

Optimal Control for Energy Management of Connected Hybrid Electrical Vehicles

Predictive Connectivity Compared to an Adaptive Algorithm

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Abstract: For fuel consumption and CO_2 emissions reduction, an optimal predictive control strategy for connected hybrid electrical vehicles is proposed, and evaluated through a comparison to an adaptive strategy. The predictive strategy relies on the future driving conditions that can be predicted by intelligent navigation systems with real-time connectivity. The theory proposed for such real-time optimal predictive algorithm is based on Pontryagin minimum principle, a mathematical principle that provides general solutions for dynamic systems optimization with integral criteria, under given constraints. In this work, the energy management problem is mathematically modeled as an optimal control one, and optimal solutions are synthesized. The predictive optimal real-time algorithm is confronted to the adaptive method. Both control strategies are simulated on different driving cycles. The simulation results show the interest of predictive approaches for hybrid electrical vehicles energy management.

1 INTRODUCTION

Hybridization has been introduced in car industry essentially to reduce fuel consumption, which leads to a reduction of CO_2 and pollutants emissions. The concept of hybridization is to add another (clean) energy source to the classical fossil fuel bringing another energy converter. In this paper, attention is focused on Hybrid Electrical Vehicle where besides the Internal Combustion Engine (ICE), the power-train is also mechanically connected to an electrical machine. One of the major interests of electrical hybridization is the reversible aspect of the energetic flow on the Electrical Machine Actuator (EMA). The electrical machine can convert the vehicle kinetic energy into electrical energy stored in the battery. In a hybrid vehicle, the combination of two energy sources creates a free energetic node. Regardless of the drivers behavior, the combination ratio of the two energy sources is a new degree of freedom which must be set by real-time embedded control.

The question that constitutes the energy management problem of this paper is : How can the fuel en-

ergy, consumed by ICE, be minimized in a way to obtain a certain electrical energy balance over the trip? In other words, knowing the future path, the objective is to use electrical on-board energy to minimize the fuel consumption over the path, and at the same time, retrieve a targeted state of charge of the battery. A problem that is also referred to as the TorqueSplit problem.

Researches on this energy management topic started many years ago (Sciarretta and Guzzella, 2007). Two approaches have been adopted. First, heuristic methods for real-time use, such as rule based methods, fuzzy logic (Caux et al., 2010), stochastic strategies or other strategies such as the equivalent consumption minimization strategy (Sciarretta et al., 2004). Although this type of methods has the advantage of being compatible with real-time use, it does not guarantee a rigorous optimal solution. On the other hand, model-based methods using optimal control theory and dynamic programming algorithms, in off-line computation (Tribioli and Onori, 2013), (Delprat et al., 2003), can guarantee optimal solutions without being real-time compatible.

In strategies based on optimal control theory, a calibration of the optimization parameters is necessary. Existing real-time embedded strategies perform this calibration either by using adaptive heuristics (Kim et al., 2011), (Kermani et al., 2011), or by recalculating the optimal control periodically using short term prediction, as in MPC (Model Predictive Control) methods (He et al., 2015). This on-line adaptation of the optimization parameters aims to avoid unexpected scenarios which can yield high fuel consumption. The drawback of these solutions is an alteration of the time-global optimization over the whole trip. They can only assure sub-optimal solution over separated parts of the trip.

The purpose of this paper is to show the interest of a predictive model based optimal control through a comparison with an adaptive method. The gain from such predictive method is evaluated in the context of a connected vehicle which is able to acquire data about the future circumstances. Both predictive and adaptive methods use the same on-line structure: the minimization of a weighted criteria of Fuel and Electrical Power. The difference between these two methods is the calibration of the weight coefficient. While the adaptive method uses a rule based strategy to compute this coefficient, the predictive method uses information about the future path in a shooting algorithm to determine the same coefficient.

Devices like Electronic-Horizon can be used to provide a robust estimation of the necessary information about the future trip. They can provide dynamic information about red light stops, speed limitations, traffic information etc... Using an embedded drivers model (that could be shaped by AI algorithm), an estimation of torque demand and speed for the whole future trip can be deduced. This gives the opportunity of long prediction horizons, unlike existing MPC methods where the prediction horizons are in an order of magnitude of 10s. The prediction part is not in the scope of the paper. We consider the vehicle equipped with such a system, that provides reliable data.

