A Pooling Strategy for Flexible Repair Shop Designs

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Abstract: We discuss the design problem of a repair shop in a single echelon repairable multi-item spare parts supply system. The repair shop consists of several parallel multi-skilled servers, and storage facilities for the repaired items. The effectiveness of repair shops and the total cost of a spare part supply system depend highly on the design of repair facility and the management of inventory levels of the spare parts. In this paper, we concentrate on a design scheme known as pooling. A repair shop can be considered as a pooled structure if the spare parts can be divided into clusters such that each part type is unambiguously assigned to a single cluster (cell). Nonetheless, it is both an important and tough combinatorial optimization question to determine which type of spares to pool together. We propose a sequential solution heuristic to find the best pooled design by considering inventory allocation and capacity level designation of the repair shop. The numerical experiments show that the suggested solution approach has a reasonable algorithm run time and yields considerable cost reductions.

1 INTRODUCTION

Maintenance costs can constitute up to 60% of the production costs of manufacturing firms (Keizer et al., 2016). Therefore, thoughtful planning of maintenance operations not only leads to a decrease in the total cost but also significant improvements in the reliability of systems (López-Santana et al., 2016). Maintenance planning includes the determination of the maintenance strategy, time interval between maintenance operations, and quantity and quality of maintenance resources such as technicians, supplies and spare parts (Duffuaa, 2000). In this paper, the corrective maintenance of high-valued assets in particular decisions regarding the amount of spare part inventory, capacity and design of repair facilities are analyzed.

The dominant inventory model for repairable spare part is called METRIC (Multi-Echelon Technique for Recoverable Item Control), developed by Sherbrooke (1968). The METRIC based models assume that the repair capacity is infinite. This assumption may not be appropriate in most industrial settings. Thus, some researchers have relaxed ample repair capacity assumption (Diaz and Fu, 1997; Rappold and Van Roo, 2009; Sleptchenko et al., 2003; Srivathsan and Viswanathan, 2017).

The performance of repairable spare part system also depends on the design of the repair facility. There exist several different design alternatives for repair shops. The two extreme situations are the dedicated (no cross-training) and fully flexible (full cross-training) designs. In a dedicated design, each cluster of servers is responsible for repairing a specific spare part type. On the other hand, in a fully flexible architecture all of the servers are merged into a single cluster that serves all of the failed parts. Nevertheless, in many real-life situations, cross-training all servers to handle all failed item types may not be feasible due to cost and/or quality penalties arising from cross-training and/or scarcity of servers capable of handling all of the item types (Tekin et al., 2009; Qin et al., 2015).

Pooling is an intermediate level design between the dedicated and fully flexible systems. Pooling means clustering of repairable items in the repair shop by using some measure of similarity. In other words, a repair shop has a pooled structure if the spare parts

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can be divided into clusters such that each part type is unambiguously assigned to a single cluster (cell). Nonetheless, it is both an important and tough combinatorial optimization question to determine as to which type of spares to pool together.

Even though pooling of resources are extensively studied, to the best of our knowledge, no results have been presented on pooled repair shops designs in spare part supply systems integrated with capacity decision. Thus, this study will contribute to the current literature by addressing following fundamental issues: (i) how many clusters to pool; (ii) which spare part types to pool together, and; (iii) how to assign servers (capacity) to each cluster.

The rest of the paper is organized as follows. In Section 2, problem definition and the mathematical model are presented. The solution algorithm for the proposed model is discussed in Section 3. Section 4 provides a computational study under different scenarios and input settings. Conclusions and future research directions are summarized in Section 5.

2 PROBLEM DESCRIPTION AND FORMULATION

The design problem of a repair shop in a single echelon repairable multi-item spare part supply system being considered here is illustrated in Figure 1. The repair shop consists of several parallel multi-skilled servers, and storage facilities for the repaired items. Once a failed part received from technical system at the installed base, it is queued to be served by a suitable server with the required skills. At the same time, if a repaired (as-good-as-new) part is available in the inventory, it is sent back to the installed base. If the item is not available in the stock, the request is backordered. In this case, the technical system goes down and a downtime cost occurs till the requested ready-for-use part is delivered.

