

A Lock-free Algorithm for Parallel MCTS

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Abstract: In this paper, we present a new lock-free tree data structure for parallel Monte Carlo tree search (MCTS) which removes synchronization overhead and guarantees the consistency of computation. It is based on the use of atomic operations and the associated memory ordering guarantees. The proposed parallel algorithm scales very well to higher numbers of cores when compared to the existing methods.

1 INTRODUCTION

In the last decade, there has been much interest in the MCTS algorithm. The start was by a new, adaptive, randomized optimization algorithm (Coulom, 2006; Kocsis and Szepesvári, 2006). In fields as diverse as Artificial Intelligence, Combinatorial Optimization, and High Energy Physics, research has shown that MCTS can find approximate answers without domain-dependent heuristics (Kocsis and Szepesvári, 2006; Kuipers et al., 2013). The strength of the MCTS algorithm is that it provides answers for any fixed computational budget with a random amount of error (Goodfellow et al., 2016). Typically, the amount of error can be diminished by expanding the computational budget for more running time. In the last ten years, much effort has been put into the development of parallel algorithms for MCTS. The domain of research contains a broad spectrum of parallel systems; ranging from small shared-memory multi-core machines to large distributed-memory clusters. The goal is to reduce the running time.

One of the approaches for parallelizing MCTS for shared-memory systems is tree parallelization (Chaslot et al., 2008a). The method is called so because a search tree is shared among multiple parallel threads. Each iteration of the MCTS has four operations (SELECT, EXPAND, PLYOUT, and BACKUP). They are executed on the shared tree simultaneously (Chaslot et al., 2008b). The MCTS algorithm uses the tree for storing the states of the domain and guiding the search process. The basic premise of the tree in MCTS is relatively straightforward: (a) nodes are added to the tree in the same order as they were expanded and (b) nodes are updated in the tree in the same order as they were selected. Therefore the fol-

lowing holds, if two parallel threads are performing the task of adding (EXPAND) or updating (BACKUP) the same node, there are potentially *race conditions*. Thus, one of the main challenges in tree parallelization is the prevention of *race conditions*.

In a parallel program a race condition shows a non-deterministic behavior that is generally considered to be a programming error (Williams, 2012). This behavior occurs when parallel threads perform operations on the same memory location without proper *synchronization* and one of the memory operations is a write. A program with a race condition may operate correctly sometimes and fail other times. Therefore, proper synchronization helps to coordinate threads to obtain the desired runtime order and avoid a race condition.

There are two lock-based methods to create synchronization in tree parallelization: (1) a coarse-grained lock (Chaslot et al., 2008a), (2) a fine-grained lock (Chaslot et al., 2008a).

Both methods are straight forward to design and to implement. However, locks are notoriously bad for parallel performance, because other threads have to wait until the lock is released. This is called *synchronization overhead*. It is shown that the fine-grained lock has less synchronization overhead than the coarse-grained lock (Chaslot et al., 2008a). Yet, even fine-grained locks are often a bottleneck when many threads try to acquire the same lock. Hence, a *lock-free* tree data structure for parallelized MCTS is desirable and has the potential for maximal concurrency. A tree data structure is lock-free when more than one thread must be able to access its nodes concurrently. Here, the problem is that the development of a lock-free tree for parallelized MCTS is shown to be non-trivial. The difficulty of designing an adequate

data structure stimulated the researchers in the community to come up with a spectrum of ideas (Enzenberger and Müller, 2010; Baudiš and Gailly, 2011). As a case in point, Enzenberger et al. compromised over the correctness of computation. They accepted faulty results to have a lock-free search tree (Enzenberger and Müller, 2010). In Below, we propose a new lock-free tree data structure without compromises together with the corresponding algorithm that uses the tree for parallel MCTS.

The remainder of this paper is structured as follows. Section 2 briefly provides the required background information. Section 3 discusses related work. Section 4 presents the proposed lock-free algorithm. Section 5 shows implementation details. Section 6 gives the experimental setup, and Section 7 provides the experimental results. Finally, a conclusion is given in Section 8.

2 BACKGROUND

Below we discuss MCTS in Section 2.1, the UCT algorithm in Section 2.2, and tree parallelization in Section 2.3.

2.1 The MCTS Algorithm

The MCTS algorithm iteratively repeats four steps (also called operations) to construct a search tree until a predefined computational budget (i.e., time or iteration constraint) is reached (Chaslot et al., 2008b; Coulom, 2006). Algorithm 1 shows the general MCTS algorithm.

At the beginning, the search tree has only a root (v_0) which represents the initial state (s_0) in a domain.

