Parallel Bubbles
Evaluation of Three Techniques for Representing Mixed Categorical and Continuous Data in Parallel Coordinates

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Abstract: Parallel Coordinates are a widely used visualization method for multivariate data analysis tasks. In this paper we discuss the techniques that aim to enhance the representation of categorical data in Parallel Coordinates. We propose *Parallel Bubbles*, a method that improves the graphical perception of categorical dimensions in Parallel Coordinates by adding a visual encoding of frequency. Our main contribution consists in a user study that compares the performance of three variants of Parallel Coordinates, with similarity and frequency tasks. We base our design choices on the literature review, and on the research guidelines provided by Johansson and Forsell (2016). *Parallel Bubbles* are a good trade-off between Parallel Coordinates and Parallel Sets in terms of performance for both types of tasks. Adding a visual encoding of frequency leads to a significant difference in performance for a frequency-based task consisting in assessing the most represented category. This study is the first of a series that will aim at testing the three visualization methods in tasks centered on the continuous axis, and where we assume that the performance of Parallel Sets will be worse.

1 INTRODUCTION

Parallel Coordinates (Inselberg and Dimsdale, 1990; Wegman, 1990) are a popular visualization method for representing multivariate data. The parallel axes represent dimensions, and each multivariate item corresponds to a polyline crossing the axes. This visualization method allows to reveal patterns quickly (Siirtola et al., 2009) and is thus a good tool for exploratory analysis tasks. A recurring problem of Parallel Coordinates is the visual clutter that occurs with large numbers of polylines: it hinders data analysts from identifying clusters and trends, and from extracting relevant information from the data. Moreover, representing categorical dimensions in Parallel Coordinates is not ideal since it increases the visual confusion: "either the frequency information is not visible or a ranking is imposed on the visual mapping transformation, influencing perception of the data" (Kosara et al., 2006). This is due to the fact that categorical dimensions are represented in the same way as continuous dimensions: the continuous design model used by Parallel Coordinates does not match the discrete user model of the data (Kosara et al., 2006). Another problem arising from the representation of categorical data is overplotting (Dang et al., 2010): it occurs when the discrete number of categorical values is significantly smaller than the size of the dataset, resulting in many samples sharing a given categorical value. This leads to "the increased likelihood of multiple lines passing successively through the same points" (Havre et al., 2006). In the frequent case that multiple items have identical values along neighboring axes, their lines overlap exactly, therefore leaving visible only the last line segment drawn, thus hiding the frequency information (Dang et al., 2010; Kosara et al., 2006). In summary, clutter and overplotting should be avoided as much as possible since they hide frequency information and prevent the detection of patterns. Reducing them is one of the main challenges in the design of Parallel Coordinates (Heinrich and Weiskopf, 2013). Researchers propose several methods to address these challenges, but very few present results form user-centered evaluations (Johansson and Forsell, 2016). There still lacks an evidence of measurable benefits that would encourage the use of a variant of Parallel Coordinates over standard ones. For example, frequency-based ap-

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proaches (Kosara et al., 2006) seem promising, but it is unclear whether these techniques positively affect the ability of users to quickly and reliably identify patterns in the data. Do the users correctly interpret categorical dimensions? Do these methods allow for a better performance in typical data analysis tasks?

In this paper, we first give an overview on the current state of the art of clutter reduction techniques for Parallel Coordinates. Next, we present an enhanced version of Parallel Coordinates, *Parallel Bubbles*, that aims to tackle the problem of overplotting instances when dealing with categorical dimensions. By adding a visual encoding of frequency, we assume that *Parallel Bubbles* will allow to detect more effectively the most represented categorical values, and thus should enhance the user performance in similarity and frequency tasks. In order to measure the gain of performance, we lead a controlled user experiment consisting of three data analysis tasks. We use randomly generated datasets composed of one continuous dimension and one categorical dimension. In the last section, we describe and discuss the outcomes of this user experiment.

2 STATE OF THE ART

Parallel Coordinates (Inselberg and Dimsdale, 1990; Wegman, 1990) were initially designed to represent continuous dimensions. Kosara et al. (2006) state that when dealing with multivariate datasets, the divergence between the user's mental model and the visual representation of the data can be eliminated by the use of frequency-based techniques. These techniques represent categories by visual entities that are scaled proportionally to their corresponding frequency (Kosara et al., 2006). In this chapter, we give an overview on the existing overplotting reduction methods, designed to enhance the visual representation of categorical data. We group these methods in the four categories proposed by Heinrich and Weiskopf (2013), namely filtering, aggregation, spatial distortion, and the use of colors.

