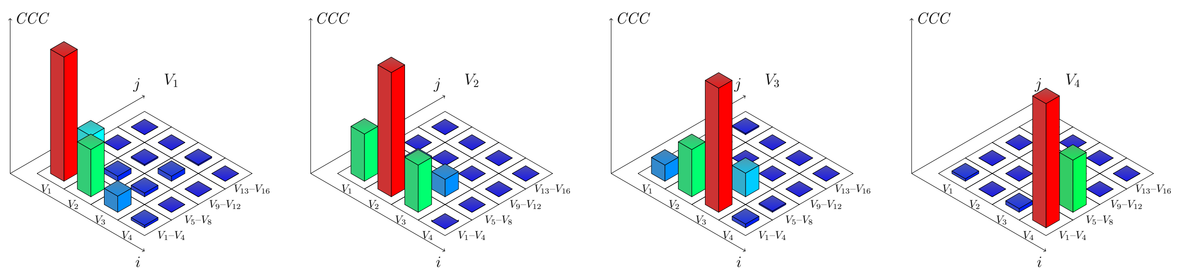


Figure 4: Plot of circular correlation coefficients for V_1 to V_4 between each two CRVs for SIFT features (out of 16).

Algorithm 2: Off-line: Build hash tree.

```
parent ← CreateHashTreeRoot();
foreach descriptor $D \in DB$ do
    $[s_0, s_1, \dots, s_{k-1}]$ ← SplitSegments($D, l$) ;
    for $j = 0$ to $k$ do
        $u_j$ ← computeCRV($s_j$);
        if $Node(u_j, j) = NULL$ then
            $Node(u_j, j)$ ←
              createChildNode($parent$)
        parent ← $Node(u_j, j)$;
    Store $D_i$ in hash node $Node(u_{k-1}, k-1)$
```

Then the CRVs are classified into one or several sub-groups, such that in each sub-group they are as far as possible uniformly-distributed and pairwise uncorrelated. The idea behind this is to make the hashing method useful regardless of the distribution in the database. The second stage is shown in Algorithm 2 and runs off-line to index the searched database. The sub-groups of the uniformly-distributed and pairwise uncorrelated CRVs are computed for each point, and used to hash dataset points into one or several hash trees (depending on the number of sub-groups).

The last, third process explained in Algorithm 3, is the same as the second one, but it is run on-line and applied to each query point in order to determine the address of its possible candidate neighbours in the hash trees.

In the Algorithms 1, 2, and 3 we assume that only one sub-group is found and hence only one hash tree is constructed and explored.

In the next section we will describe how this method can be applied to SIFT features, one of the most popular local descriptors used in computer vision and image processing applications.

4 THE CRVB MATCHER ON SIFT FEATURES

To apply CRV method on SIFT descriptors, we choose the CRV period equal to 8. Then, from the

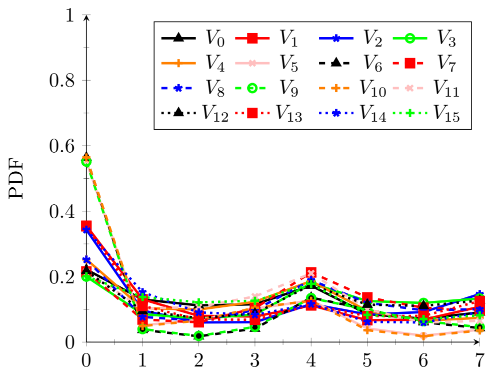
Algorithm 3: Online: Querying hash tree.

```
foreach descriptor $Q$ in the query image do
    Bucket ← $HashTreeRoot()$;
    $[s_0, s_1, \dots, s_{k-1}]$ ← SplitSegments($Q, l$);
    for $j = 0$ to $k$ do
        $u_j$ ← computeCRV($s_j$);
        if $Node(u_j, j) \neq NULL$ then
            Bucket ← $Node(u_j, j)$
        else
            break
    foreach descriptor $D$ in Bucket do
        match($Q$, $D$);
```

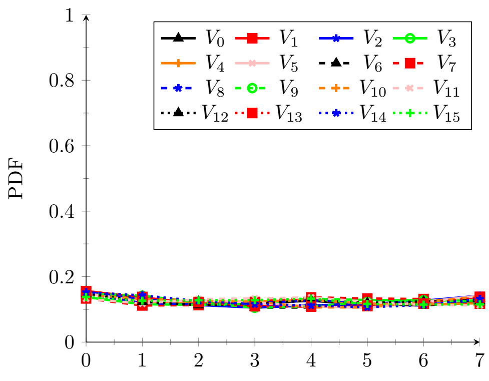
128-dimensional vector we can obtain 16 different CRVs. A subset of these CRVs have to be chosen, so that the CRVs meet the pairwise uncorrelated and uniformly-distribution conditions. To this end, we analyze the dataset of the SIFT features statistically. Statistically we found that the SIFT feature have a special signature, so that some components are always larger than some others. The signature of SIFT features is represented in Fig. 6 by the mean value of each component. For example, the 41th, 49th, 73th and the 81th components are always significantly larger than their neighbours.