Each node in the search tree resembles a state of the domain. The edges directed to the child nodes represent actions leading to succeeding states. Figure 1 illustrates one iteration of the MCTS algorithm on a search tree that already has nine nodes. The non-terminal and internal nodes are represented by circles.

Algorithm 1: The general MCTS algorithm.

```

1 Function MCTS( $s_0$ )
2    $v_0 :=$  creat root node with state  $s_0$ ;
3   while within search budget do
4      $\langle v_l, s_l \rangle :=$  SELECT( $v_0, s_0$ );
5      $\langle v_l, s_l \rangle :=$  EXPAND( $v_l, s_l$ );
6      $\Delta :=$  PLYOUT( $v_l, s_l$ );
7     BACKUP( $v_l, \Delta$ );
8   end
9   return action  $a$  for the best child of  $v_0$ 

```

Squares show the terminal nodes.

1. **SELECT:** A path of nodes inside the search tree is selected from the root node until a non-terminal leaf with unvisited children is reached (v_6). Each of the nodes inside the path is selected based on a predefined *tree selection policy* (see Figure 1a).
2. **EXPAND:** One of the children (v_9) of the selected non-terminal leaf (v_6) is generated randomly and added to the tree and also to the selected path (see Figure 1b).
3. **PLAYOUT:** From the given state of the newly added node, a sequence of randomly simulated actions is performed until a terminal state in the domain is reached. The terminal state is evaluated using a utility function to produce a reward value Δ (see Figure 1c).
4. **BACKUP:** For each node in the selected path, the number $N(v)$ of times it has been visited is incremented by 1 and its total reward value $Q(v)$ is updated according to Δ (Browne et al., 2012). These values are required by the tree selection policy (see Figure 1d).

As soon as the computational budget is exhausted, the best child of the root node is returned (e.g., the one with the maximum number of visits).

2.2 The UCT Algorithm

This section explains the most common algorithm in the MCTS family, the Upper Confidence Bounds for Trees (UCT) algorithm. The UCT algorithm addresses the exploitation-exploration dilemma in the selection step of the MCTS algorithm using the UCB1 policy (Kocsis and Szepesvári, 2006). A child node j is selected to maximize:

$$UCT(j) = \bar{X}_j + 2C_p \sqrt{\frac{2 \ln(N(v))}{N(v_j)}} \quad (1)$$

Where $\bar{X}_j = \frac{Q(v_j)}{N(v_j)}$ is an approximation of the game-theoretic value of node j . $Q(v_j)$ is the total reward of all playouts that passed through node j , $N(v_j)$ is the number of times node j has been visited, $N(v)$ is the number of times the parent of node j has been visited, and $C_p \geq 0$ is a constant. The left-hand term is for exploitation and the right-hand term is for exploration (Kocsis and Szepesvári, 2006). The decrease or increase in the amount of exploration can be adjusted by C_p in the exploration term. It has profound effect on the behavior of the algorithm (see Section 7).

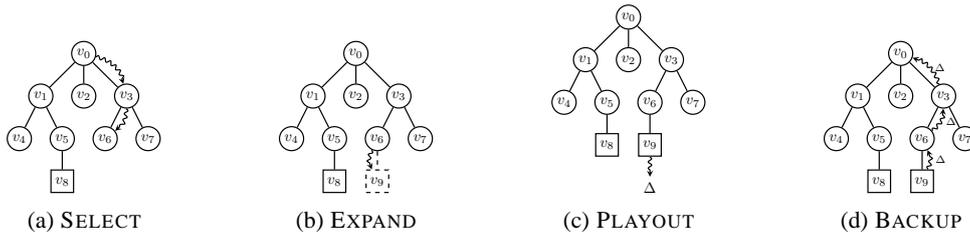


Figure 1: One iteration of MCTS.

2.3 Tree Parallelization

There are three parallelization methods for MCTS (i.e., *root parallelization*, *leaf parallelization*, and *tree parallelization*) that belong to two main categories: (A) parallelization with an ensemble of trees, and (B) parallelization with a single shared tree.

The root parallelization method belongs to category (A). It creates an ensemble of search trees (i.e., one for each thread). The trees are independent of each other. When the search is over, they are merged, and the action of the best child of the root is selected.

The leaf parallelization and tree parallelization methods belong to category (B). In the leaf parallelization, the parallel threads perform multiple **PLAYOUT** operations from a non-terminal leaf node of the shared tree. These **PLAYOUT** operations are independent of each other, and therefore there is no race condition. In tree parallelization, parallel threads are potentially able to perform different MCTS operations on a same node of the shared tree (Chaslot et al., 2008a). These shared accesses are the source of the potential *race conditions*.