2.1 Clutter Reduction Methods

2.1.1 Filtering

Filtering consists in removing signals from the input. Brushing (Shneiderman, 1994) is one example: specifying values ranges and using logical operators (Martin and Ward, 1995), allows to render only a portion of the polylines, reducing visual clutter and overplotting. However, brushing is of little use with non-

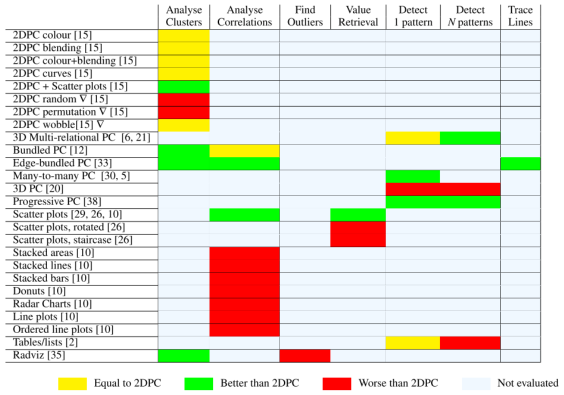


Figure 1: Comparison of evaluations of 26 visualization techniques in relation to standard Parallel Coordinates (2DPC). "A yellow colour indicates no significant difference in performance. A green colour means that the technique outperforms 2DPC for the specific task. A red colour means that the technique performs worse than 2DPC. A light blue colour shows that no evaluation has been found in the literature. ∇ denotes that the technique is based on animation" (Johansson and Forsell, 2016). Source: Johansson and Forsell (2016).

ordered categorical dimensions: selecting a range of categories is not useful since their order does not have any meaning. A good way to implement filtering for non-ordered dimensions is to allow the user to select a discrete amount of categories instead of a range, by using checkboxes.

2.1.2 Aggregation

Aggregation techniques are based on the principle of grouping data, and representing aggregate items instead of individual samples (Heinrich and Weiskopf, 2013). Such aggregates include mean, median or cluster centroid of a subset of samples. Density functions are another method, and allow to reveal dense areas and clusters in the data: the mathematical model of density in parallel coordinates proposed by Heinrich and Weiskopf (2009) is an example of this technique. The Angular Histogram (Geng et al., 2011) is another example of this approach: a vector-based binning depicts the distribution of the data. For their part, geometry-based techniques map the clusters to envelopes (Moustafa, 2011) or to bounding boxes (Fua et al., 1999).

Frequency-based representations show the distribution of data samples in each category, like the parallel coordinate dot plot (Dang et al., 2010), which overcomes the problem of overplotting by adding a dot plot to each axis. Kosara et al. (Kosara et al., 2006) developed the Parallel Sets: they represent each category with a parallelogram scaled according to its corresponding frequency (Figure 2c) and connecting the axes. This method is best suited for data containing exclusively categorical dimensions: it does not provide a good way to integrate continuous data since each continuous dimension axis is divided into bins, ”and thus, transformed into a categorical dimension.” Moreover, outliers are impossible to detect with this method. The authors note that ”showing continuous axes as true Parallel Coordinates dimensions would of course be the most useful display of this data”.

2.1.3 Spatial Distortion

Spatial distortion techniques consist in scaling the distance between the ticks on an individual axis according to a meaningful criterion, or in modifying the distance between two axes. A few examples of spatial distortion are the fisheye view and the linear zoom (Heinrich and Weiskopf, 2013), or the method proposed by Teoh and Ma (2003) defining frequency-based intervals on categorical axes. Spatial distortion can help in several ways: it reduces clutter, clarifies dense areas and facilitates the brushing of individual lines with a pointing device (Heinrich and Weiskopf, 2013). Rosario et al. (2003) present a way to give a meaning to the spacing between the levels of a categorical dimension: similar levels are closer to each other. Their approach transforms levels into numbers with techniques similar to Multiple Correspondence Analysis. By doing so, the spacing among levels conveys semantic relationships. Their method consists in three steps:

1. The **Distance** step consists in identifying a set of independent dimensions that allow to calculate the distance between their nominal levels.
2. The **Quantification** step uses the distance information to assign order and spacing among nominal levels.
3. The **Classing** step uses results from the previous step to determine which levels within a dimension are similar to each other, grouping them together.

This method can also be used at the dimensions scale, to assign spacing among the axes given a similarity measure. Heinrich and Weiskopf (2013) advocate precaution regarding the use of this method at the dimensions scale. They state that ”horizontal distortion affects angles and slopes of lines, which can have an impact on the accuracy of judging angles”, and hence the correlation levels. Like filtering methods, spatial distortion does not restore the missing frequency information since overplotting is still present.

2.1.4 Colors

Using colors is an efficient way to represent and identify a small set of categories. However, this approach does not scale well with large amounts of categories, since it is difficult for the human visual system to reliably distinguish more than twelve colors (Heinrich and Weiskopf, 2013).