The repair shop may have pooled structure with one or more cells/clusters or an arbitrary structure. In Figure 1(a), an example of a pooled repair shop with five types of failed stock keeping units (SKUs), two clusters and four servers is shown. The first cluster has a dedicated server with the ability to serve one type of SKU. On the other hand, the second cluster is obtained by pooling remaining four SKUs, and this cluster has three cross-trained servers to serve all (four) different types of repairables in that cluster. On the other hand, in Figure 1(b), an arbitrary design is presented, where the first and the second clusters have 'N' and 'W' structures, respectively. It is should be noted that in arbitrary designs, not all of the servers in a cluster are fully flexible; i.e., some servers are partially cross-trained to repair only a subset of all SKUs in the cluster. In this paper, we restrict design alternatives limited to only pooled repair shops as in Figure 1(a), and formulate a stochastic mixed-integer mathematical programming model to find the minimum cost spare part supply system. The sets, parameters and decision variables for the developed formulations and solution procedures are presented as follows.

Decision variables:

- S_i: Amount of initial inventory (basestock level) kept on stock for SKU type i ($i = 1,\ldots,N$), where $\mathbf{S} = (S_1,\ldots,S_N)$.
- z_k: Number of the operational servers in the cluster k ($k = 1,\ldots,y$), and where $\mathbf{Z} = (z_1,\ldots,z_y)$.
- x_{ik}: Binary variable indicating that whether the cluster k has a skill to repair SKU type i ($i = 1,\ldots,N$) or not, where $\mathbf{X}_k = (x_{1k},\ldots,x_{Nk})^T$ and $\mathbf{X} = [\mathbf{X}_1|\ldots|\mathbf{X}_y]$.
- y: Number of clusters in the repair shop.

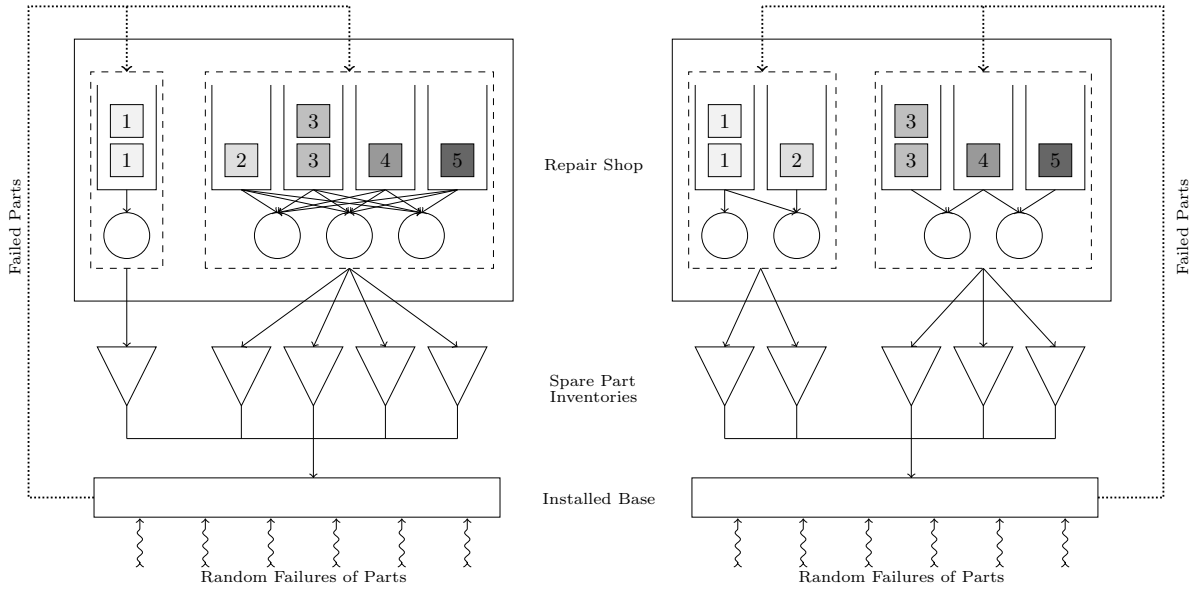
Problem parameters:

- N: Number of distinct type of repairables (SKUs).
- λ_i Failure rate of SKU type i ($i = 1,\ldots,N$).
- μ_i: Service rate of SKU type i ($i = 1,\ldots,N$).
- h_i: Inventory holding cost of SKU type i per unit time per part ($i = 1,\ldots,N$).
- b: Penalty cost for each back ordered demand per unit time, which is equivalent to paying per unit time per technical system that is down because of a lack of spare parts.
- f: Operation cost of a server per unit time (e.g., annual wage).
- c_i: Cost of having skills to repair SKU type i per unit time per server ($i = 1,\ldots,N$) (e.g., annual qualification bonus).
- ε: Very small positive real number.

The objective function in Eq.(1) has four cost terms namely server (capacity), cross-training, holding and backorder costs. Objective function considers several trade-offs between the cost terms such as the cost of holding excess inventory and the cost of downtime, and also the trade-off between the cost of having single or several clusters that include dedicated or cross-trained servers.

$$\min_{\mathbf{S},\,\mathbf{X},\,\mathbf{Z}} \sum_{k=1}^{y} f z_k + \sum_{k=1}^{y} z_k \left(\sum_{i=1}^{N} c_i x_{ik} \right) + \sum_{i=1}^{N} h_i S_i + b \sum_{i=1}^{N} \mathbb{EBO}_i [S_i, \mathbf{X}, \mathbf{Z}] \quad (1)$$

The *penalty (backorder)* cost term is calculated using the penalty cost b and the expected total number of backordered parts $\mathbb{EBO}_i [S_i, \mathbf{X}, \mathbf{Z}]$ for each SKU type



(a) Pooled Design with Two Clusters (b) 'N' and 'W' Designs inside Two Clusters

Figure 1: A comparative example for single echelon spare part supply systems with different (pooled vs. not pooled) repair shop designs.

i in the steady-state; under the given initial inventory level S_i, pooling scheme of the repair shop $\mathbf{X}$ and the server assignment policy $\mathbf{Z}$. The variable $\mathbf{X}$ represents the $(N \times y)$ matrix of the binary decision variables x_{ik} denoting how SKUs are pooled in the repair shop, and the variable $\mathbf{Z}$ represents a $(1 \times y)$ row matrix of integer decision variables z_k denoting the number of servers in the each cluster of the repair shop.

Constraints (2) and (4) ensure that pooling scheme $\mathbf{X}$ satisfies mutually exclusive and total exhaustive condition for each cluster, i.e., any SKU type being repaired by exactly one cluster. Queues (number of waiting failed spares) in each cluster have to be in finite queue length at steady-state to prevent overloading of repair shop. Thus, the stability of system is guaranteed by constraint (3) and (5) by assigning sufficient number of servers to each cluster. Constraints between Eqs.(4)-(7) are required for non-negativity and integrality of the variables.

$$\sum_{k=1}^{y} x_{ik} = 1 \qquad i = 1,\ldots,N \quad (2)$$

$$\sum_{i=1}^{N} x_{ik} \frac{\lambda_i}{\mu_i} \leq z_k - \varepsilon \qquad k = 1,\ldots,y \quad (3)$$