2.4 The Race Conditions

A race condition occurs when concurrent threads perform operations on the same memory location without proper synchronization, and one of the memory operations is a write (McCool et al., 2012). Consider the example search tree in Figure 2. Three parallel threads (1, 2, and 3 from v_0 to v_3) attempt to perform MCTS operations on the shared search tree. There are three race condition scenarios.

- **Shared Expansion (SE):** Figure 2b shows two threads (1 and 2) concurrently performing **EXPAND**(v_6). In this SE scenario, synchronization is required. Obviously, a race condition exists if two parallel threads intend to add node v_9 to v_6 simultaneously. In such an SE race, the child node should be created and added to its parent only once.
- **Shared Backup (SB):** Figure 2c shows two threads (1 and 3) concurrently performing **BACKUP**(v_3).

In the SB scenario, synchronization is required because there are two data race conditions when parallel threads update the value of $Q(v_3)$ and $N(v_3)$ simultaneously. There are two dangers: (a) the value of either $Q(v_3)$ or $N(v_3)$ could be corrupted due to concurrently writing them, and (b) the variable $Q(v_3)$ and $N(v_3)$ could be in an inconsistent state when the writing of their values does not happen together at the same time (i.e., the state of one variable is ahead of the other one).

- **Shared Backup and Selection (SBS):** Figure 2d shows thread 2 performing **BACKUP**(v_3) and thread 3 performing **SELECT**(v_3). In the SBS scenario, synchronization is required. Otherwise, a race condition may occur between (i) thread 3 reading the value of $Q(v_3)$, and (ii) before thread 3 can read the value of $N(v_3)$, thread 2 updates the value of $Q(v_3)$ and $N(v_3)$. Thus what happens is that when thread 3 reads the value of $N(v_3)$, the variables $Q(v_3)$ and $N(v_3)$ are not in the same state anymore and therefore thread 3 reads an inconsistent set of values ($Q(v_3)$ and $N(v_3)$).

3 RELATED WORK

In this section, we present the related work for two categories of synchronization methods for tree parallelization: (1) lock-based methods and (2) lock-free methods.

3.1 Lock-based Methods

As already mentioned, one of the main challenges in tree parallelization is to prevent data race conditions using synchronization. Figure 3 shows the tree parallelization where two threads (1 and 2) simultaneously perform the **EXPAND** operation on a node (v_6) of the tree. There are two methods to create synchronization in this case for tree parallelization: (1) coarse-grained lock (Chaslot et al., 2008a), (2) fine-grained lock (Chaslot et al., 2008a):

1. The coarse-grained lock method uses one lock

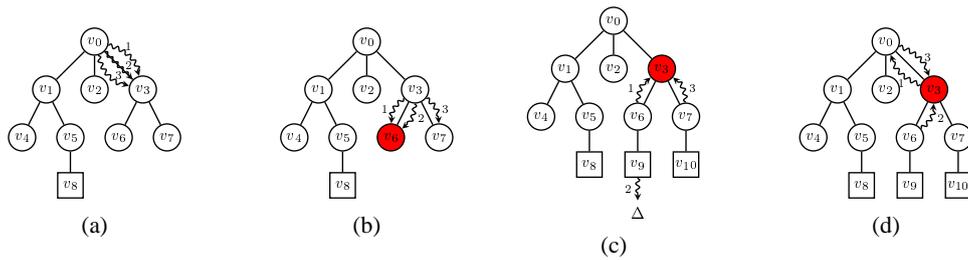


Figure 2: (2a) The initial search tree. The internal and non-terminal leaf nodes are circles. The terminal leaf nodes are squares. The curly arrows represent threads. (2b) Thread 1 and 2 are expanding node v_6 . (2c) Thread 1 and 2 are updating node v_6 . (2d) Thread 1 is selecting node v_3 while thread 2 is updating this node.



Figure 3: Tree parallelization with coarse-grained lock.

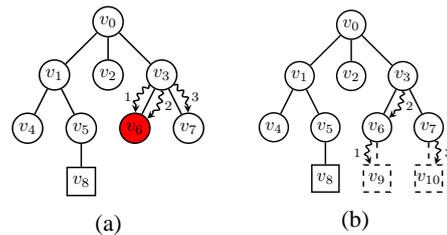


Figure 4: Tree parallelization with fine-grained lock.

to protect the entire search tree (Chaslot et al., 2008a). For example in Figure 3a, both thread 1 and 2 want to expand node v_6 , then thread 1 first acquires a lock; subsequently, it performs the EXPAND operation and finally releases the lock. During this process thread 2 also wanting to perform the EXPAND operation on node v_6 should wait for the release of the lock (see Figure 3b). This method is called coarse-grained because the access to the tree for performing the EXPAND operation will be given to one and only one thread. Even if multiple threads want to expand different nodes inside the tree. For example in Figure 3a, thread 3 also wants to perform the EXPAND operation but on node v_7 . However, the lock is already acquired by thread 1. Therefore, thread 3 should wait until the lock is released (see Figure 3b).