2.2 Research Guidelines

In a recent paper, Johansson and Forsell (2016) performed a survey on 23 existing papers that present user-centered evaluations of the standard 2D Parallel Coordinates technique and its variations. They highlight the fact that despite the large number of publications proposing variations of Parallel Coordinates, only a limited number present results from user-centered evaluations. Figure 1 gives an overview on the performance of 26 techniques in relation to standard Parallel Coordinates (2DPC), for 7 tasks. We therefore conclude that up to now, many types of tasks have not been assessed yet (configuration, visual mining) and some visualization methods are missing (Trellis Plot, Scatterplot matrix). Moreover, the comparison lacks information about the nature of the data (continuous, categorical, mixed) that was visualized.

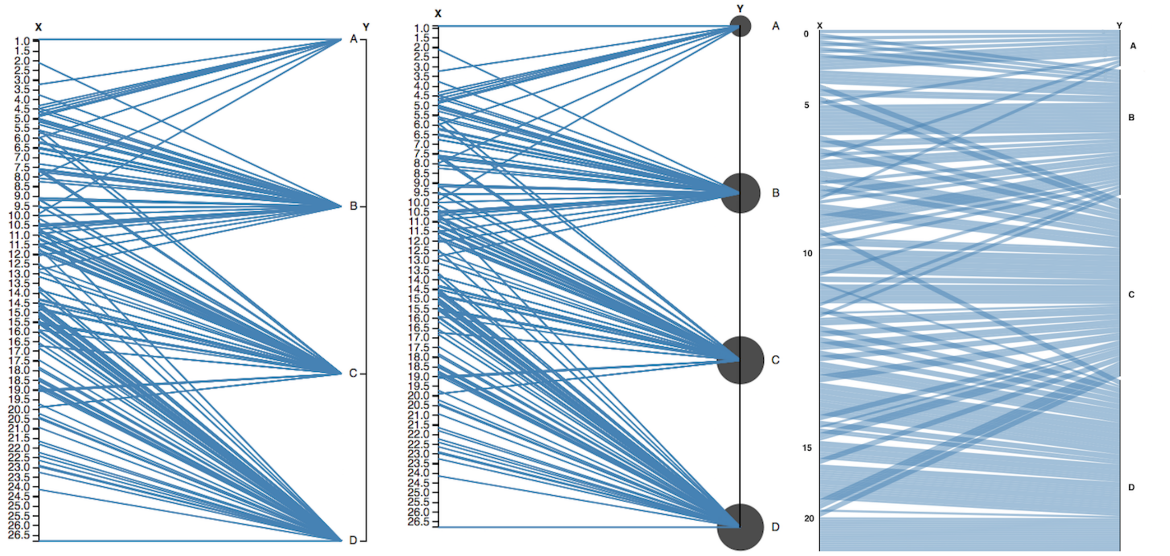
They categorize the evaluations as follows :

1. Evaluating axis layouts of Parallel Coordinates.
2. Comparing clutter reduction methods.
3. Showing practical applicability of Parallel Coordinates.
4. Comparing Parallel Coordinates with other data analysis techniques.

The enhancement of the visualization of mixed categorical and continuous data falls into the categories 1 and 2. Evaluating axis layouts of Parallel Coordinates includes ”techniques for arranging axes in 2D Parallel Coordinates in order to highlight specific types of relationships, or for reducing clutter”.

They discuss seven studies that evaluated the axis layouts of Parallel Coordinates. They note that ”the 2D Parallel Coordinates axis layout is both effective and efficient for tasks involving comparing relationships between variables”. This layout is qualified as intuitive ”and novice users learn it without effort”. They underline the need to investigate and study systematically the differences between axis layouts with different tasks and users.

To sum up, there still lacks an evidence of measurable benefits that would encourage the use of a variation of Parallel Coordinates over standard ones (Johansson and Forsell, 2016). It is unclear whether



(a) Parallel Coordinates. (b) *Parallel Bubbles.* (c) *Parallel Sets.*

Figure 2: The three variants of Parallel Coordinates that we compared. The left axis is continuous, the right axis is categorical. Here, the dataset 2 (medium correlation) is represented.

frequency-based techniques positively affect the ability of users to quickly and reliably identify patterns in the data: do they correctly interpret categorical dimensions? Do these methods allow for a better performance in standard data analysis tasks?

Therefore, we focus our research on testing the effect of adding a frequency encoding for categorical dimensions on basic data analysis tasks. The outcomes of this study should allow to tell if a a frequency-enhanced version of Parallel Coordinates allows the user to accomplish configuration tasks significantly faster than with regular Parallel Coordinates. The next chapter describes the protocol of our user experiment.

3 THE STUDY: PARALLEL COORDINATES, *PARALLEL BUBBLES* AND PARALLEL SETS

The goal of our study is to compare the performance of three variants of Parallel Coordinates in similarity and frequency tasks, in order to verify if the addition of a visual encoding of frequency results in a significant difference in performance. Our approach, *Parallel Bubbles*, aims to enhance the visual perception of categorical dimensions by adding a visual encoding of frequency.

