A Multi-stage Centralized Approach to Formation Flight Routing and Assignment of Long-haul Airline Operations

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Abstract: This paper describes the development of an optimization-based multi-stage centralized planning system for the efficient routing and assignment of extended flight formations in commercial airline operations. In an extended formation, where aircraft are longitudinally separated by 5-40 wingspans, a trailing aircraft can attain a reduction in induced drag at fixed lift, and consequently in fuel burn, by flying in the upwash of the leading aircraft's wake. To organize the assembly of flight formations on a network-wide scale essentially two distinct approaches can be taken, viz., a centralized approach and a decentralized approach. Both approaches have distinct advantages and disadvantages. In this study a novel multi-stage method for flight formation assignment is proposed that combines the advantages of the decentralized approach (fast computation and reduced vulnerability to flight delays) with the main benefit of the centralized approach (a near-global optimum in terms of fuel savings). The multi-stage centralized approach that we propose is validated and subsequently demonstrated in a case study involving a wave of 267 eastbound transatlantic flights. In the case study fuel savings of 6.8% are recorded (relative to flying "solo"), while flying in formations comprising up to 16 aircraft.

1 INTRODUCTION

Over the past decades, a variety of studies has been devoted to exploring the use of extended formation flight as a means to improve the fuel efficiency in commercial aviation. In an extended formation, where aircraft are longitudinally separated by 5-40 wingspans, a trailing aircraft can attain a reduction in induced drag at fixed lift by flying in the upwash of the leading aircraft's wake (Bower et al, 2009). Evidently, a reduction in induced drag translates directly into a reduction in fuel burn for aircraft trailing in a formation. An experimental study involving a formation flight of two military C-17 transport aircraft revealed that fuel savings of 5-10% for the trailing aircraft are possible, increasing with mission length (Flanzer&Bieniawski, 2014).

With the drag reduction mechanism in formation flight now relatively well understood, the focus in research on flight formation has shifted in recent years into the direction of the organization and planning of flight formations of commercial aircraft operations on an airline (alliance) network-wide scale (Ning et al, 2014).

To organize the assembly of flight formations on a network-wide scale essentially two distinct approaches can be taken, viz., a centralized approach and a decentralized approach (Visser et al, 2016). In a centralized approach formations are determined pre-flight through concurrent optimization of the routing and assignment of formation flights for an entire fleet, whilst in a decentralized (agent-based) approach the assembly of flight formations is conducted in-flight through local coordination of flights. Both approaches have distinct advantages and disadvantages. The main benefit of using a centralized formation planning method is the ability to provide globally optimal (fuel-saving) solutions. However, a centralized approach is computationally expensive compared to the fast local solutions that are obtained via a decentralized approach, in a realistically sized network. The huge computational burden of a centralized approach is a direct result of the combinatorial complexity of the associated (NP-hard) assignment problem. Another disadvantage of a centralized approach is that it does not readily accommodate schedule disruptions, thus providing a global optimum only in the absence of flight delays. In contrast, decentralized approaches make it

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possible to manage delayed flights by simply searching for alternative formation flight partners. However, decentralized approaches typically exhibit a number of shortcomings as well, notably the sub-optimal fuel savings due to the local nature of the employed coordination method.

Excellent examples of a centralized approach concern the studies presented by Kent & Richards (2015) and by Xu et al (2014). In both studies, the centralized approach taken to formation routing and assignment concerns a so-called two-stage (or bi-level) method. In the first stage, the routing/mission problem is considered for each candidate set of two or three long-haul origin/destination flights that might join in, respectively, a two or three aircraft formation. The first stage routing problem essentially deals with locating the rendezvous and splitting points for the flights involved in each potential formation and with scheduling the associated altitude/speed profiles such that the overall mission (fuel) cost is minimized. Assessment of the mission fuel burn is based on the Breguet-range equation for given speed and altitude. The second stage concerns the assignment problem, in which the network is optimized by selecting the best subset of formation and solo missions given the complete set of all possible combinations of individually optimized formation and solo missions obtained in the first stage.

Both studies involving centralized approaches (Kent & Richards, 2015; Xu et al, 2014) were faced with the necessity to deal with the highly-combinatorial assignment process, while keeping the computational burden within check. The combinatorial problem emerges when enumerating the number of possible combinations of aircraft formations for a set of *n* (unidirectional) O/D flights, to be grouped into formations of size *m*. Therefore, given a formation of size *m* and a set of *n* flights, the number of possible formation combinations N_m can be computed from the binomial coefficient:

$$N_m = \binom{n}{m} = \frac{n!}{m!(n-m)!} \qquad (1)$$

The number of possible formations N_m grows rapidly with increasing values of either *m* or *n*. For example, when considering a fleet size $n = 500$, the number of possible formation combinations is equal to $N_m = 124{,}750$ for a formation size $m = 2$, while $N_m = 2{,}573{,}031{,}125$ for a formation size $m = 4$. Since all possible combinations need to be evaluated in the first stage of the flight scheduling process, i.e., the routing/mission analysis problem, it is readily clear that introducing formations of large size renders the scheduling problem computationally intractable. In order to keep the computational burden in check, in both studies (Kent & Richards, 2015; Xu et al, 2014) flight formations up to size 3 only are considered, while other simplifying measures have been introduced as well. In Kent & Richards (2015) it is proposed to partition the global lists of flights in a number of ways (e.g. by airline or alliance) to keep *n* relatively low. Moreover, a computationally efficient geometric method for formation routing and mission analysis is employed in the first stage, which makes it possible to evaluate a large number of combinations very quickly. In contrast, in Xu et al (2014) a relatively high-fidelity, computationally expensive routing/mission analysis is employed in the first stage of the scheduling process; here the computational savings are obtained through the introduction of heuristics that allow the number of formations to be considered in the first stage of the scheduling process to be reduced, by filtering out all non-viable flight formation combinations upfront.