2. The fine-grained lock method uses one lock for each node of the tree to protect a smaller part of the search tree and to allow a greater level of concurrency in accesses to the search tree (Chaslot et al., 2008a). For example in Figure 4a, thread 3 also wants to perform the EXPAND operation but on node v_7 . It can acquire the lock in v_7 and should not wait (see Figure 3b).

Both lock-based methods use locks to protect shared data. However, these approaches suffer from synchronization overhead due to thread contentions and do not scale well (Chaslot et al., 2008a). A lock-free method can remove these problems.

3.2 Lock-free Methods

A lock-free implementation exists in the FUEGO package (Enzenberger and Müller, 2010). However, the method in (Enzenberger and Müller, 2010) does not guarantee the computational consistency of the multithreaded program with the single-threaded program. To address the SE race condition, Enzenberger et al. assign to each thread an own memory array for creating nodes (Enzenberger et al., 2010). Only after the children are fully created and initialized, they are linked to the parent node. Of course, this causes memory overhead. What usually happens is the following. If several threads expand the same node, only the children created by the last thread will be used in future simulations. It can also happen that some of the children that are lost in this way already received some updates; these updates will also be lost. It means that Enzenberger et al. ignore the SB and SBS race conditions. They accept the possible faulty updates and the inconsistency of parallel computation.

In the PACHI package (Baudiš and Gailly, 2011), the method in (Enzenberger and Müller, 2010) is used for performing lock-free tree updates. Again, it means that both SB and SBS race conditions are neglected. However, to allocate children of a given node, PACHI does not use a per-thread memory pool as FUEGO does, but uses instead a pre-allocated global node pool and a single atomic increment instruction updating the pointer to the next free node. This solves the memory overhead problem in FUEGO. However, there are still two other issues with this method: (1) the number of required nodes should be known in advance, and (2)

the children of a node may not be assigned in consecutive memory locations which results in poor *spatial locality* (i.e., if a particular memory location is referenced at a particular time, then it is likely that nearby memory locations will be referenced in the near future). The spatial locality is specifically important for the SELECT operation.

4 OUR PROPOSED LOCK-FREE TREE DATA STRUCTURE AND ALGORITHM

Algorithm 2 shows our new lock-free tree data structure of type *Node*. The UCT algorithm that uses the proposed data structure is given in Algorithm 3 (for the difference, see the end of this section).

Algorithm 2 uses the new multithreading-aware memory model of the C++11 Standard (Williams, 2012). To avoid the race conditions, the ordering of memory accesses by the threads has to be enforced (Williams, 2012). In our lock-free approach, we use the synchronization properties of the *atomic* operations to enforce an ordering between the accesses. We have used the atomic variants of the built-in types (i.e., *atomic_int* and *atomic_bool*); they are lock-free on all most popular platforms. The standard atomic types have different member functions such as *load()*, *store()*, *exchange()*, *fetch_add()*, and *fetch_sub()*. The differences are subtle. The member function *load()* is a load operation, whereas the *store()* is a store operation. The *exchange()* member function is special. It replaces the stored value in the atomic variable by a new value and automatically retrieves the original value. Therefore, we use two memory models for the memory-ordering option for all operations on atomic types: (1) *sequentially consistent* ordering (*memory_order_seq_cst*) and (2) *acquire_release* ordering (*memory_order_acquire* and *memory_order_release*). The default behavior of all atomic operations provides for *sequentially consistent* ordering. This implies that the behavior of a multithreaded program is consistent with a single threaded program. In the *acquire_release* ordering model, *load()* is an *acquire* operation, *store()* is a *release* operation, *exchange()* or *fetch_add()* or *fetch_sub()* are either *acquire*, *release* or both (*memory_order_acq_rel*).

In Algorithm 2 each node v stores nine different pieces of data: (1) a the action to be taken, (2) p the current player at node v , (3) w_n (a 64-bit atomic integer) that stores both the total simulation reward $Q(v)$ and the visit count $N(v)$, (4) the list of children, (5) the *is_parent* flag (an atomic boolean) that