3.1 Visual Metaphor

The cornerstone of *Parallel Bubbles* is a "bubble" (a circle) of variable radius that represents a categorical level. It was inspired by the "bubble plot" invented more than two centuries ago by Playfair (1801). A similar feature has already been available in Parabox[1] by Advisor Solutions and is presented in Few (2006). The area of the bubble is linearly proportional to the amount of items that belong to the said value: the radius r_b of each bubble b is defined by computing the square root of the number of items n_b belonging to one category, and then multiplying this number by two:

$$r_b = 2\sqrt{n_b} \tag{1}$$

Sets of vertically aligned bubbles are arranged on each categorical axis. Continuous dimensions are represented as regular Parallel Coordinates axes, and are easily distinguishable from the categorical ones. As in Parallel Coordinates, each data item is represented by a polyline passing through each of the continuous and categorical axes. This approach gives an overview of the distribution of categories inside a dimension, and restores the frequency information that was lost because of overplotting: the size of a bubble gives an information about the amount of lines that intersect at a given point on the axis.

[1] https://www.advizorsolutions.com/articles/parabox

3.2 Advantages

This method offers several advantages: first, it has a low computational complexity when compared to density models (Heinrich and Weiskopf, 2009) and kernel density estimators, making it a suitable solution even for large datasets. Secondly, bubbles offer a minimal learning curve: it is a simple add to Parallel Coordinates, which are widely used nowadays. Third, *Parallel Bubbles* are easy to implement: one bubble has to be added to each categorical value, which is a trivial task with today's available data visualization javascript libraries, like D3.js[2] for example.

3.3 Hypothesis

Our hypothesis is that the three types of visualizations induce a significant difference in performance, in frequency and similarity tasks. We also formulate the following sub-hypotheses: Parallel Coordinates (abbreviated "ParaCoord") offer the best performance for similarity tasks, because the representation of references as lines allow to assess the level of correlation easily. Next, Parallel Sets ("ParaSet") offer the best performance for frequency tasks because of the visual encoding of frequency they implement. Finally, *Parallel Bubbles* ("ParaBub") should be a good compromise in terms of performance for both types of tasks, with a significantly better performance than Parallel Coordinates in frequency tasks due to the add of a frequency encoding, and a significantly better performance than Parallel Sets in similarity tasks thanks to the aforementioned advantages of representing links between dimensions in the form of polylines. We describe the three tested variants of Parallel Coordinates as follows (Figure 2):

- **ParaCoord** – standard Parallel Coordinates (Figure 2a). This method is sensible to clutter and overplotting problems, making the frequency tasks harder to complete.
- **ParaBub** – Parallel Coordinates, with the addition of bubbles on axes, a visual encoding of frequency for categorical values ("Y" axis) (Figure 2b). This makes the frequency of categorical values easier to estimate. The similarity between dimensions is still easy to assess too, thanks to the graphical representation of links between dimensions in the form of polylines.
- **ParaSet** – a variant of Parallel Coordinates, encoding frequencies according to the height of the categorical segment on the axis (Figure 2c). The continuous axis, "X", is only showing a few values on the graduation. The height between two given values on the continuous axis is proportional to the number of items belonging to the range. With this visualization method, similarity tasks should be harder to complete because the polylines are replaced by colored areas, adding a layer of complexity (namely the height of each area) that is not directly relevant to the task. This visualization method should offer the best results for frequency tasks.

In order to evaluate the performance of participants on the three visualizations, we established three data analysis tasks. We describe them in the next section.

3.4 Tasks

The tasks that we submitted to the participants only represent a subset of the tasks commonly carried out by data analysts – these include, among others, clusters and outliers identification, classification, or selection of single data points. According to Fernstad and Johansson (2011), "the overall task of data analysis is to identify structures and patterns within data. Most patterns, such as correlation and clusters, can be defined in terms of similarity. Hence, the most relevant general tasks to focus on are, in our opinion, the identification of relationships in terms of similarity." They also state that "when it comes to analysis of categorical data the frequency of categories, i.e. the relative number of items belonging to specific categories or combinations of categories, is often of major interest and is, as mentioned previously, the main property of focus in categorical data visualization." With this in mind we defined the two types of tasks as follows:

1. **Similarity** – identify structures in data. To succeed, the user needs to be able to estimate the strength of the correlation (Figure 4a).
2. **Frequency** – the user has to estimate the amount of items belonging to a given categorical value. Being able to identify the single most represented category is important too (Figures 4b and 4c).

On this basis, we defined the three following data analysis tasks:

- **(T1) Similarity task** – *Are the X and Y axes correlated?* Possible answers: Strongly, mildly, not at all (Figure 4a).
- **(T2) Frequency task** – *What proportion of lines have the value B for the Y axis?* Possible answers: a range of values from 0% to 100%, by step of 10% (Figure 4b).