Motivated by concerns regarding the combinatorial complexity of the assignment of large fleets in a centralized approach, an alternative decentralized (agent-based) approach to the coupled problem of formation flight routing and assignment was conceived in (Visser et al, 2016). In this approach formation flight is not anticipated pre-flight but rather treated as an in-flight option based on local coordination. More specifically, this study implements a `proposal-marriage' type greedy-algorithm for decentralised assignment of flights into formations, similar to the one proposed in (Ribichini&Frazzoli, 2003) for formation scheduling of UAVs. However, the coupled routing and mission analysis required for each formation combination option is treated differently in both studies. While the study reported in (Visser et al, 2016) relies on the - slightly adapted - geometric formation flight routing method as proposed by Kent & Richards (2015), in combination with the Breguet-range equation for fuel-burn assessment, the work in (Ribichini&Frazzoli, 2003) implements a graph search over possible rendezvous and splitting points to find the optimal routing for each formation combination, in conjunction with the assumption of constant fuel-burn rates.

Both studies (Ribichini&Frazzoli, 2003; Visser et al, 2016) clearly demonstrate that by resorting to an agent-based, decentralized approach the inherent weaknesses, notably their vulnerability to delayed flights and the huge computational burden resulting from the highly-combinatorial assignment process, can essentially be eliminated. In (Visser et al, 2016) it is clearly shown that in the proposed decentralized approach flight (departure) delays can be readily accommodated, as delayed aircraft are simply able to search alternative formation flight partners within the network. Another beneficial feature of the approach proposed in (Visser et al, 2016) is that it permits building formations of (arbitrarily) large size. This is achieved by considering two aircraft that have just joined in a two-ship formation as a single "entity" that then becomes eligible for further assignment in subsequent down-route stages.

Despite its ability to overcome the typical weaknesses of centralized approaches, there is also a major downside associated to a decentralized approach, in the sense that the global fuel savings potential of flight formation is not fully achieved, due to the greedy nature of the flight formation assembly decision making process. Indeed, the numerical experiments conducted in (Doole, 2016) show that the decentralized approach presented in (Visser et al, 2016) results in fuel burn savings that are only fraction of the global fuel burn savings potential (50 to 60% in large-scale transatlantic scenarios).

In this paper a novel method for flight formation routing is proposed that aims to combine the advantages of the decentralized approach (fast computation and reduced vulnerability to flight delays) with the main benefit of the centralized approach (a near-global optimum in terms of fuel savings). The main aim of this study is to evaluate the fuel saving potential of this novel multi-stage centralized approach to formation flight routing, which combines some elements of the decentralized approach presented in (Ribichini &Frazzoli, 2003) with the geometric formation flight routing method as proposed by Kent & Richards (2015) in their bi-level centralized approach. The sections that follow provide a problem formulation and the proposed operational concept. Next, this paper discusses the employed routing method along with the modelling of fuel burn. This is then followed by a description and validation of a multi-stage fleet assignment model. Finally, a transatlantic case study is presented, from which the conclusions of this paper originate.

2 OPERATIONAL CONCEPT

Inspired by (Visser et al, 2016) and (Kent & Richards, 2015), a novel multi-stage centralized approach has been conceived for formation routing and assignment. The new approach that is proposed is based on a multi-stage assignment algorithm that updates the assignment of flight formations each time a certain event occurs. An event in this context is defined as: a departure, joining, or splitting of a flight within the network. As soon as one these events occurs, a flight formation assignment is made involving all eligible en-route flights, while considering the current aircraft positions as the "origin" nodes of the flights. The key feature of the multi-stage centralized approach is that in each assignment stage, only formations of size two are assembled, reducing the combinatorial complexity of the assignment in each step.

The initial assignment evaluation is made as soon as the first two aircraft that have departed to their destinations (using great circle routes) reach cruise altitude. The most likely outcome of this first assignment evaluation is that an assembly of a flight formation is not favourable and the two flights will continue on their solo routes towards their destination. As soon as the next aircraft reaches its cruise level, a new assignment evaluation is triggered, which includes the new flight. This process is repeated until at a certain point, the first two-ship flight formation is assigned. The two aircraft involved in the formation assignment then change course and speed so as to rendezvous at the calculated joining point. During their transition towards this joining point, the two flights involved are not eligible for any subsequent assignment. However, once these two aircraft have joined, the resulting two-ship formation is regarded as a single "entity" that then again becomes eligible for assignment, thus allowing the assembly of larger formations in subsequent assignment stages.