In section 2, a mathematical modelling of the energy management problem into an optimal control problem is exposed. Then its solution is developed into a control structure that characterizes the two compared methods. Section 3 opposes predictive and adaptive methods by exposing the core practical difficulty of the problem which is the calibration of the control structure. The predictive method is detailed, and its differences with the adaptive heuristics are emphasized. An evaluation of the methods by simulation results on different driving cycles is presented in section 4. The conclusions of the paper are in section 5.

2 PROBLEM MODELLING

2.1 Hybrid Electrical Vehicle: 48V P2 Architecture

The vehicle considered is a 48V P2 Hybrid Ford Focus. The P2 architecture has the advantage of engine-off electrical traction. The ICE is connected to the power-train through a clutch that can be opened when the engine is not required. The power-train is connected to an electrical machine using a transmission belt. A 48V Battery feeds the electrical machine, and also an on-board 12V electrical net as shown in Figure 1.

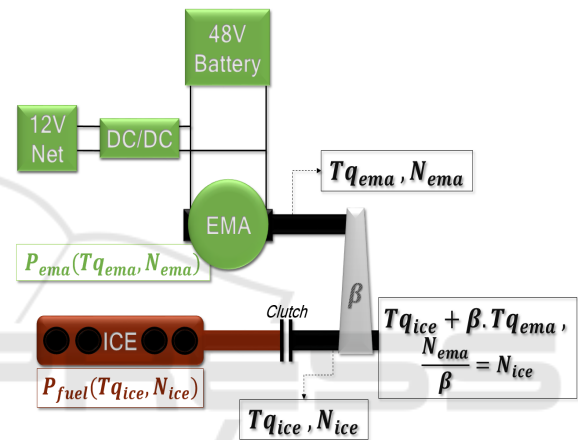


Figure 1: Hybrid 48V P2 architecture.

The two electrical circuits are interfaced by a DC/DC converter controlled using a PI controller. The strategy proposed will not control this low voltage net. The power demand from the 12V net is written P_{12V} .

ICE consumes a power given by $P_{fuel}(Tq_{ice}, N_{ice})$ to supply a torque Tq_{ice} at the speed N_{ice} , a power which is proportional to the necessary amount of fuel. EMA takes $P_{elec}(Tq_{ema}, N_{ema})$ from the battery to supply Tq_{ema} at the speed N_{ema} . P_{fuel} and P_{elec} are functions depending on respectively ICE and EMA torque and speed. These functions are available as numerical maps from experimental testing of ICE and EMA over different operating points of the engines. The electrical power is transmitted to the clutch using a transmission belt. The belt ratio is considered to be constant β . The mechanical transmission equations are:

$$N_{ema} = \beta \cdot N_{ice} = \beta \cdot N_{clu} \quad (1)$$

$$Tq_{clu} = Tq_{ice} + \beta \cdot Tq_{ema} \quad (2)$$

The battery energy is characterized by its State of Charge (SoC). SoC dynamic is:

$$\frac{dSoC}{dt} = \frac{I_{bat}}{Q_{max}} \quad (3)$$

Where I_{bat} is the input current of the battery, and Q_{max} is its maximum capacity. Knowing that: $P_{bat} = I_{bat} \cdot V_{bat} = -(P_{elec} + P_{12V})$, it can be concluded that:

$$\frac{dSoC}{dt} = -\frac{P_{elec} + P_{12V}}{V_{bat} \cdot Q_{max}} \quad (4)$$

Knowing the speed profile and the torque demand from the driver over a future trip, the objective is to minimize the fuel consumption under a constraint on the SoC final value. The targeted SoC value is denoted SoC_{tg} .

2.2 Optimal Control Model

The equations developed in the subsection above are already sufficient to synthesize an optimization problem that describes the needs of the energy management problem:

$$\begin{aligned} \min_{Tq_{ice}, Tq_{ema}} \quad & J = \int_0^T P_{fuel}(Tq_{ice}, N_{ice}) dt \\ \text{s.t.} \quad & SoC(T) = SoC_{tg} \\ & Tq_{clu} = Tq_{ice} + \beta \cdot Tq_{ema} \\ & \frac{dSoC}{dt} = -\frac{P_{elec} + P_{12V}}{V_{bat} \cdot Q_{max}} \end{aligned} \quad (5)$$

where T corresponds to the trip duration.