[2] D3.js: https://d3js.org/

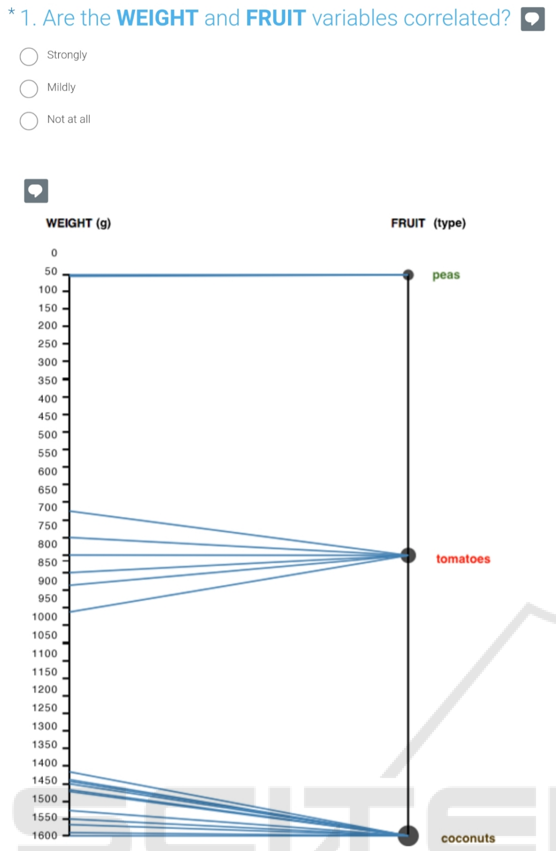


Figure 3: First task asked to participants in the tutorial. This figure was shown to participants assigned to the *Parallel Bubbles*.

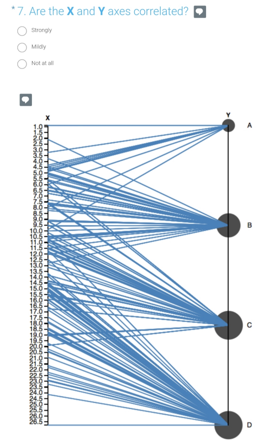
- **(T3) Frequency task** – *What is the most represented value on Y axis?* Possible answers: A, B, C, D (Figure 4c).

We reduced interaction capacities as much as possible in order to minimize the amount of dependent variables and to obtain more robust results. Thus, we did not allow the user to manually reorder the axes, to change the position of categorical values on the axis and to delete, add or group dimensions together.

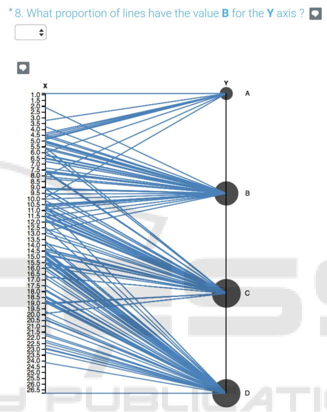
3.5 Study Material

We submitted the tasks in the form of an online survey[3] and participants were recruited via Prolific[4], a crowdsourcing platform dedicated to research surveys. In total, 367 participants filled out the survey. Each of them was paid 0.70£. A tutorial was proposed in the beginning of the survey. It explained the assigned visualization method and the two types of

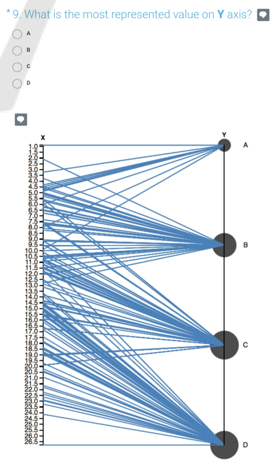
[3]SurveyMonkey: https://www.surveymonkey.com
[4]Prolific: https://www.prolific.ac/



(a) *Task 1: similarity.*



(b) Task 2: frequency.



(c) *Task 3: frequency.*

Figure 4: The three tasks, as asked to the participants using the *Parallel Bubbles*, on the dataset 2.

tasks. At the end of it, participants had to complete three questions about a fictive dataset with the highest level of correlation (the dataset and task 1 are shown in Figure 3). These tasks allowed to screen out participants who didn't meet basic criteria for our study:

- **(T1) Similarity task** – *Are the Weight and Fruit variables correlated?* Possible answers: Strongly, mildly, not at all.
- **(T2) Frequency task** – *How many of the fruits do you estimate to be tomatoes (in %)?* Possible answers: a range of values from 0% to 100%, by step of 10%.
- **(T3) Frequency task** – *Which fruit type is the most represented?* Possible answers: Pea, Tomato, Coconut.