Algorithm 2: The new lock-free tree data structure.

```

1 type
2   type a : int;
3   type p : int;
4   type w_n : atomic_int_64;
5   type children : Node*[];
6   type is_parent := false : atomic_bool;
7   type n_nonexpanded_children := -1 : atomic_int;
8   type is_expandable := false : atomic_bool;
9   type is_fully_expanded := false : atomic_bool;
10  type parent : Node*;
11  Function CREATECHILDREN(actions) : <void>
12    if is_parent.exchange(true) is false then
13      j := 0;
14      while actions is not empty do
15        choose a' ∈ actions;
16        add a new child v' with a' as its action
17        and p' as its player to the list of
18        children;
19        j := j+1;
20      end
21      n_nonexpanded_children.store(j);
22      is_expandable.store(
23        true, memory_order_release);
24    end
25  Function ADDCHILD() : <Node*>
26    index := -1;
27    if is_expandable.load(memory_order_acquire) is
28    true then
29      if (index :=
30        n_nonexpanded_children.fetch_sub(1)) is 0
31      then
32        is_fully_expanded.store(true);
33      end
34      if index < 0 then
35        return current node;
36      else
37        return children[index];
38      end
39    else
40      return current node;
41    end
42  Function ISFULLYEXPANDED() : <bool>
43    return is_fully_expanded.load();
44  Function GET() : <int,int>
45    w_n' := w_n.load();
46    w := high 32 bits of w_n';
47    n := low 32 bits of w_n';
48    return <w,n>;
49  Function SET(int Δ)
50    w_n' := 0;
51    high 32 bits of w_n' := Δ;
52    low 32 bits of w_n' := 1;
53    w_n.fetch_add(w_n');
54  Function UCT(int n) : <float>
55    <w',n'> := GET();
56    return w'/n + 2C_p √(2ln(n)/n)
57  Node;

```

Algorithm 3: The Lock-free UCT algorithm.

```

1  Function UCTSEARCH(Node* v0, State s0, budget)
2  |   while within search budget do
3  |   |   < vl, sl > := SELECT(v0, s0);
4  |   |   < vl, sl > := EXPAND(vl, sl);
5  |   |   Δ := PLAYOUT(vl, sl);
6  |   |   BACKUP(vl, Δ);
7  |   end
8  Function SELECT(Node* v, s) : <Node*, State>
9  |   while v.ISFULLYEXPANDED() do
10 |   |   < w, n > := v.GET();
11 |   |   vl := argmaxvj ∈ children of v vj.UCT(n);
12 |   |   s := v.p takes action vl.a from state s;
13 |   |   v := vl;
14 |   end
15 |   return < v, s >;
16 Function EXPAND(Node* v, State s) : <Node*, State>
17 |   if s is non-terminal then
18 |   |   actions := set of untried actions from state s;
19 |   |   v.CREATECHILDREN(actions);
20 |   |   v' := v.ADDCHILD();
21 |   |   if v' is not v then
22 |   |   |   v := v';
23 |   |   |   s := v.p takes action v.a from state s;
24 |   |   end
25 |   end
26 |   return < v, s >;
27 Function PLAYOUT(State s)
28 |   while s is non-terminal do
29 |   |   choose a ∈ set of untried actions from state s
30 |   |   |   uniformly at random;
31 |   |   s := the current player p takes action a from state s;
32 |   end
33 |   Δ(p) := reward for state s for each player p;
34 |   return Δ
35 Function BACKUP(Node* v, Δ) : void
36 |   while v is not null do
37 |   |   v.SET(Δ(v.p));
38 |   |   v := v.parent;
39 |   end

```

shows whether the list of children is already created, (6) *n_{nonexpanded_children}* the number of children that are not expanded yet, (7) the *is_expandable* flag (an atomic boolean) that shows whether *v* is ready to be expanded, (8) the *is_fully_expanded* flag (an atomic boolean) that shows whether all children of *v* are already expanded and (9) *parent* that points to the parent of *v*. By using (a) the atomic variables, (b) the atomic operations, and (c) the associated memory models, we can solve all the three above cases of race conditions (SE, SB, and SBS).

- SE: To solve the SE race condition, the EXPAND operation in Algorithm 3 consists of two separate

sub-operations: (A) the CREATECHILDREN operation and (B) the ADDCHILD operation. The first operation has four key steps (A-1, A-2, A-3, A-4) which are given in Algorithm 2. (A-1): Exchanging the value of *is_parent* from *false* to *true* prevents the other threads to create the list of children (Line 12). Thus, the problem that the list of children is created by two threads at the same time is solved. (A-2): Creating the list of children (Line 14-18). (A-3): Set the value of *n_{nonexpanded_children}* to counter *j* (Line 19), (A-4): Set the value of *is_expandable* to *true* (Line 20). After a node successfully has become a parent, one of the non-expanded children in its list of children can be added using the ADDCHILD operation. The ADDCHILD operation in Algorithm 2 has three key steps (B-1, B-2, B-3). (B-1): Read the value of *is_expandable* (Line 24), if it is *true*, try to expand a new child (Line 25-32). Otherwise, return the current node (Line 34). (B-2): The value of *index* is calculated (Line 25), if it is zero, then node *v* is fully expanded (Line 26). (B-3): *index* shows the next child to be expanded (Line 31), if *index* becomes negative, the current node is returned (Line 29).