$$x_{ik} \in \{0,1\} \qquad i = 1,\ldots,N \; k = 1,\ldots,y \quad (4)$$

$$z_k \in \mathbb{Z}^+ \qquad k = 1,\ldots,y \quad (5)$$

$$S_i \in \mathbb{N}_0 \qquad i = 1,\ldots,N \quad (6)$$

$$y \in \{1,\ldots,N\} \quad (7)$$

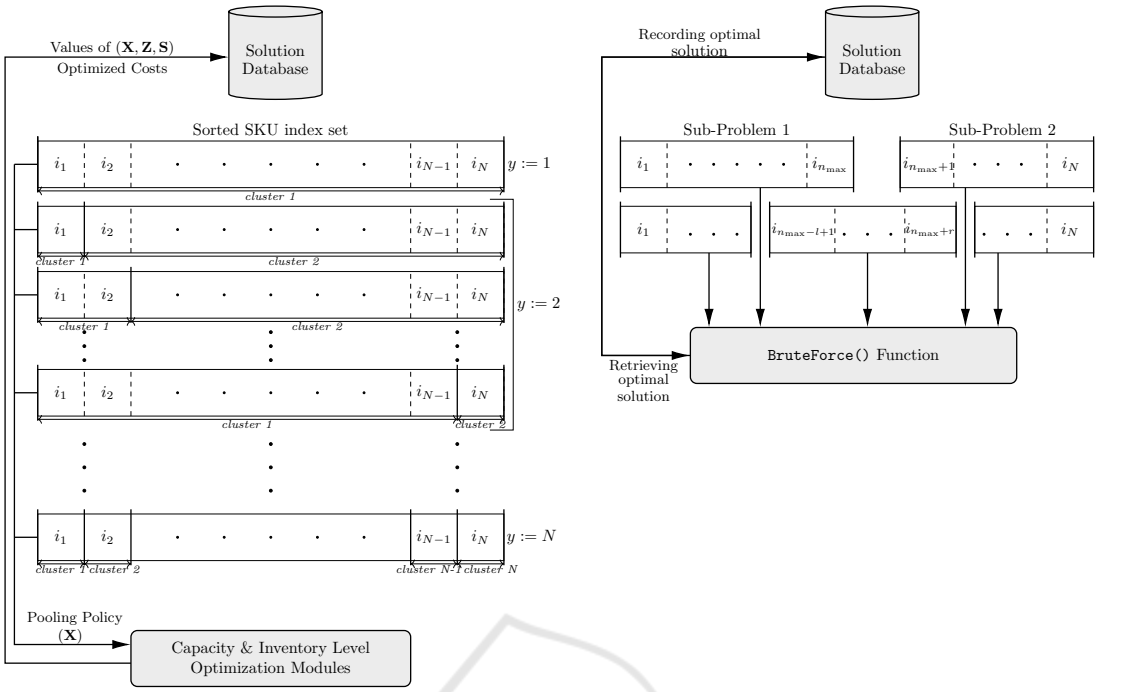
For not an overloaded system, the overall utilization rate of a particular cluster k ($\sum_{i=1}^{N} x_{ik}\lambda_i/\mu_i$) must be strictly smaller than the capacity (total number of servers in the cluster z_k) of that cluster, which is ensured by the parameter, ε.

3 SOLUTION ALGORITHM

The optimal values of decision variables are searched sequentially by fixing the values of some decision variables and optimizing the remaining ones. First, feasible partitions of SKUs, i.e., pooling policies $\mathbf{X}$ are generated. Afterwards, capacity levels $\mathbf{Z}$ and basestock inventory levels $\mathbf{S}$ are optimized under the given pooling scheme for each cluster. The visual flow of the proposed solution heuristic together with its sub-routines and their interactions with each other are depicted in Figure 2.

To find partitions of SKUs into clusters, all SKUs are sorted in ascending order by their service rates μ_i so that SKUs closer in service rates are likely to be in the same cluster. This is aimed for decreasing variations in the service times of SKUs in clusters. Decrease in the variation of service times usually leads to decrease in number of failed parts waiting for repair in the cluster and eventually lowering the number of backorders and the total cost.