3 FUEL BURN ASSESSMENT

In this study we consider all aircraft operating in the network to be of the same type The parameters of the particular aircraft model employed in this study relate to a Boeing B777, and have been extracted from (Visser et al, 2016). The transatlantic routes are modeled in this study as great circle paths from origins to destinations. Moreover, it is assumed that the entire route is flown in cruise at a single constant altitude and at constant speed. The fuel consumption along the routes is estimated using the well-known Breguet-range equations for flight at constant altitude and constant speed (Vinh, 1993), assuming the absence of wind:

$$R = \frac{2V}{c_t}\left(\frac{C_L}{C_D}\right)_{\max}\left(\tan^{-1}\frac{C_{L_1}}{C_{L_{md}}} - \tan^{-1}\frac{C_{L_2}}{C_{L_{md}}}\right), \quad (2)$$

where :

$$C_{L_{md}} = \sqrt{\frac{C_{D_0}}{K}}; C_{L_i} = \frac{W_i}{\frac{1}{2}\rho V^2 S} \;, \; i = 1,2 \tag{3}$$

are, respectively, the values of the lift coefficient for minimum drag, and for the initial and final weight of the aircraft (at the considered altitude and speed). The parameter :

$$\left(\frac{C_L}{C_D}\right)_{\max} = \frac{1}{2\sqrt{C_{D_0} K}} \tag{4}$$

in Eq.(2) represents the maximum lift over drag ratio. The above relationships assume a traditional drag polar of the form $C_D = C_{D_0} + KC_L{}^2$. The parameter c_t in Eq.(2) represents the Specific Fuel Consumption (SFC) of the aircraft type considered.

Equation (2) can be resolved for the aircraft weight at the end of the flight stage, W_2, given an initial weight W_1 :

$$W_2 = W_1 \left(\frac{\left(\frac{C_{L_1}}{C_{L_{md}}}\right) - \tan\psi}{\left(\frac{C_{L_1}}{C_{L_{md}}}\right)^2 \tan\psi + \left(\frac{C_{L_1}}{C_{L_{md}}}\right)} \right) \tag{5}$$

where :

$$\psi = \frac{Rc_t}{2V\left(\frac{C_L}{C_D}\right)_{\max}} \tag{6}$$

Alternatively, given a final weight W_2, the required initial weight W_1 needed to fly a distance R can be estimated using Eq.(6). Note that Eq.(6) can be used to assess fuel burn in both solo flight legs and formation flight legs. To allow for the formation drag reduction, a discounting factor λ is used in the induced drag coefficient K of a trailing aircraft in a formation, i.e. $K_{trailing} = \lambda K$. In the present study, a discount factor $\lambda = 0.867$ has been adopted (Visser et al, 2016), which results in a fuel flow rate reduction of about 10% for each trailing aircraft in a formation. It is noted that several aerodynamic studies related to formation flight point out that the formation induced drag reduction increases as the number of aircraft in the formation string increases (Ning et al, 2014; Xue&Hornby, 2012). However, in this study this particular effect is ignored and each trailing aircraft in a formation essentially enjoys the same induced drag discount factor, regardless of the size of the formation string. Note that organizing the traffic flow into larger formation flights can still be very rewarding in the sense that the number of formation flight leaders that are needed (and that don't enjoy any drag reduction benefit) can be reduced in this way.

The constant speed that is adopted along a route is generally taken as M=0.82 in this study. This Mach number is slightly lower than the typical cruise value for the aircraft type considered, in order to mitigate compressibility effects in a formation which may impede aircraft safety (Ning et al, 2014). For reasons of simplicity, this Mach number is also adopted in all solo flight legs, with one exception. This exception relates to aircraft flying the shortest flight leg towards the joining point in a formation rendezvous. In order to rendezvous at the joining point, the aircraft flying the shortest leg needs to reduce its speed relative to its partner aircraft flying the longer leg. Note that the lowest speed in a synchronization flight leg is the maximum endurance speed, which is the speed at which the lowest fuel consumption per unit of time is attained (Vinh, 1993). Assuming a drag polar as defined in Eq.(4), the maximum endurance speed corresponds to the speed for minimum drag and is given by :

$$V_{md} = \sqrt{\frac{2W}{\rho S}}\left(\frac{K}{C_{D_0}}\right)^{1/4} \tag{7}$$

The actual take-off weight for a specific (great circle route) flight is calculated using the Breguet-range equation assuming that at destination the weight of the aircraft is equal to the zero-fuel weight plus the weight of the reserve fuel. Given the aircraft weight at destination, and the distance covered along the route, the aircraft weight at the origin can be assessed using the Breguet-range equation, for the assumed speed and altitude. To allow for the fact that aircraft may have to fly detours and thus longer routes in order to engage in flight formation, the initial fuel load is increased by 10%; the take-off weight of the aircraft is increased accordingly.

4 FORMATION ROUTING

To generate routes for the assembly of formation flights, a routing method was used based on (Kent & Richards, 2015). In this approach, a formation flight route for two aircraft is obtained through the minimization of weighted distance. The time-free routing method from Kent & Richards was extended in (Visser et al, 2016) to ensure rendezvous of timed flights in the formation assembly process.

4.1 Geometric Routing Method by Kent & Richards

In (Kent & Richards, 2015), a simple geometric method to construct a formation flight routing is presented based on a classical mathematical problem posed by Fermat in the 17th century. The problem, illustrated in Figure 1, is posed as follows: given a triangle ABC (Fig.1.a), find a point P such that the sum of the distances $\|AP\|$, $\|BP\|$ and $\|CP\|$ is minimized. The geometric approach to construct the solution to this problem is illustrated in Fig.1.b.