The signature of SIFT features influences the distribution of proposed CRVs. In order to neutralise the signature effect, the SIFT descriptors must be weighted with a constant weight vector before computing of CRVs. The weight vector is calculated from signature vector by inverting its components. $S = [s_1, s_2, \dots, s_n] \Rightarrow W = \left[\frac{1}{s_1}, \frac{1}{s_2}, \dots, \frac{1}{s_n}\right]$. Fig. 5 illustrates the PDF functions of the CRVs before and after removing the feature signature effect. As shown in Fig. 5(b), after normalising the PDFs, all CRVs meet the uniformly-distributedness condition.

To study the dependence between the CRVs, we compute the CCC between each two CRVs. The estimated correlation coefficients are explained in Fig. 4. As becomes evident from Fig. 4, neighbouring CRVs in the descriptor are highly-correlated, whereas there are no or only very weak correlations between non-



(a) PDFs of CRVs



(b) PDFs of CRVs after removing signature effect

Figure 5: The probability density function of the CRVs before and after removing signature influence.

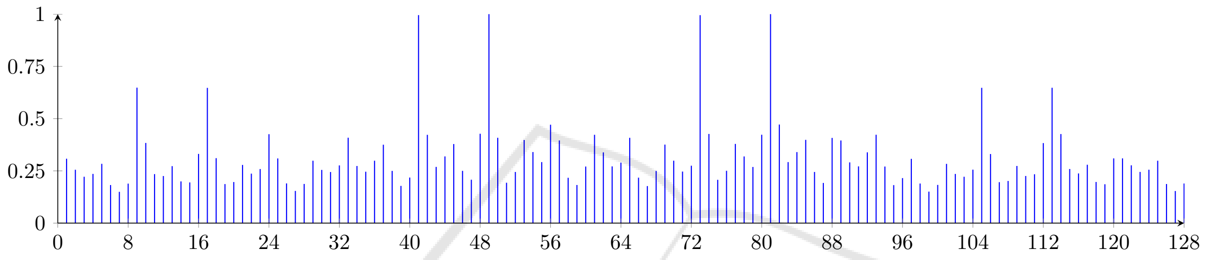


Figure 6: The signature of a SIFT descriptor; the mean values of the SIFT descriptor components were computed from a dataset of 100,000 features.

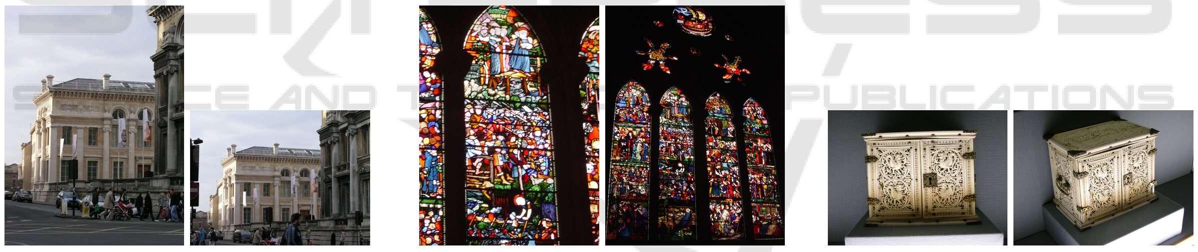
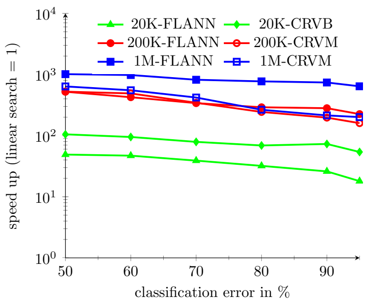
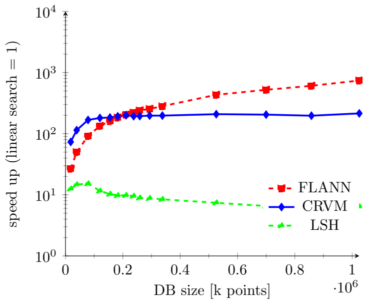


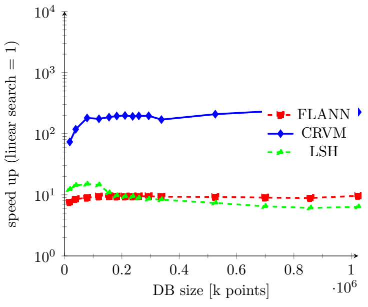
Figure 7: Some examples of images used for the experiments; the first image of each pair belongs to the query set, the second one is the corresponding image in the database.



(a) Trade-off between speed-up and classification precision for different database sizes.



(b) Static database



(c) Dynamic database

Figure 8: Speed-up and precision comparison between CRVM and FLANN; baseline is linear search for static and dynamic databases.