The battery voltage depends on SoC. Moreover, $V_{bat}(SoC)$ is non-linear, which makes the battery model difficult to manipulate and the analytic resolution of this problem becomes difficult. Two simplifications are introduced in order to deal with a more affordable optimization problem for real-time use.

First, the power demand from the low-voltage on-board net is considered as a constant, and null in the following development $P_{12V} = 0$. The other assumption concerns the voltage of the battery, which is considered constant $V_{bat} = 48V$. Although the second assumption is not realistic, it has been shown in previous studies that the effect of the internal dynamic of the battery can be negligible for hybrid electrical vehicles energy management, (Sciarretta and Guzzella, 2007) and (Steinmayer and del Re, 2001).

In addition to the two simplifications, equation (2) shows that only one of the two torque set-points can be used as decision variable or command. The SoC dynamic equation can then be rewritten as a function of the ICE torque:

$$\frac{dSoC}{dt} = -\frac{P_{elec} \left(\frac{Tq_{req} - Tq_{ice}}{\beta}, N_{ema} \right)}{V_{bat} \cdot Q_{max}} \quad (6)$$

Accordingly, the optimization problem is rewritten :

$$\begin{aligned} \min_{Tq_{ice}} \quad & J = \int_0^T P_{fuel}(Tq_{ice}, N_{ice}) dt \\ \text{s.t.} \quad & SoC(T) = SoC_{tg} \\ & \frac{dSoC}{dt} = -\frac{P_{elec}(Tq_{ice}, N_{ema})}{V_{bat} \cdot Q_{max}} \end{aligned} \quad (7)$$

The control variable is the ICE torque $Tq_{ice}(t)$, and the state to control is SoC .

2.3 Solution of the Optimal Control Problem

In optimal control theory, the purpose is to take a system from an initial state A to a final state B, in a way that minimizes an integral criterion. In this case, the state would be the state of charge of the battery.

Optimal control theory (Pontryagin Minimum Principle) is applied to the problem (7). SoC is the state to control from an initial point $SoC(0)$ to a final point SoC_{tg} .

Pontryagin principle asserts that the optimal solution minimizes the Hamiltonian of the problem at each instant. The Hamiltonian is constructed introducing the co-state variable λ which is a function depending on time that verifies the following differential equation:

$$\frac{\partial H}{\partial SoC} = -\frac{d\lambda}{dt} \quad (8)$$

The Hamiltonian of the problem is then written as follows:

$$H_{\lambda}(Tq_{ice}) = P_{fuel}(Tq_{ice}, N_{ice}) - \lambda \cdot \frac{P_{elec}(Tq_{ice}, N_{ema})}{V_{bat} \cdot Q_{max}} \quad (9)$$

From the differential equation (8) of the co-state variation, we obtain:

$$\frac{d\lambda}{dt} = 0 \quad (10)$$

Hence, λ is constant.

According to optimal control theory, the optimal Tq_{ice} minimizes at each instant the Hamiltonian:

$$Tq_{ice}^{opt} = \arg \min_{Tq_{ice}} (H_{\lambda}(Tq_{ice})) \quad (11)$$

At instant t , knowing the value of N_{ice} and N_{ema} , and the functions P_{fuel} and P_{elec} , the solution is found by minimizing the Hamiltonian (11). Only the co-state variable λ needs to be determined. Its value will determine the admissibility of the solution, or in other words the final value of SoC .

As a conclusion of these developments, we deduce that the optimal control theory applied to the hybrid energy management problem yields a very intuitive solution. The result is to minimize, at each instant, a criteria involving the two electrical and fuel powers, with a pondering coefficient λ . Other types of strategies do not directly use optimal control and Pontryagin Maximum Principle. However, the most of them minimize the same hybrid criteria by using a weight coefficient that is adapted following the vehicle state. It is the case of the adaptive method that we chose to compare to the predictive optimal control approach.

In the following, we expose the two approaches of calibration: predictive and adaptive.

3 ENERGY MANAGEMENT CALIBRATION: PREDICTIVE VS ADAPTIVE

3.1 Predictive Shooting Algorithm to Find λ

The predictive method proposed in this paper, is a continuity of the optimal control theory. Optimal control problems often imply high non-linearity which makes the integration of the differential equations impossible. The classical solution to this problem is to develop a shooting algorithm which is the predictive approach proposed. It consists of doing iterations of numerical integration of the system, until the targeted SoC is reached with a satisfactory error.