The method shown in Fig.1.b is based on constructing outwardly three equilateral triangles along the sides AB, BC and CA. Then the lines from the outer vertex of each new triangle to its opposite vertex of the original will intersect at a single point, which is the desired point P. Equivalently, the point P can be found as the intersection point of the circumscribed circles of each of the three new equilateral triangles.

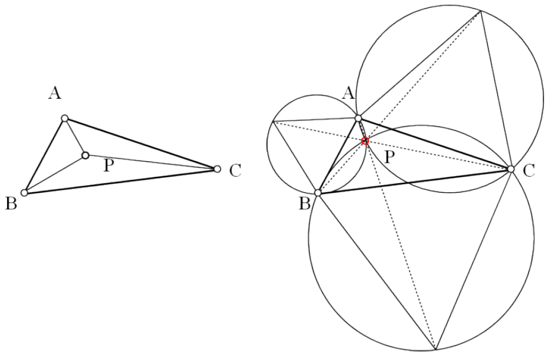
Fermat's problem provides a good analogy to the formation flight assembly problem, if it is assumed that fuel consumption is proportional to the distance covered. However, it is readily clear that the fuel consumption per unit distance along the solo arcs $\|AP\|$ and $\|BP\|$ differs from that on the formation flight arc $\|PC\|$. To resolve this issue, Kent & Richards formulated a weighted-arc version of the problem, where the arc weights reflect the different fuel consumption per unit distance. More specifically, to represent the cost of flying a unit of distance, the arc weights w_A, w_B, and w_C are introduced for the segments AP, BP and PC, respectively. Note that the value of w_C is typically set equal to the combined values of w_A and w_B, while applying some discount factor to represent the fuel savings due to induced drag reduction of the trailing aircraft.

Thus, in the modified problem the location of the joining point P has to be selected such that the total cost of distance, expressed by Eq.(1) is minimized:

$$\underset{P}{Minimize:} \quad f(P) = w_A \cdot \|AP\| + w_B \cdot \|BP\| + w_C \cdot \|PC\| \tag{8}$$

Following Kent & Richards, the location of point P that minimizes Eq.(9) must satisfy the vectorial equilibrium condition expressed by Eq.(2):

$$w_A \cdot \frac{AP}{\|AP\|} + w_B \cdot \frac{BP}{\|BP\|} + w_C \cdot \frac{PC}{\|PC\|} = 0 \,\| \tag{9}$$



(a) (b)

Figure 1: Geometric construction to locate the optimal joining point P (Kent & Richards, 2015).

Application of the law of cosines to Eq.(9) yields expressions for the intersection angles ∠APB, ∠APC, and ∠BPC. Since the angle ∠APB represents the intersection angle between the two solo legs AP and BP, it is referred to as the "formation angle". Equation (10) gives the expression for the resulting formation angle θ_f:

$$\theta_f = \cos^{-1}\left(\frac{-w_A{}^2 - w_B{}^2 + w_C{}^2}{2w_A w_B}\right) | \tag{10}$$

Note that the formation angle θ_f only depends on the routing weights w_A, w_B, and w_C. The formation angle is illustrated in Figure 2. It is noted that as long as the weights w_A, w_B, and w_C are not altered, also the formation angle θ_f remains unaffected. As a result, the point C can be shifted freely along the line PC, without altering the solution.

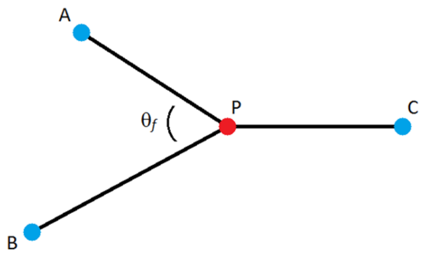


Figure 2: Illustration of formation angle.

The method for locating point P can be extended to a scenario in which the two flights do not have a common destination C. Figure 4 illustrates two solo routes connecting origins A and B to destinations C and D, respectively. The joining point J and the splitting point S are to be determined. Since the formation angle condition must be satisfied at both J and S in order to minimize the weighted distance, one can draw two circular arcs from A to B and from C to D along which the formation angle is constant and equal to the value obtained from Eq.(11). These arcs are displayed in Figure 4.

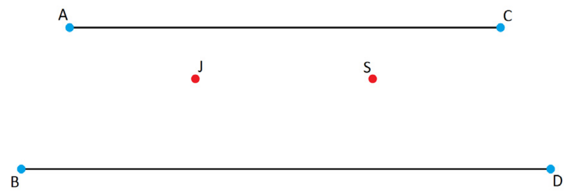


Figure 3: Illustration of example solo routes AC and BD.

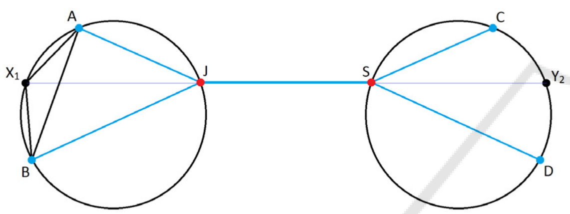


Figure 4: Geometric construction of the formation route.