In the next step, sorted list of SKUs is divided into smaller lists that have a size of n_{max} or less. The trade-off between the run time of algorithm and the output solution quality has to be taken into account when determining the value of n_{max}. We set n_{max} as 10 for our



(a) Pooling policy generation and optimization by `BruteForce()` function for $N \leq n_{\max}$

(b) Calling `BruteForce()` function and generating Sub-Problems for $N > n_{\max}$

Figure 2: The proposed sequential solution heuristic.

experimental runs. For the smaller list ($N \leq n_{max}$), `BruteForce()` function is invoked as depicted in Figure 2(a). `BruteForce()` takes an array of SKU indexes as an input and slices it into sub-arrays for given number of clusters y from 1 to the length of the input array. Each slice/sub-array corresponds to a cluster in a pooled repair shop, and each slicing scheme corresponds to a particular pooling policy $\mathbf{X}$. For the larger sorted SKU index sets, it is not possible to enumerate all slicing schemes with `BruteForce()` function. Thus, we divide the problem into sub-problems that have the maximum size of n_{max} or less as in Figure 2(b) and call `BruteForce()` for each sub-problem obtained after division. Then, we generate new sub-problems by combining the last and the first elements of adjacent sub-problems. At each iteration, we insert a new SKU index to newly generated sub-problem till the size of the problem reaches n_{max}.

After the generation of the pooling policy $\mathbf{X}$ via above described heuristic, capacity and inventory level optimization modules are called. These modules rely on the fact that for every feasible policy $\mathbf{X}$, each cluster can be analyzed and optimized separately due to the clusters being mutually exclusive and independent from each other. The decomposition of the repair shop in sub-systems by pooling reduces the complexity of the problem and enables the use of queue-theoretical approximations to optimize the inventory and capacity levels. In this direction, each cluster k in the repair shop for given number of servers can be analyzed as a multi-class multi-server $M/M/z_k$ queuing system.

The probability distribution of the number of failed SKU type i at the steady-state, $p_i(q)$, is required to evaluate $\mathbb{EBO}_i[S_i, \mathbf{Z}, \mathbf{X}]$. To calculate the probability distribution of the number of failed SKU type i, the approach proposed by Van Harten and Sleptchenko (2003) is used. Nonetheless, computational burden arises when number of SKU types and number of servers increases in the cluster. To overcome this issue, queuing approximation discussed in Altiok (1985) and Van Der Heijden et al. (2004) is used. In this approximation, marginal probability distribution (and several performance characteristics) of the SKU type i in the cluster k is derived by aggregating all other SKUs in the cluster k into a single SKU type (class). To obtain the remaining distributions for the other SKUs in the cluster, the procedure is repeated. Basically, a multi-class multi-server queuing system including two-class (two SKUs) is solved for each SKU in the cluster rather than solving one multi-class multi-server with larger number (total number of SKUs in the cluster) of SKUs.

By using the approximated distributions, $\tilde{p}_i(q)$,

the sum of holding and backorder costs $h_i S_i + b\mathbb{EBO}_i[S_i, \mathbf{Z}, \mathbf{X}]$ can be minimized by the smallest S_i for which Eq.(8) holds.

$$\sum_{q=0}^{S_i} \tilde{p}_i(q) \geq \frac{b-h_i}{b} \quad i = 1,\ldots,N \tag{8}$$

A detailed discussion of capacity and inventory level optimization modules can be found in Turan et al. (2017).

4 NUMERICAL STUDY

A full factorial design of experiment (DoE) with 7 factors and 2 levels per factor is used to generate the testbed with total of 128 test instances (Sleptchenko et al., 2016). The number of SKUs, N, and the initial total number of servers, M, are the first two DoE factors with levels 10 and 20 for the numbers of SKUs, and 5 and 10 for the initial numbers of servers. The failure rates and the service rates are generated based on the system (repair shop) utilization rate with the assumption that all SKUs are processed on all servers, i.e., a repair shop design with one cluster and fully flexible servers. The system utilization rate, ρ, is the third design factor with levels 0.65 and 0.80. For the chosen utilization rate, we randomly generate two sets of parameters:

(a) the failure rates λ_i, such that $\sum_{i=1}^{N} \lambda_i = 1$, and
(b) workload percentages δ_i, such that $\sum_{i=1}^{N} \delta_i = 1$.