We followed the design recommendations of Kittur et al. (2008) in order to maximize the usefulness of the data we collected: answers to the questions are explicitly verifiable, and filling them out accurately and in good faith requires the same amount of effort than a random or malicious completion. In the next part of the questionnaire, we tested the visualization methods on three datasets presenting different levels of correlation. Using different datasets should allow to get more robust results. In order to fully control the correlation levels, we generated the data ourselves: we generated three datasets with a Python script, with a size large enough to generate overplotting (465 to 480 data items). We used two-dimensional datasets because the tasks that we defined are comparison tasks, i.e. requiring the user to assess two dimensions at a time; this type of task focuses the user's attention on the smallest level of granularity offered by Parallel Coordinates. As explained above, multidimensional data exploration is based upon a subset of tasks of this type. Therefore, we chose to use one continuous dimension X and one categorical dimension Y for our datasets. The function `random.multivariate_normal(mean,cov,size)` from the `numpy` library allowed us to generate random samples from a multivariate normal distribution. For each dataset we ran this function three times using overlapping input ranges (e.g. for one dataset: $[0,20]$, $[10,30]$ $[20,40]$) in order to make the distribution more even. The `cov` argument contained the covariance matrix which allowed to control the variation of the variable X in regard to the variable Y, by multiplying the elements $C_{x,y}$ and $C_{y,x}$ by an index taking the values 0.0 (no correlation), 0.8 (medium correlation) and 1.0 (strong correlation) for each dataset.

3.6 Experimental Design

We designed the study as a "between-group". The independent variables were the type of visualization (ParaCoord, ParaBub or ParaSet) and the type of data (no correlation, medium correlation and strong correlation). The presentation order of the three datasets was counterbalanced using the Latin square procedure (Graziano and Raulin, 2010), giving 6 data order variants for each visualization method, for a total of 18 questionnaires. Each participant was assigned one visualization method. To avoid any learning bias, each of the three datasets was shown only once to each participant, in the order defined by the Latin square method. Each participant had to follow a tutorial explaining the visualization method assigned to him, and then had to perform 9 tasks: 3 tasks on each of the 3 datasets.

3.7 Procedure

In order to grade the visual analysis capacities of participants on each visualization method, we placed a written tutorial in the beginning of the survey, using a two-dimensional dataset of fruits and vegetables, along with their respective weights. The categorical value was the fruit or vegetable type (pea, tomato, coconut, strawberry), and the continuous value was the weight of each data item. At the end of the tutorial, participants had to perform a set of explicitly verifiable qualification tasks. We used the results of these tasks to exclude negligent participants and keep the remaining for further analysis. Participants were informed in the beginning of the survey that we would evaluate the performance of the visualization techniques rather than their individual performance. Once the tutorial was completed, each participant had to fill in the main part of the survey. Using the selected visualization method, they had to complete the three tasks described above, for each dataset.

3.8 Results

This section aims to compare the results of the participants who present the best expertise in data analysis tasks. We selected the participants who completed the tutorial questions with the highest scores. The aim is to get the most representative results for a usage by expert data analysts.

Before computing the results, we preprocessed the data by deleting the answers of 5 participants who had completed the first task only (1 for ParaCoord and 4 for ParaSet), 10 participants who timed out, and 1 participant who completed the questionnaire too quickly

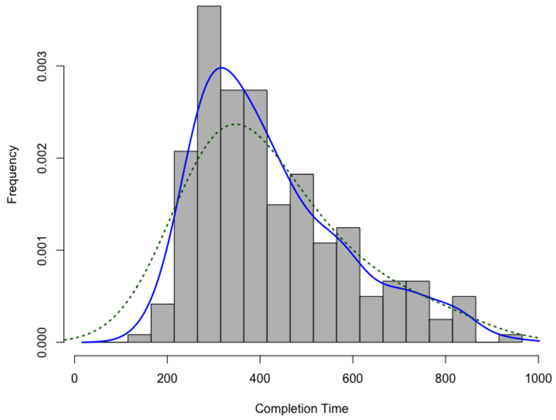


Figure 5: Frequency histogram for the distribution of the completion time of the full questionnaire.

Post Hoc Comparisons – Visu

		Prior Odds	Posterior Odds	$BF_{10,U}$	error %
parabub	paracoord	0.587	0.130	0.221	5.557e −6
	paraset	0.587	0.259	0.441	4.857e −6
paracoord	paraset	0.587	0.904	1.539	1.490e −6

(a) *Factor: Visualization method*

Figure 6: The Post Hoc tests performed on the results of the task 1.

(6 seconds), leaving a total of 356 participants.