The (back vertex) point X_1 in Fig. 4 is obtained by making use of the fact that Eq.(11) holds in triangle ABX_1:

$$|AB|:|BX_1|:|X_1A| = w_C : w_A : w_B| \qquad (11)$$

Mirroring the described steps at the destinations C and D provides the (back vertex) point Y_2. The locations of J and S that minimize the weighted distance from A to C and from B to D are obtained from the intersections of the line from X_1 to Y_2 and the arcs of constant formation angle.

4.2 Accommodating Synchronization in the Basic Routing Method

The geometric routing procedure presented in Section 4.1 is inherently time-free, essentially assuming that the flight schedules are synchronized such that the two flights joining in formation are able to rendezvous at the calculated joining point. However, the approach proposed herein considers flight formation to be an in-flight option, with the current aircraft positions serving as the origin nodes of the flights. Therefore, in order to realize formation flight, the route must be constructed such that the two aircraft are able to arrive at the joining point simultaneously. For solo flight legs to the joining point that are not too dissimilar in size, synchronization is typically accomplished by slightly slowing down the aircraft that is flying the shortest leg. However, if the formation flight route does not permit synchronization in this way, the joining point must be relocated to enable a stretch of the connecting flight legs. It is conceivable that even relocating the joining point may not be sufficient to ensure a rendezvous. In this case, excess delay time needs to be absorbed by holding one of the aircraft at the joining point. When the latter situation occurs, usually the formation option turns out to be less favorable then flying solo. Details regarding the synchronization process can be found in (Visser et al, 2016).

4.3 An Extension to Larger Formation Sizes

Kent&Richards (2015) demonstrated that the geometric routing approach developed for the assembly of two-ship formations could be extended to formations of any size. Here, we will illustrate this extension for the assembly of a four-ship flight formation. Figure 5.a shows the assembly of two two-ship formations; in the first two-ship formation Flights 1 and 2 are joined, while in the second two-ship formation Flights 3 and 4 are joined. These two-ship formations are now regarded as two "pseudo flights" (respectively, Flight_{12} and Flight_{34}), with, respectively, the back vertices $X_{flight_{12}}$ and $X_{flight_{34}}$ serving as origin nodes and the back vertices $Y_{flight_{12}}$ and $Y_{flight_{34}}$ as destination nodes. Next, as shown in Fig.6 the two pseudo flights Flight_{12} and Flight_{34} join in formation, and a joining and splitting point are computed in the same way as for the assembly of a two-ship formation. The resulting four-ship formation can then then regarded as a new pseudo flight (Flight_{1234}). It is noted that the four-ship formation assembly sketched in Fig.6 presumes a certain order of joining in formation (e.g., Flight 1 first joins with Flight 2) and therefore represents just one of the options in the four-ship formation assembly process for these four flights. All alternative joining order options (e.g., Flight 1 first joins with Flight 3, or Flight 1 first joins with Flight 4) need to be explored as well, including the possibility of three-ship formations (pseudo flights) joining with single aircraft flights. It is readily clear that the assembly of larger formations represents is also problem that is highly combinatorial in nature. In (Doole, 2016) the time-free four-ship assembly process proposed by Kent & Richards has been extended to include flight synchronization.

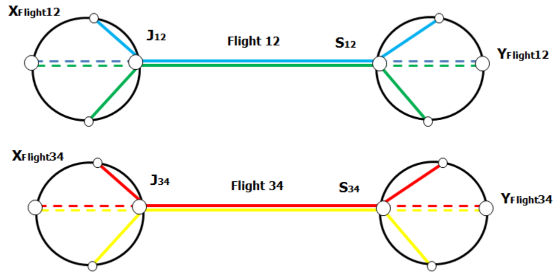


Figure 5: Assembly of two two-ship formations (pseudo flights, $Flight_{12}$ and $Flight_{34}$) from four solo flights.

4.4 Formation Leaders

When two, or more, flights have successfully completed their rendezvous, one of the aircraft involved needs to be assigned as the formation leader. A flight that does not lead a formation, is referred to as a 'trailing'. In the present set-up, where only aircraft of the same type are considered, it is readily clear what the best choice is from a collective perspective: the least heavy aircraft of the two is designated as the lead aircraft, as the heavy aircraft can benefit relative more from an induced drag reduction. It is noted that the formation leader does not gain any direct benefit from flying in an extended formation; however, in this study it is assumed that formation partners will somehow share the overall economic benefits.

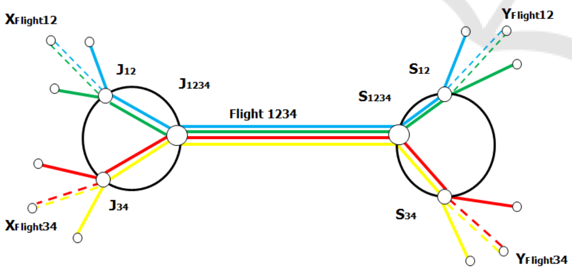


Figure 6: Assembly of a four-ship formations (pseudo flight, $Flight_{1234}$) from two two-ship formations.