Using the generated λ_i and δ_i, we produce the service rates μ_i as $\mu_i = \frac{\lambda_i}{\delta_i \rho M}$, where $\delta_i \rho M$ is the total workload of SKU type i. The pattern of the holding costs, h_i, is the fourth design factor with two variants (levels): (i) IND: completely randomly (independent) within a range $[h_{min}, h_{max}]$, and (ii) HPB: hyperbolically related to the workloads $w_i = \lambda_i/\mu_i = \delta_i \rho M$:

$$h_i = \frac{h_{max} - h_{min} + 10}{9\frac{w_i - w_{min}}{w_{max} - w_{min}} + 1} - 10 + h_{min} + \xi_i$$

where

$$\xi_i \in U[-\frac{h_{max} - h_{min}}{20}, \frac{h_{max} - h_{min}}{20}],$$

$$w_{min} = \min_{i=1,\ldots,N} w_i \text{ and } w_{max} = \max_{i=1,\ldots,N} w_i$$

The parameters of the hyperbolic relation are chosen such that it replicates some of the real-life scenarios where more expensive repairables are repaired less frequently. The minimum holding cost, h_{min}, is the fifth factor with levels 1 and 100. The maximum holding cost is fixed at 1,000. The server cost, f, and the skill cost, c_i, are the last two factors in our DoE. The server cost levels are set as 10,000 and 100,000 ($10h_{max}$ and $100h_{max}$). The skill cost is assumed as 1% or 10% of the chosen server cost for all SKUs. The penalty cost, b, is set as fifty-fold of the average holding cost so that about 98% of requests can be met from spare stocks. That means the probability of backorder is only 0.02. The overview of all factors and levels are presented in Table 1.

Table 1: Problem parameter variants for test bed.

Factors	Levels
No. of SKUs (N)	[10, 20]
No. of Initial servers (M)	[5, 10]
Utilization Rate (ρ)	[0.65, 0.80]
Minimum Holding Cost (h_{min})	[1, 100]
Maximum Holding Cost (h_{max})	1000
Holding cost/Workload relation	[IND, HPB]
Server Cost (f)	[$10h_{max}$, $100h_{max}$]
Cross-Training Cost (c_i)	[$0.01f$, $0.10f$]
Penalty Cost (b)	$50\frac{\sum_{i=1}^{N}\lambda_i h_i}{\sum_{i=1}^{N}\lambda_i}$,

The total system costs found by the proposed pooling heuristic are compared with the costs obtained from fully flexible (a single cluster where any SKU can be processed on any server) and dedicated (where number of clusters equal to number of SKUs) designs. Table 2 summarizes cost reductions for different problem factors. On an average, the heuristic that produce only pooled designs can produce ∼45% and ∼25% savings in comparison with dedicated and fully flexible designs, respectively. In some extreme settings, average cost reductions reach to 55% and 43% compared with dedicated and fully flexible systems, respectively.

Table 2: Average cost reductions by pooling under different factors.

Factor	Levels	Dedicated	Fully Flexible
Number of SKUs (N)	10	35.19%	22.00%
	20	55.65%	28.11%
Number of Initial Servers (M)	5	53.76%	23.43%
	10	37.08%	26.68%
Utilization Rate (ρ)	0.65	48.01%	24.30%
	0.80	42.83%	25.81%
Minimum Holding Cost (h_{min})	1	45.84%	24.81%
	100	45.00%	25.30%
Holding Cost/Work Load Relation	IND	44.42%	24.04%
	HPB	46.42%	26.08%
Server Cost (f)	$10h_{max}$	39.95%	22.44%
	$100h_{max}$	50.89%	27.67%
Cross-Training Cost (c_i)	$0.01f$	50.22%	9.93%
	$0.1f$	40.62%	40.18%
Average		**45.42%**	**25.06%**