- SB: To solve the SB race condition, Algorithm 2 uses a single 64-bit atomic integer *w_n* for storing both variables *Q(v)* and *N(v)*. The value of *Q(v)* is stored in the high 32 bits of *w_n*, while the value of *N(v)* is stored in low 32 bits. This compression technique preserves the correct state of the variables *Q(v)* and *N(v)* in all threads because they should always be written together using a SET operation. Therefore, we have no faulty updates and guarantee consistency of computation.
- SBS: To solve the SBS race condition, Algorithm 3 always reads *w_n* variable by a GET operation in the SELECT operation. The GET operation always reads the value of *Q(v)* and *N(v)* together. If a BACKUP operation wants to update the *w_n* variable in the same time, it happens through a SET operation which writes the value of *Q(N)* and *N(v)* together. Therefore, the value of *Q(v)* and *N(v)* are always correct, in the same state, and consistency of computation is guaranteed.

In Algorithm 3, each node *v* is also associated with a state *s*. The state *s* is recalculated as the SELECT and EXPAND steps descend the tree. The term $\Delta(p)$ denotes the reward after simulation for each player.

Algorithm 4: The pseudo-code of GSCPM algorithm.

```

1 Function GSCPM(State  $s_0, nPlayouts, nTasks$ )
2    $v_0 :=$  create a shared root node with state  $s_0$ ;
3    $grain\_size := nPlayouts / nTasks$ ;
4    $t := 1$ ;
5   for  $t \leq nTasks$  do
6      $s_t := s_0$ ;
7     fork UCTSEARCH( $v_0, s_t, grain\_size$ ) as task  $t$ ;
8      $t := t + 1$ ;
9   end
10  wait for all tasks to be completed;
11  return action  $a$  of best child of  $v_0$ ;

```

5 IMPLEMENTATION

We have implemented the proposed lock-free data structure and algorithm in the **ParallelUCT** package (Mirsoleimani et al., 2015). The implementation is available online as part of the package. The ParallelUCT package is an open source tool for parallelization of the UCT algorithm.¹ It uses *task-level parallelism* to implement different parallelization methods for MCTS. We have used an algorithm called *grain-sized control parallel MCTS* (GSCPM) to implement and measure the performance of the proposed lock-free UCT algorithm. The pseudo-code for GSCPM is given in Algorithm 4. The GSCPM is implemented by multiple methods from different parallel programming libraries such as C++11 STL, thread pool (*TP-FIFO*), TBB (*task_group*) (Reinders, 2007), and Cilk Plus (*cilk_for* and *cilk_spawn*) (Robison, 2013) in the ParallelUCT package. More details about each of these methods can be found in (Mirsoleimani et al., 2015).

6 EXPERIMENTAL SETUP

Section 6.1 discusses our case study, Section 6.2 explains the performance metrics, and Section 6.3 provides the details of hardware.

6.1 The Game of Hex

The performance of the lock-free algorithm is measured by using the game of Hex. Hex is a board game with a diamond-shaped board of hexagonal cells (Arneson et al., 2010). The game is usually played on a board of size 11 on a side, for a total of 121 hexagons, as illustrated in Figure 5 (Weisstein, 2017). Each player is represented by a color (Black or White).

¹<https://github.com/mirsoleimani/paralleluct/>

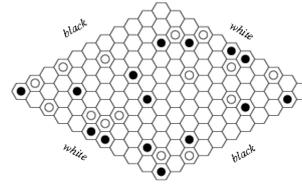


Figure 5: A sample board for the game of Hex

Players take turns by placing a stone of their color on a cell on the board. The goal for each player is to create a connected chain of stones between the opposing sides of the board marked by their colors. The first player to complete this path wins the game. The game cannot end in a draw since no path can be completely blocked except by a complete path of the opposite color. Since the first player to move in Hex has a distinct advantage, the swap rule is generally implemented for fairness. This rule allows the second player to choose whether to switch positions with the first player after the first player has made the first move.

In our implementation of Hex, a disjoint-set data structure is used to determine the connected stones. Using this data structure the evaluation of the board position to find the player who won the game becomes very efficient (Galil and Italiano, 1991).

6.2 Performance Metrics

One important metric related to performance and parallelism is speedup. Speedup compares the time for solving the identical computational problem on one worker versus that on P workers:

$$speedup = \frac{T_1}{T_P}. \quad (2)$$

Where T_1 is the time of the program with one worker and T_P is the time of the program with P workers. In our results we report the scalability of our parallelization as *strong scalability* which means that the problem size remains fixed as P varies. The problem size is the number of playouts (i.e., the search budget) and the P is the number of tasks. In the literature this form of speedup is called *playout-speedup* (Chaslot et al., 2008a).