We wanted to determine with which visualization method the most ”expert” users obtained the best performance. In order to keep only answers made by users that completely understood the tasks, we removed the answers of the participants who had made at least one mistake in the questions 1 and 3 of the tutorial, and who answered more than 10% away from the correct percentage in the question 2. The odds of answering correctly by chance to the 3 questions of the tutorial were thus 1/3 * 3/11 * 1/3, about 3%. We then conducted a χ^2 test that revealed no significant difference between the three visualization methods ($p < 0.05$). This means that the results of the qualification questions were not mainly influenced by the visualization method, but rather by the analysis capabilities of the participants. This qualification filtering left us with the answers of 241 participants' to analyze: 78 for Parallel Coordinates, 76 for *Parallel Bubbles* and 87 for Parallel Sets.

We did not take the completion time into account for our analysis, because too many variables can affect the time spent on the questionnaire for an online questionnaire. Establishing a threshold for the time spent on tasks ”[...] may not adequately identify non-conscientious participants, and may inadvertently disqualify many others” (Downs et al., 2010). Figure 5 shows the distribution of completion time for all participants. It has a positively skewed unimodal shape, which is typical to response time distributions (Heathcote et al., 1991).

3.8.1 Tasks Performance

For the task T1 (similarity) and T3 (frequency), whose answers are dichotomous (true or false), we computed the error rate for each participant. For task T2 (frequency), the answer was given in %, by step of 10%. It should be noted that the true percentage value could only be approximated with the proposed answers – for example, the real proportion of B was 16.77% in the Data 1, and the closest answer the user could give was 20%. For consistency with other studies (Heer and Bostock, 2010; Skau and Kosara, 2016), we computed the log absolute error of accuracy in this way: $log_2(|judgedvalue - truevalue| + \frac{1}{8})$. We performed an analysis of variance (ANOVA) (Graziano and Raulin, 2010) on the results of the three tasks, followed by a Bayesian ANOVA and Post Hoc tests.

3.9 Task 1

The average error rate of T1 can be seen in Figure 7. The Parallel Sets returned the best error rate, followed by the *Parallel Bubbles*. We performed an ANOVA which revealed no significant difference between the three factors ”Visualization method”, with a p-value of 0.0809. The Parallel Sets are returning the best performance, which is surprising considering the fact that they present the largest visual clutter due to their area representation. We performed a Bayesian ANOVA followed by Post Hoc tests that confirmed the results of the ANOVA (Figure 6). For the levels ”*Parallel Bubbles*” and ”Parallel Coordinates” of the factor ”Visualization method”, the Bayes Factor corresponds to substantial evidence for the null hypothesis (H0). For the levels ”*Parallel Bubbles*” and ”Parallel Sets”, the Bayes Factor corresponds to anecdotal evidence for H0. For the levels ”Parallel Coordinates” and ”Parallel Sets” of the factor ”Visualization method”, the Bayes Factor corresponds to substantial evidence for the alternative hypothesis (HA). These results suggest that for a similarity task, there is no significant difference in error rate between the *Parallel Bubbles* and the two other visualization methods. There is an anecdotal evidence for an effect of the factors ”Parallel Coordinates” and ”Parallel Sets” for HA.

3.10 Task 2

For T2 (frequency), we computed the mean of quartiles 1 and 3 for the log absolute error, and the 95%

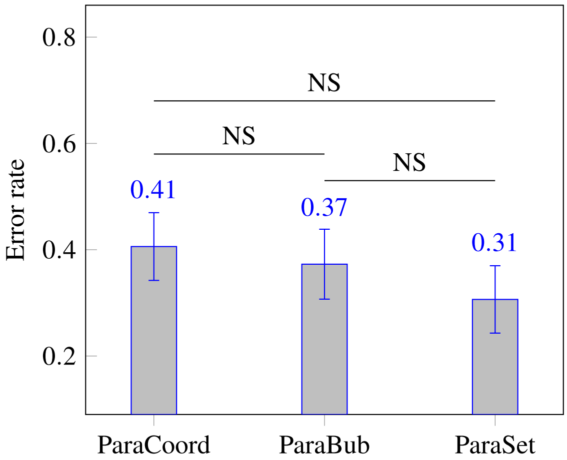


Figure 7: Average error rate for T1 (similarity) on all datasets. Error bars show 95% CI. The ANOVA indicates that the means of the factor "Visualization method" are not significantly different.

confidence interval (Table 1 and Figure 8). We constructed the 95% percentile interval based on 1000 bootstrap iterations. As for the first task, the Parallel Sets give the minimal error rate. A decomposition by type of data (1 = no correlation, 2 = medium correlation, 3 = strong correlation) shows that the strongly correlated data lead to the smallest error rate. We performed a multiway ANOVA to test the effects of the multiple factors ("visualization method", "data") on the mean of the vector "Score". It results that only the mean responses for levels of the factor "data" are significantly different with a p-value smaller than 0.05. The multiple comparison shows that two groups can be made based on the factor "data" (see Figure 9): the first group is composed of the levels 1 and 2 (strong and medium correlation), and the second group is composed of the level 3 (strong correlation). The Bayesian ANOVA, followed up with a Post Hoc test, confirms these results (Figure 10). For the factor "data", the Bayes Factor corresponds to substantial evidence for H0 for the levels 1 and 2 (inexistent and medium correlation). Between the levels 1 and 3, and 2 and 3, there is decisive evidence for an effect of the data type for HA. The dataset 3 (strongly correlated), with no lines intersecting, leads to a significantly lower error rate than the two other datasets.