The analysis also shows that when the cost of having an extra skill is relatively small compared to that for having an extra server (i.e., the case of cross-

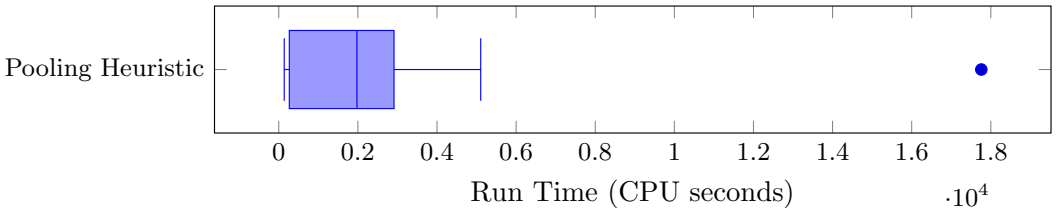


Figure 3: Run time performance of pooling heuristic.

training cost being equivalent $0.01f$), fully flexible design is as good as a pooled system. On the other hand, dedicated and fully flexible systems perform similarly where there is a high cost of cross-training.

Table 3 shows the average number of servers used, skills assigned to a server and the percentage of cross-training per server under different factors and solution algorithms. The percentage of cross-training is obtained from a ratio of the number of skills allocated into a server to the total number distinct SKUs, N, in the system. The dominate factor affecting the number of servers used is the number of initial servers, M. This factor sets an average value of the required number of operational servers to achieve a predetermined overall utilization rate for the whole system. Thus, we observe a nearly doubled increase from 5.03 to 9.17 in the average number of operational servers when the number of initial servers increases from 5 to 10.

Table 3: Capacity and cross-training analysis under different factors.

Factor	Levels	# server used	# skill per server	%cross training
Number of SKUs (N)	10	7.03	3.81	38.16
	20	7.17	5.58	27.93
Number of Initial Servers (M)	5	5.03	5.24	37.20
	10	9.17	4.16	28.89
Utilization Rate (ρ)	0.65	6.51	4.67	32.64
	0.80	7.68	4.73	33.45
Minimum Holding Cost (h_{min})	1	7.06	4.69	33.09
	100	7.14	4.70	33.00
Holding Cost/Work Load Relation	IND	7.23	4.88	34.17
	HPB	6.96	4.51	31.92
Server Cost (f)	$10h_{max}$	7.65	5.06	35.48
	$100h_{max}$	6.54	4.34	30.61
Cross-Training Cost (c_i)	$0.01f$	6.93	6.06	42.66
	$0.1f$	7.26	3.33	23.43
Average		**7.10**	**4.70**	**33.05**

At the optimal design, %cross-training per server fluctuates between 30%-40%, which shows that partial flexibility; i.e., partial cross-training is usually sufficient for optimal system performance.

All the experiments are implemented on a computer with 16 GB RAM and 2.8 GHz i7 CPU. Figure 3 shows a boxplot of run time performances for all cases. The presented heuristic converges quite fast in most of the cases and provides the final solution within 5000 cpu seconds with a median run time of 2000 seconds, which is still acceptable for tactical/operational levels decisions in real-life spare part supply systems.

5 CONCLUSIONS

When designing a spare part supply network for repairable parts that balances cost efficiency with effectiveness, several questions in both strategic and tactical nature have to be answered. In this article, the joint problem of resource pooling, inventory allocation and capacity level designation of the repair shop is analyzed and a solution heuristic is developed. Performed extensive numerical experiments conclude that the pooled designs result in cost savings of around 45% and 25% in comparison to dedicated and fully flexible designs, respectively. Besides, we observe that the optimal repair shop designs can be achieved by partially cross-training servers.

As further research possibilities, designing new clustering heuristics or meta-heuristics that will generate better pooling schemes with less computational complexity would be invaluable contribution. It would be also worthwhile to investigate effects of priority rules by taking into account service rates and costs characteristics of the SKUs.

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