It is necessary for this method to design a function that, by integrating its equations, simulates the vehicles behavior over some trip with some value of λ . The simulation is based on the same model as the one used to synthesize the optimal control problem in the first section. Such function is called a shooting function. It takes for input, the predicted information from the future path, as well as an entry value of λ .

The function computes the optimal Tq_{ice} by minimizing the expression (2.3) at each instant. Then the optimal Tq_{ema} is deduced by fulfilling the torque demand on the clutch, which allows the integration of $P_{elec}(Tq_{ice}, N_{ema})$ over the time of the trip, to find the corresponding final SoC .

The value of λ determines the value of the final state SoC_{tg} . A random value of λ would lead to a random final value of the SoC . The idea of the shooting algorithm is to search for the constant value of λ which renders the final state of charge targeted, by iterating the simulation of the system, using for each

iteration, a λ that is updated by the result of the previous simulation (see Figure 3).

This co-state can be seen from another point of view: the minimization of the Hamiltonian (2.3) can be understood as a weighted minimization of fuel energy and electrical energy. λ is then the weight of the electrical energy in the minimization. This perspective of the problem gives an idea of the adequate optimization method for the shooting algorithm to find λ : the final state of charge grows as λ increases, which means that the function, for which the shooting algorithm is finding a zero, would be monotonous.

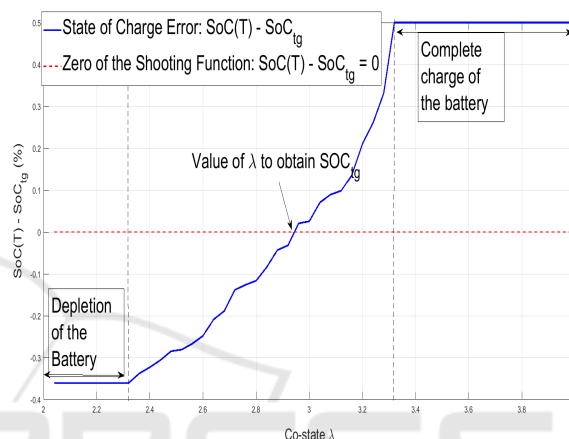


Figure 2: Variation of the SoC error: 50% SoC target on an NEDC driving cycle.

As an example, on the NEDC driving cycle, with $SoC_{tg} = 50\%$, an increasing, monotonous variation is noticed, as illustrated in Figure 2. The final value of SoC increases when λ increases.

The assumption that $SoC(T) = f(\lambda)$ is a monotonous increasing function, allows the reduction of the number of simulations to find the appropriate λ . Finding the λ that guarantees SoC_{tg} is equivalent to finding the zero of the function $SoC(T) - SoC_{tg} = f(\lambda) - SoC_{tg} = g(\lambda)$. Thanks to its monotonicity we can use simple, fast, and efficient ways to find its zero. The method used here is a simple bisection algorithm.

A shooting algorithm is designed under Matlab® as in Figure 3. The algorithm uses the shooting function previously designed to perform simulations using a value of λ as an entry. The search of the appropriate λ is done using bisection.

As an output of the shooting algorithm, the value of λ that reaches the targeted final SoC is returned.

The embedded shooting algorithm is ran at the beginning of a trip. It takes the predictions that connectivity provides and returns the optimal λ which will be used in real-time Hamiltonian minimization during the trip.

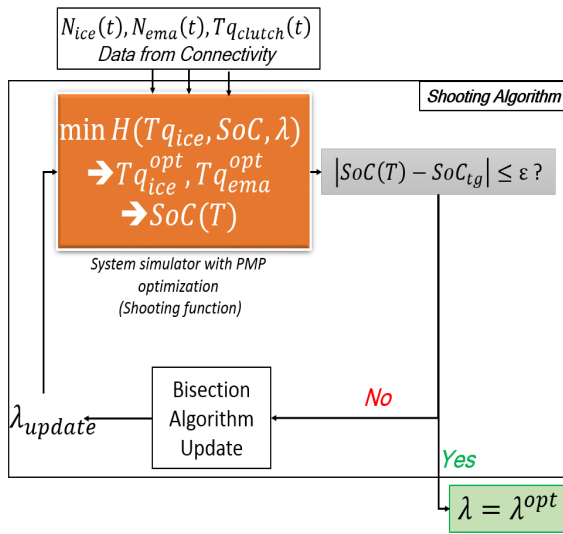


Figure 3: Shooting algorithm scheme.