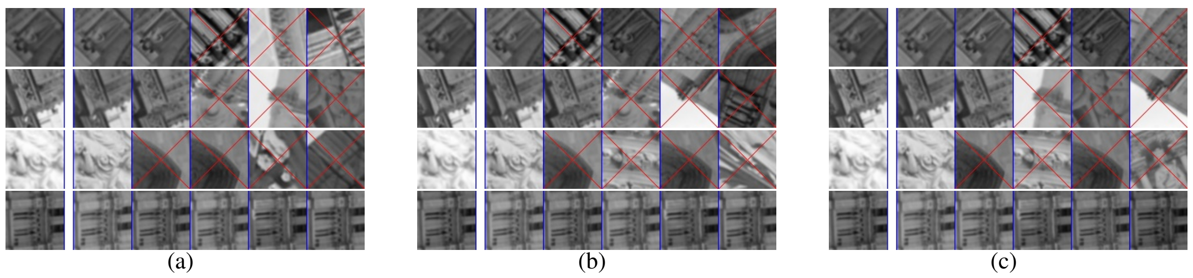


Figure 9: Some examples of patch matching results using (a)linear, (b) FLANN and (c) CRVB matchers; first patch of each raw is the query, while the others are its NNs from 100k patches. False NN are labelled with red X

contiguous CRVs. We omitted the other 12 diagrams showing the correspondences for the other CRVs. From the 16 CRVs we get two groups of pairwise uncorrelated and uniformly-distributed groups of CRVs: $g_1 = \{V_0, V_2, V_5, V_7, V_8, V_{10}, V_{13}, V_{15}\}$ and $g_2 = \{V_1, V_3, V_4, V_6, V_9, V_{11}, V_{12}, V^{14}\}$. From these two groups, two hash trees can be constructed.

5 EMPIRICAL EVALUATION

In this section, we compare the performance of our method with the state-of-the-art NN matcher FLANN and LSH. The experiments are carried out using the *Oxford Buildings Dataset* of real-word images (Philbin et al., 2007) and the *Learning Local Image Descriptors Data* dataset. The first dataset consists of about 5000 images. Among them there are several pairs that show the same scene from different viewpoints. 10 images of that pairs are deleted from the database and used as query set. Fig. 7 shows some examples of used pairs. The second dataset consists of 1024 x 1024 bitmap images, each containing a 16 x 16 array of corresponding patches. The corresponding patches are obtained by projecting 3D points from Photo Tourism reconstructions back into the original images (Winder et al., 2009).

We compare our method with FLANN and LSH in terms of both speedup over the linear search (base line) and the percentage of correctly sought neighbours (precision). Again, linear search is taken as the base line algorithm. To evaluate the performance of our method, two experiments were conducted. The first experiment was carried out with different database sizes (20K, 200K, 1000K) and with varying precision parameters. We measured the trade-off between the speed-up and the precision. For the FLANN matcher, the precision was adjusted by varying the FLANN respective parameters (number of trees and checks), whereas for the CRV matcher, the precision is changed by varying ratio threshold. The obtained results are shown in Fig. 8(a). As can be seen from the figure, our matcher outperforms the FLANN matcher for databases with a size of less than 200K features. In the second experiment, the performance is compared against FLANN and LSH for two different settings, a static and a dynamic database, respectively. In the static setting, the image database remains unchanged, while in the dynamic one, the database needs to be updated on-line by adding or deleting images. In this experiment, we keep the precision level at 90% and vary the size of database. Fig. 8 shows the obtained results for both database settings. Fig. 8(c) shows that in the case of a dynamic databases, the CRV matcher outperforms both the LSH and FLANN matcher for all database sizes significantly. It reaches speed-up factor of 20 over FLANN for database sizes up 100K features. The reason of this outcome can be explained by the FLANN matcher constructs a specific NN search index for a specific database; when the database is updated by adding or removing some data, the search index has to be updated as well, otherwise the search speed decrease. Conversely, the CRV matcher works independently from the database contents and its performance is not influenced by adding or removing data points. Fig. 9 shows some examples of matching results using brute force, FLANN and our proposed matcher. The three compared matchers return always the same KNN if correct corresponding patches are available. For incorrect KNN they returned often different patches.

6 CONCLUSIONS

In this paper, we presented a new hashing method for Nearst Neighbour (NN) search for high-dimensional spaces. The idea is to extract a set of Circular Random Variables (CRVs) from data vectors by splitting it in several segments of a certain length. A CRV is constructed for each segment and gets assigned the value of the relative position of the peak value in that respective segment. The length of the segment determines the period of a CRV. The CRVs are grouped together in such a way that in each group, they are all pairwise uncorrelated. The CRVs in each group are used to compute an integer hash value that index the data points in a one-dimensional hash table. In the query phase, the same process is applied on the query points and the obtained hash value is used to fetch only the database points that can be candidate neighbours for the query point. The proposed method was tested on a standard dataset of real-world images and compared with LSH and FLANN Matcher. The presented experimental results show that, in case of a static database, our CRV matcher is faster than FLANN for smaller databases (less than 200K features). For a dynamic database, the CRV Matcher is (10–20) time faster than FLANN depending on the size of the database.

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