4.5 Azimuthal Equidistant Projection Method

The geometric approach presented herein is inherently planar in nature, however it can be extended to hold for problems on a sphere (Kent & Richards, 2015). This latter option has not been pursued in this study; rather the great circle routes connecting the various O/D pairs were projected on a plane by means of the so-called azimuthal equidistant projection method (Tobler, 1962), so that the original planar geometric approach can be retained. The azimuthal equidistant projection method has the property that all distances from the center are rendered correctly to scale and that all points on the map are at the correct azimuth (direction) from the center point. By using this particular projection method, the relative locations of the origins and destinations, as well as the route lengths and relative flight headings remain reasonably well preserved in the transformation. Since these route characteristics are most relevant for the success of formation flight implementation, the selected projection method is considered appropriate for this study.

5 FLEET ASSIGNMENT PROBLEM

5.1 Basic BMILP Formulation

The fleet assignment model that has been adopted selects a compatible set of possible flight formations in order to achieve global minimum fuel burn. The optimization problem used to solve the formation flight assignment is formulated as a Binary Mixed-Integer Linear Program (BMILP), using a similar formulation as adopted by Kent & Richards (2015).

Given a fleet of size $n = N$, and flight formations up to size $m = M$ (including solo flights, which are regarded as flight formations of size one), the total possible number of flight formations N_F is given by:

$$N_F(N,M) = \sum_{m=1}^{m=M} N_m(N) \quad , \qquad (12)$$

where N_m is the binomial coefficient defined in Eq.(1) that gives the number of possible formation combinations of size m. To give an example, suppose we have a fleet of $N = 5$ aircraft and we consider formations up to size $M = 3$, we then have: $N_1 = 5$; $N_2 = 10$; $N_3 = 10$; $N_4 = 5$; $N_5 = 1$, and thus $N_F(5,3) = N_1 + N_2 + N_3 + N_4 + N_5 = 31$ possible formation combinations are found.

The variables and (cost and pairing) coefficients that are used to define the fleet assignment problem are as follows:

- x_j A binary decision variable that has the value one if configuration j is selected in the solution and zero otherwise.
- c_j A real-valued cost coefficient that represents the total incurred fuel cost of formation j.

p_{ij} A binary pairing coefficient that indicates whether flight i is included in formation j ($p_{ij} = 1$) or not ($p_{ij} = 0$).

The BMILP formulation that is used to optimally assign each flight to a formation is then given by :

$$\text{minimize} \quad J = \sum_{j=1}^{j=N_F} c_j \cdot x_j \quad , \tag{13}$$

subject to :

$$\sum_{j=1}^{j=N_F} p_{ij} \cdot x_j = 1 , \quad \forall i \in \{1,...,N\} \tag{14}$$

$$x_j = \begin{cases} 1 \\ 0 \end{cases} , \quad \forall j \in \{1,...,N_F\} \tag{15}$$

The number of binary variables is equal to the number of possible flight formations N_F, which increases dramatically with the number of flights N, and especially with the maximum formation string size M. It is readily clear that for larger problems (in terms of maximum formation string size M), the combinatorial complexity of the MILP becomes significant and a huge computational effort is required. In the multi-stage centralized assignment approach the number of decision variables is kept low in each assignment step basically by restricting the formation string size to just two. In this case the number of potential formation combinations amounts to:

$$N_2 = \frac{N(N-1)}{2} \tag{16}$$

5.2 Multi-Stage Fleet Assignment Model

Figure 7 presents a schematic diagram of the developed multi-stage fleet assignment model. A new assignment is made every time an event (i.e., a departure, joining, or splitting) occurs. When an event takes place, the location of each eligible flight (or pseudo flight) that is en-route is determined and subsequently all potential flight formation missions for the eligible fleet are assessed in terms of routing and fuel burn performance, and the associated cost and pairing coefficients are determined. Next, the BMILP problem is formulated and solved (using CPLEX) and the resulting formations are assigned; the aircraft (strings) involved in the assignment are declared ineligible for further assignment until they have joined. Note that when such a joining takes place a new event is triggered.

6 VALIDATION OF THE MULTI-STAGE FLEET ASSIGNMENT MODEL

To prove the effectiveness and efficiency of the proposed multi-stage centralized approach to flight formation routing and assignment a validation study was undertaken. In this validation study the (computational and fuel burn) performance of the newly developed multi-stage fleet assignment approach is compared against that of the original bi-level centralized approach of Kent & Richards (2015), albeit that both approaches are somewhat modified in order to provide a proper basis of comparison. More specifically, since the centralized approach of Kent & Richards is essentially time-free, it has been augmented with the rendezvous synchronization mechanism for formation flight routing developed in (Visser et al, 2016), so that it can be used in conjunction with a given flight schedule. Since the centralized approach of Kent & Richards is computationally expensive for large problems, a relatively small fleet assignment problem is considered in the validation scenario. More specifically, in the validation scenario the assignment problem is limited to formations comprising strings up to four aircraft ($M = 4$) only, whilst considering a fleet of 50 aircraft ($N = 50$) that seek to traverse the North Atlantic. Even for this relatively small problem, the enumeration time is significant as the number of potential formations is already quite large. Using Eq.(1), it is readily established that there are 1,225 two-ship, 19,600 three-ship, and 230,300 four-ship formation combination possibilities, in addition to the 50 solo flights. Therefore, aside from the 50 solo flights, there are 251,125 possible formation combinations in total, each requiring individual formation routing and fuel burn predictions. It is recalled that the routing/mission analysis itself is also plagued by combinatorial complexity issues, leading to computational times that increase dramatically with the maximum size of the formation string.