3.2 Adaptive Costbased Tuning of λ

The adaptive method, that we chose to compare to the predictive optimal control, has been developed by Continental Automotive engineers. It is a rule based strategy for Torque Split Energy management, and does not use future predictions. This method is based on an instantaneous minimization of a weighted criterion combining electrical energy and fuel energy. The weight coefficient λ is tuned on-line based on the states of the vehicle, essentially the battery state of charge. The coefficient varies with the state of charge by using heuristic rules.

Although the method is not directly derived from optimal control theory, it uses the same online control structure: it is an on-line minimization of a pondered criteria. The practical advantage of this method is its independence from connectivity and prediction.

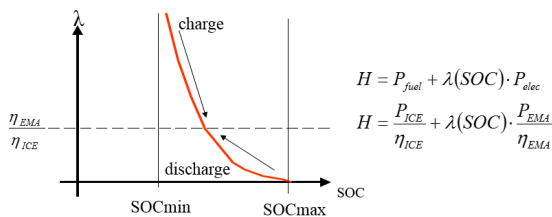


Figure 4: Calibration of the criteria using the CostBased adaptive method.

λ is computed on-line using the value of $\frac{\eta_{EMA}}{\eta_{ICE}}$. Where η_{EMA} and η_{ICE} are respectively the EMA and ICE efficiency. The electrical machines efficiency varies when the battery SoC varies. This variation determines at each instant the value chosen for λ .

The value $\frac{\eta_{EMA}}{\eta_{ICE}}$ represents a cost of electrical power in terms of fuel power. The CostBased adaptive method computes it off-line for all operating points of the engine and all driving modes. The values of $\frac{\eta_{EMA}}{\eta_{ICE}}$ are stored in maps and used in real-time to determine λ .

The CostBased method is the most efficient method developed by Continental Automotive for the TorqueSplit problem, in terms of CO_2 emissions and fuel consumption performance. It has been chosen specifically for that reason as an adaptive method to be compared to the predictive method proposed in this paper, in order to demonstrate the possible gain from such predictive methods.

4 SIMULATIONS ON FORD FOCUS 48V P2 MODEL: COMPARISON OF PERFORMANCES

The model of Ford Focus P2 48V hybrid vehicle, furnished by the CostBased method developers, is used for the next validation. It is representative of all the energetic flows on the vehicle which are not taken into account in the optimization model developed in the first section (such as thermal dynamics, mechanical transmission losses,...). Note that P_{12V} is not constant in this reference model. The optimal control applied here uses a constant value of $P_{12V} = 280W$ synthesized as mean value from NEDC driving cycle simulations.

The reference vehicle model was developed under the modeling software AMESim[®]. The control model using predictive and adaptive approaches is developed under Matlab/Simulink[®]. Therefore, a co-simulation is performed, a real-time exchange of data between AMESim and Simulink is implemented. The simulation is performed on multiple driving cycles:

- NEDC (New European Driving Cycle): CO_2 emissions certification cycle of 11km.
- WLTP (Worldwide harmonized Light vehicles Test Procedures): CO_2 emissions certification cycle of 23km.
- RCC (Regensburg City Cycle): An urban driving cycle recorded in Regensburg/Germany of 18km.
- Falkenstein Cycle: An extra-urban driving cycle recorded in Regensburg/Germany of 59km.

The simulation results are summarized in Figure 5. The predictive shooting algorithm gives for

	Predictive Shooting Algorithm			Adaptive CostBased Method		
	Fuel Consumption [g]	Overall SoC variation [%]	Equivalent CO ₂ emissions [g/km]	Fuel Consumption [g]	Overall SoC variation [%]	Equivalent CO ₂ emissions [g/km]
NEDC	327	+4	92.38	332	+0.5	94.67
WLTP	869.32	+2.16	117.79	872.17	+0.72	118.33
Falkenstein	2279.94	-0.4	123.17	2297.6	+0.85	123.97
RCC	584.19	+0.74	100.15	591.3	-0.16	101.55

Figure 5: Simulation Results using 48V P2 Hybrid Ford Focus targeting a final SoC of 50% : Predictive TorqueSplit vs Adaptive TorqueSplit.