The second important metric in two-player games, such as Hex, is the percentage of win for method a versus method b :

$$win(\%) = \frac{W_a}{W_a + W_b} * 100. \quad (3)$$

Where W_a is the number of wins for method a and W_b is the number of wins for method b . If there is a draw, it will be counted as a win for both players. In

Hex, there is always a winner. We note that we have used the swap rule. Each method played half of the games as Black and the other half as White.

6.3 Hardware

Our experiments were performed on a dual socket Intel machine with 2 Intel Xeon E5-2596v2 CPUs running at 2.4 GHz. Each CPU has 12 cores, 24 hyper-threads, and 30 MB L3 cache. Each physical core has 256KB L2 cache. The peak TurboBoost frequency is 3.2 GHz. The machine has 192GB physical memory. We compiled the code using the Intel C++ compiler with a `-O3` flag.

7 EXPERIMENTAL RESULTS

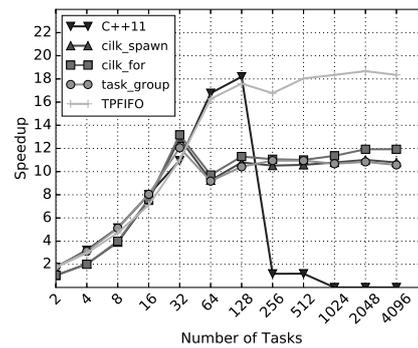
In Section 7.1, the scalability is studied and the achieved playout-speedup is reported. The effect of differences in values of C_p parameters on the speedup of the parallel algorithm is measured in Section 7.2. The performance of the proposed lock-free algorithm for tree parallelization when playing against root parallelization is reported in Section 7.3.

7.1 Playout-speedup

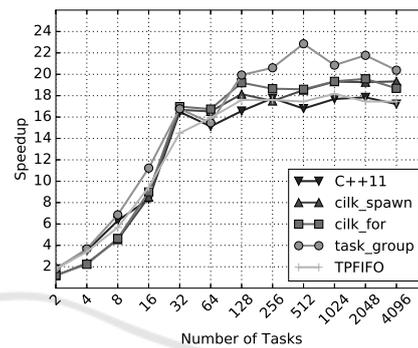
As mentioned before, we are interested in strong scalability. Therefore, the search budget is fixed to 1,048,576 playouts as the number of tasks are increasing. Figure 6 shows the scalability of the algorithm for different parallel programming libraries when the first move on the empty board is made. Each data point is the average of 21 games. Figure 6a illustrates the scalability when a coarse-grained lock is used (The graph is taken from (Mirsoleimani et al., 2015)) and Figure 6b demonstrates the scalability when the proposed lock-free method is used. There are three main improvements when the lock-free tree is used: (1) the maximum speedup increased from 18 to 23. (2) the scalability of all methods is improved (It shows the notoriously bad effect of locks on the scalability for Cilk Plus, TBB, and C++11). (3) 32 tasks are sufficient to reach near 17 times speedup, while for the lock-based method at least 64 tasks are required.

7.2 The Effect of C_p on Playout-speedup

Table 1 shows the execution time of the sequential UCT algorithm for three different C_p values. It is observed that the execution time is decreasing as the value of C_p is increasing. There is an obvious explanation for this behavior. When the algorithm uses



(a)



(b)

Figure 6: The scalability of tree parallelization for different parallel programming libraries when $C_p = 1$. 6a Coarse-grained lock. 6b Lock-free.

high exploitation (i.e., low value for C_p), it constructs a search tree that is deeper and more asymmetric. In Figure 7b, the depth of the tree is 56 when the number of tasks is 1 and $C_p = 0$. When the shape of the tree is more asymmetric, each iteration of the algorithm must traverse a deeper path of nodes inside the tree using the SELECT operation until it can perform a PLAYOUT operation. The SELECT operation consist of a *while loop* which for a tree with the depth of 56 has to perform 56 iterations in the worst case (see Algorithm 3). The BACKUP operation also consists of a *while loop* which for a deeper tree has more iterations. These two operations are also memory intensive ones (i.e., accessing the nodes of the tree which reside in memory). The results are that the execution time of the sequential algorithm becomes higher for high exploitation. Increasing the value of C_p means more exploration and thus a more symmetric tree with a lower depth. In Figure 7b, the depth of the tree is 5 when the number of tasks is 1 and $C_p = 1$. In this case, the *while loop* in the SELECT operation has to perform only 5 iterations in the worst case.