The multi-stage fleet assignment model developed herein is run in a "degraded" mode of operation in the validation scenario. In order to avoid the assembly of formations in excess of size four, an assignment step is not made after each occurrence of an event, but rather in just two discrete stages. First of all, in the validation scenario it is assumed that all 50 flights depart at exactly the same time; at this given departure time the first assignment evaluation is made. A second assignment step is then made as soon as all formations that were assigned in the first stage have actually joined up. Evidently, in the first stage only formations up to size two can emerge, while in the second stage formations up to size four can be assembled. Clearly, in the first stage there are only 1,225 possible (two-ship) flight formations, in addition to the 50 solo flights. In the second assignment stage, the "fleet" to be considered is substantially smaller than in the first stage. The reason for this is that it turns out that in the first stage 24 two-ship formations were assembled (involving 48 aircraft), so that in the second stage the fleet only comprises 24 "pseudo flights" and 2 solo flights, for a total of 26 entities. The total number of possible formations in the second stage (for formations ranging from size 1 up to size 4) is therefore just 302.

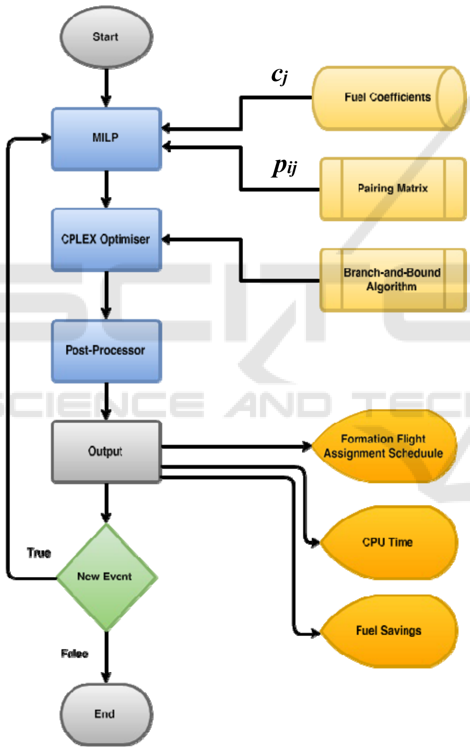


Figure 7: Flow chart of the operational multi-stage fleet assignment concept.

The performance characteristics of both assignment approaches are summarized in Table 1. The centralized approach by Kent&Richards (2015) is denoted here as the "single-stage" approach, whilst the multi-stage fleet assignment approach proposed herein is labeled as the "two-stage" approach. In Table 1, the number of potential formation combinations (and thus routing/mission analysis evaluations) that is assessed in each approach is indicated, along with the required CPU time (on a standard 2.1 GHz personal computer), and the resulting fuel savings of formation flight in absolute terms and in relative terms (as compared to executing all flights solo).

A close inspection of the results in Table 1 reveals that the two-stage approach performs almost equally well as the single-stage approach in terms of fuel savings, but at only a fraction of the computational cost. Overall, the differences in the routing and assignment solutions between the two approaches turn out to be marginal.

7 TRANSATLANTIC CASE STUDY

The proposed multi-stage fleet assignment model is applied in a case study involving 267 eastbound transatlantic flights. It is assumed in this case study that each flight can potentially join in formation with any other flight in the network (regardless of airline or alliance membership). The routes included in the case study, shown in Figure 8, are obtained from an available real-life data set (Lith et al, 2014) by means of selecting the longitude/latitude coordinates of all origins and destinations. However, in order to make the case study more challenging, it is assumed that all 267 flights depart at the same time.

Also in the transatlantic case study, an alternative fleet assignment approach is considered to obtain a basis of comparison for the multi-stage centralized approach. Given the size of the assignment problem, the agent-based decentralized approach reported in (Visser et al, 2016) is applied in the case study to provide the basis for comparison.

In the simulation of the decentralized formation flight planning method a local search for potential partners is conducted once every 5 minutes. The employed communication radius for formation assembly negotiation is set at 250 km; hence only flights within this radius are considered eligible for formation assembly.

The results for the decentralized approach bear out that the largest observed formation is a sixteen-ship formation. To establish a fair basis of comparison, a restriction on the maximum size of a formation is enforced in the multi-stage centralized approach as well, in the sense that no formations is excess of size 16 are allowed to be assembled. To this end, the multi-stage assignment model is again run in "degraded mode", limiting the assignment updates to just four discrete stages.

Table 1: Performance comparison between single-stage and two-stage fleet assignment solutions.

Approach	Formation combinations	Computation time (mm:ss)	Fuel savings (%)	Fuel saved (kg)
Single-Stage	251,175	58:43	5.72	184,381
Two-Stage	1,577	00:14	5.66	182,480

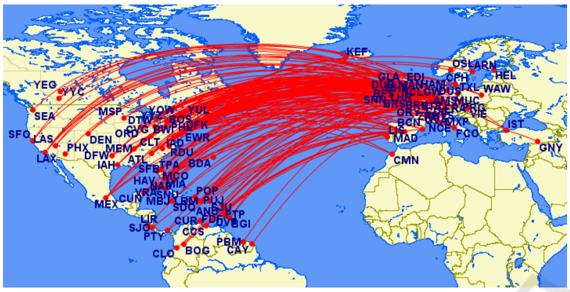


Figure 8: Route set used in the case study (created using Great Circle Mapper at www.gcmap.com).