Predictive Shooting Algorithm λ_1		Predictive Shooting Algorithm λ_2		Predictive Shooting Algorithm λ_3		Predictive Shooting Algorithm λ_4		Adaptive CostBased Method	
Overall SoC Variation [%]	Equivalent CO ₂ [g/km]	Overall SoC Variation [%]	Equivalent CO ₂ [g/km]	Overall SoC Variation [%]	Equivalent CO ₂ [g/km]	Overall SoC Variation [%]	Equivalent CO ₂ [g/km]	Overall SoC Variation [%]	Equivalent CO ₂ [g/km]
-4.14	151.92	-0.87	151.15	+1.27	150.72	+5.61	151.09	-3.9	152.77

Figure 6: Simulation Results using 48V P2 Hybrid Ford Focus targeting a final SoC of 50% with the predictive approach using slightly different Predictions.

each cycle a better result in fuel consumption. Although the adaptive CostBased method renders a better state of charge over the Falkenstein cycle, the CO₂ equivalent is always better with the predictive shooting algorithm.

Note that the adaptive method is always closer to the target state of charge while the predictive method leads to more error. This is due to the fact that the predictive method is a pure application of the optimal control theory which is an open loop control. The calibration of λ is calculated at the beginning of the trip and is not updated on-line. This λ was calculated using a prediction which is not 100% reliable. The distance from the targeted final SoC reflects the accuracy of the prediction: a more accurate prediction gets closer to SoC_{Tg} . On the other hand, the adaptive method keeps track of the SoC on-line and controls λ to stay close to the targeted value.

However, even if the prediction is not completely accurate, and the final state of charge obtained by the open loop predictive approach deviates from the targeted value, the predictive approach still has a better CO₂ equivalent than the adaptive method. Simulations on an urban cycle of 5km, with slightly different predictions that render slightly different values of λ yield the results in Figure 6.

Moreover, the SoC trajectories reveal an identifiable behavior of the predictive method: more battery

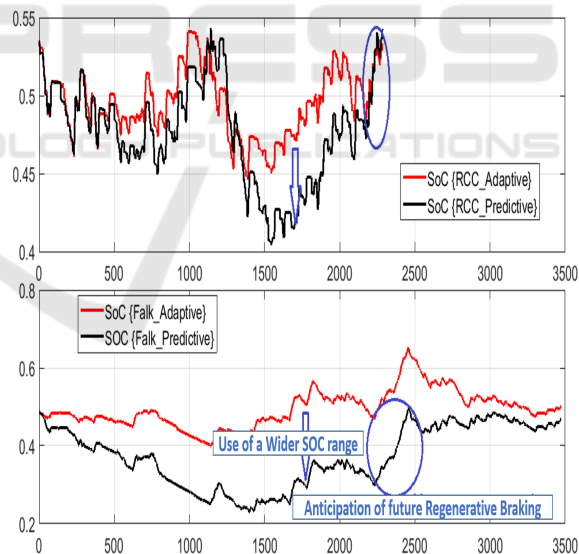


Figure 7: SoC trajectories (SoC vs Time) over the Falkenstein cycle: Predictive anticipation of future recoverable energy VS Adaptive method.

energy is used and the SoC trajectory is wider. This behavior is what is expected from the use of prediction: the use of more electrical energy that is known to be available in the future. This behavior is illustrated in Figure 7, in which are illustrated the SoC trajectories from RCC and Falkenstein cycles using predictive and adaptive methods. The results presented in

	Predictive Shooting Algorithm				Adaptive CostBased Method		
	CO_2 saved	Recuperated Electrical energy [Wh]	Pure Electrical Driving [Wh]	Hybrid Driving Electrical Energy [Wh]	Recuperated Electrical energy [Wh]	Pure Electrical Driving [Wh]	Hybrid Driving Electrical Energy [Wh]
NEDC	-2,1%	199	218	18	194	82	24
WLTP	-0,3%	455	306	199	424	155	137
Falkenstein	-0,4%	656	144	178	617	114	171
RCC	-1,4%	524	294	80	484	189	158

Figure 8: Simulation Results using 48V P2 Hybrid Ford Focus targeting a final SoC of 50% : Predictive TorqueSplit vs Adaptive TorqueSplit. Analysis of electrical Energy usage.

the table in Figure 8 confirm that tendency, and show that more electrical energy is used and recovered by using the predictive approach.