We have measured the scalability of the proposed lock-free algorithm for different C_p values (see Fig-

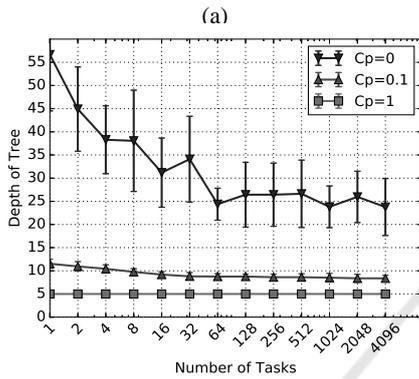
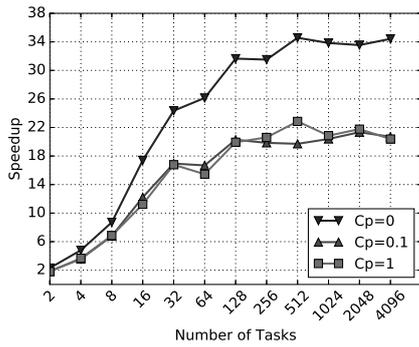


Figure 7: 7a The scalability of the algorithm for different C_p values. 7b The changes in the depth of tree when the number of tasks are increasing.

ure 7a). The sequential time for each C_p in Table 1 is used as the baseline. The maximum speedup for $C_p = 0$ is around 34. It is much higher than 23 times, the speedup when $C_p = 1$. There is a possible explanation for the higher speedup. The parallel algorithm may be more efficient than the equivalent serial algorithm, since the parallel algorithm may be able to avoid work that in every serialization would be forced to be performed (McCool et al., 2012). For example, Figure 7b shows the changes in the depth of the constructed tree with regards to the number of tasks for three different C_p . Increasing the number of tasks reduces the depth of the tree from 56, when the serial execution is exploitative (i.e., $C_p = 0$), to around 25. It means that, in parallel execution (a) threads explore different branches of the tree and (b) the tree is more symmetric compared to the serial execution. Hence, the number of iterations in both SELECT and BACKUP operations reduces in parallel execution and therefore causes a higher speedup. When the serial execution has high exploration (i.e., $C_p = 1$), increasing the number of tasks does not change the depth of the tree.

Table 1: Sequential execution time in seconds.

C_p	Time (s)	Depth of Tree (Avg.)
0	59.97 ± 10.93	56.66 ± 12.16
0.1	26.66 ± 0.81	11.52 ± 0.98
1	20.7 ± 0.3	5

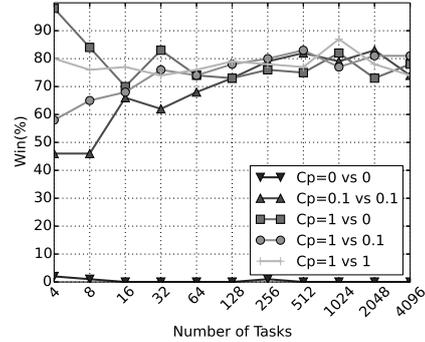


Figure 8: The playing results for lock-free tree parallelization versus root parallelization. The first value for C_p is used for tree parallelization and the second value is used for root parallelization.

7.3 Playing vs. Root Parallelization

In this section, the result of playing Hex between the proposed lock-free tree parallelization against root parallelization is presented. Root parallelization is also a parallelization method that does not use locks because it uses an ensemble of independent search trees. Therefore, it is interesting to see the performance of the proposed lock-free algorithm versus root parallelization. Figure 8 reports the percentage of win for lock-free tree parallelization for five different combinations of C_p . Both methods use a same number of tasks. For each data point, 100 games are played.

When $C_p = 0$ for both algorithms, tree parallelization cannot win against root parallelization. It shows that the high speedup for $C_p = 0$ (see Figure 7a) is not useful. However, when the value of C_p is selected to be more exploratory, the lock-free tree parallelization is superior to root parallelization, specifically for a higher number of tasks.

8 CONCLUSION

Monte Carlo Tree Search (MCTS) is a randomized algorithm that is successful in a wide range of optimization problems. The main loop in MCTS consists of individual iterations for constructing a search tree,

suggesting that the algorithm is well suited for parallelization. The existing tree parallelization for MCTS uses a shared search tree and runs the iterations in parallel. However, the shared search tree has potential race conditions. In this paper, we have presented a new lock-free algorithm that has no race conditions. It showed better scalability and playout-speedup when compared to other synchronization methods. Currently, we have used the default sequential consistency memory ordering for all atomic operations because that is the most convenient way to explain the intricacies. For future work, we will look at reducing a selected set of the ordering constraints to the relaxed-memory ordering.

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