The results for the multi-stage assignment model are summarized in Figure 9, which shows the fraction of total time spent in formations ranging in size 1 to size 16. It is readily clear that the majority of the time is spent in formations of size 4. Indeed, no less than 58 four-ship formations are assembled in this transatlantic case study, half of which later merge into larger formations. Although ultimately four sixteen-ship formations are assembled, the total time spent in these largest formations is relatively small. No formations of size 5 or 7 were assembled. Moreover, the sixteen-ship formations are the only formations in excess of size 8. Only two flights do not engage in flight formation and remain solo. It is noted that the results might be somewhat tainted by the fact that the multi-stage fleet assignment model has been run in "degraded mode". This will be explored in future research.

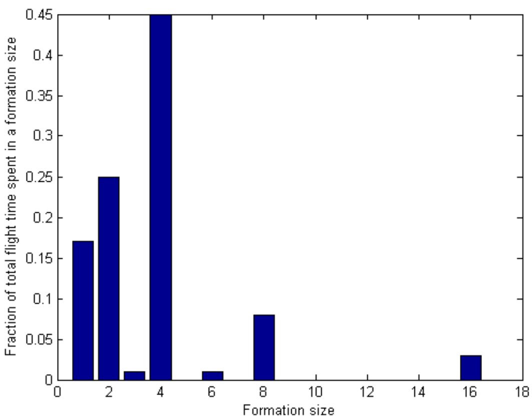


Figure 9: Formation size usage when the multi-stage fleet assignment approach is employed.

Figure 10 shows the corresponding results for the agent-based decentralized approach. It can be seen that in the decentralized approach considerably less time is spent in formations of size four. Moreover, a wider variety in the size of the formations that are formed is observed.

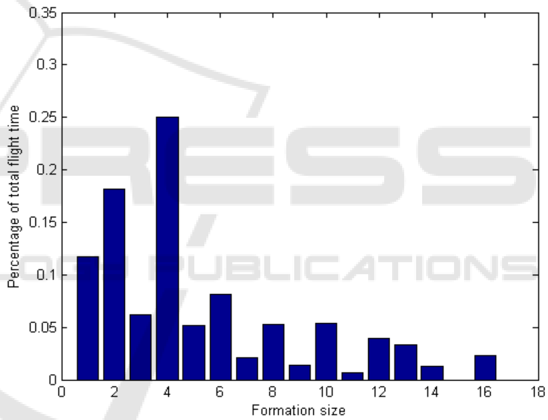


Figure 10: Formation size usage when the decentralized fleet assignment approach is employed.

In Table 2 the computational and fuel burn performances of the two approaches are compared. When comparing the results, it is evident that the multi-stage centralized approach is vastly superior is terms of fuel savings. Indeed, the savings in the multi-stage centralized approach are almost twice that of the decentralize approach. Although the total computational time for the multi-stage approach features a significantly higher computational burden relative to the decentralized approach, in absolute terms the computational time is still modest and remains well within the requirement for real-time application.

Table 2: Performance comparison between multi-stage centralized and decentralized assignment solutions.

Approach	Computation time (mm:ss)	Fuel savings (%)	Fuel saved (kg)
Multi-stage Centralized	09:24	6.88	1,202,000
Decentralized	03:23	3.90	681,200

8 CONCLUSIONS

This study proposes a multi-stage centralized cooperative planning system for the efficient routing and scheduling of flight formations. In the proposed concept, the global fleet assignment is updated every time a certain event occurs en-route, where an event is either the emergence of a new flight, the emergence of a newly assembled formation, or the emergence of a disassembly of a formation. In each assignment update, only the assembly of formations involving two flights are considered, where a flight can either be an actual solo flight or a “pseudo flight”, representing a prior assembled formation. By limiting the formation size to just two (pseudo) flights in each assignment step, the combinatorial complexity of the overall assignment process can be kept in check.

In a validation scenario involving flight formations up to size four, it was demonstrated that the proposed multi-stage fleet assignment model is capable of generating a near-optimal global solution, at a fraction of the computational cost required for the single-stage assignment of formations up to size four. In a subsequent large-scale transatlantic scenario, it was demonstrated that the multi-stage fleet assignment offers a vast improvement over a decentralized approach in terms of the fuel savings that can be achieved.

In view of the results obtained in this study, the proposed concept holds out great promise for further development towards real-world application. In future studies, many of the simplifying assumptions taken in the present study will need to be removed for this purpose. In particular, the no-wind assumption that underlies the current study has a profound impact on the solution behavior and more realistic wind scenarios will need to be considered in lieu of the no-wind assumption in future studies.

Although in an extended formation aircraft fly at safer separation distances than in close formation flight, aircraft still fly in relatively close proximity, exceeding current regulated separation minima. Many aviation regulations and standards, including the reduction of aircraft separation limits, therefore will need to be adapted. The technical, operational and regulatory challenges associated to introducing extended formation in the civil aviation domain need to be investigated in future research.

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