On a local point of view, this tendency is often translated into a longer pure electrical driving phases, especially in weak acceleration phases. It also implies longer use of electrical energy for the vehicle starting phases, as illustrated in Figure 9.

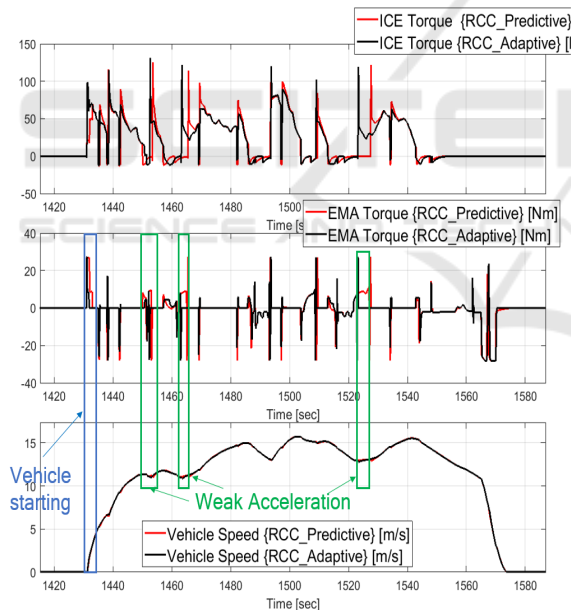


Figure 9: Local behavior analysis in a part of the Regensburg City Cycle.

Nonetheless, the predictive shooting algorithm approach is not a rule based method and does not guarantee the same behavior in every situation. It mathematically guarantees a time-global optimum for the model based optimal control synthesized from the TorqueSplit problem, without being predictable in terms of local behavior. However, it is safe to assume that the general behavior of anticipation will always be present, since it represents the most important in-

terest of using predictions about the future path.

The predictive optimal control approach proves itself to be more efficient than the adaptive one, in terms of fuel consumption and CO_2 emissions. The adaptive method has the advantage of a closed loop on-line control, that makes sure to always reach the targeted final SoC with a high precision. In addition, it has a stable behavior, that is reproducible for every driving cycle. The predictive approach adapts its behavior to reach the targeted final SoC with the less fuel possible, by using the results of optimal control theory and the prediction of the road. Being an open loop method, this method has the flaw of not reaching the exact SoC target, but it still can guarantee an optimal equivalent CO_2 . However, if the deviation from the targeted state of charge is too big, an increase of fuel consumption may occur, and the CO_2 equivalent may be worst.

5 CONCLUSIONS

Optimal control theory offers a simple control structure that guarantees a time-global minimization of the fuel consumption over the whole trip. From a control point of view the ICE torque is used to control the battery State of Charge while minimizing the fuel mass consumed. Using this formalism, we obtain a simple control structure that consists of an instantaneous minimization of a pondered criteria. The calibration of this criteria is what makes the difference between the predictive and the adaptive approaches. The predictive approach which is a continuity of the application of optimal control theory to the TorqueSplit problem, fixes a calibration for each road via a shooting algorithm. This shooting algorithm is fed with a prediction of the trip. On the other hand, the adaptive method updates this calibration using information about the vehicles actual states.

The use of future information allows the con-

trol to anticipate and use electrical power ranges that are recoverable due to a future regenerative braking. The adaptive method, as well as any non predictive method, would not have the necessary information to anticipate. Thus, the obtained solution could sometimes be wrongly careful when a recoverable amount of electrical energy is available.

The predictive strategy yields better fuel consumption and CO_2 emissions on 5 different cycles, in comparison to the CostBased adaptive method. However, the method relies on the prediction to be close enough to the real power demand in order to be able to approach the targeted state of charge.

Future works will improve the efficiency of the proposed predictive solution with less simplifications in the optimization modelling. A more representative model of the 12V net can be considered. The low voltage net model could be introduced in the optimal control modelling and the state of charge of the 12V battery can be controlled to reach a targeted final value. Dynamics such as temperature, and mechanical transmission losses can also be introduced to enhance the optimal control problem, and make the model closer to a real vehicle.

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