3.11 Task 3

The average error rate of task T3 (frequency) can be seen in Figure 11, showing the same ranking in the visualization types as for T1 and T2. The multiway ANOVA reveals a significant difference between the three visualization methods. The Post Hoc test performed after the Bayesian ANOVA suggests

Table 1: T2: Average of quartiles Q1 and Q3, for the log absolute error. 95% confidence intervals. Split by visualization type. ANOVA: $F(39.23) = 1.88021e{-}15, p < 0.01$.

Type	Log error	CI 95%
ParaCoord	2.093	±0.178
ParaBub	2.094	±0.165
ParaSet	2.018	±0.150

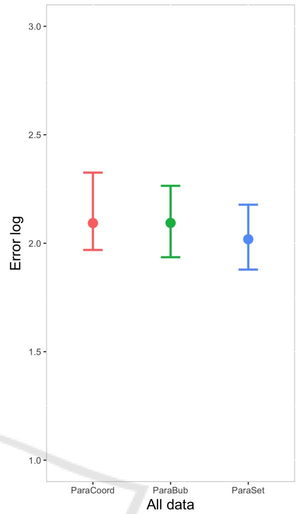


Figure 8: T2: Average of quartiles Q1 and Q3, for the log absolute error, for all datasets. 95% confidence intervals.

a very strong evidence for an effect of the Visualization method for the levels "Parallel Coordinates" and "*Parallel Bubbles*" for HA, and a decisive evidence for the levels "Parallel Coordinates" and "Parallel Sets" for HA, and "*Parallel Bubbles*" and "Parallel Sets" for HA too.

3.12 Discussion

The analysis of tasks T1 and T2 did not yield any significant difference between any of the three visualization methods. For T1, we can conclude that Parallel Sets and *Parallel Bubbles* are suitable for similarity tasks that consist in assessing the level of correlation between a categorical and a continuous dimension. The same can be said for T2: the three visualization methods are equivalent regarding the error rates. This task was the most complex, since it required to pick one answer out of the 11 available. For the task T3, Parallel Sets significantly outperformed the two other methods, and *Parallel Bubbles* are significantly better than Parallel Coordinates. This partially confirms one of our sub-hypotheses: adding a frequency encoding does have a significant influence on performance with a frequency task consisting in identifying the most represented category.

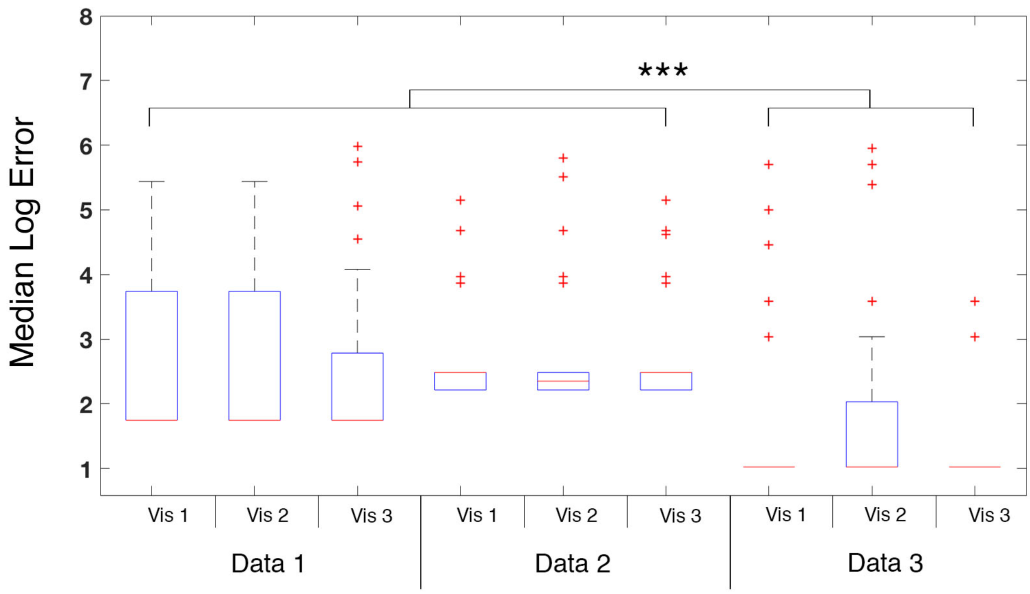


Figure 9: The distribution of Log Error per chart type, for task T2. The error bars show 95% CI and the middle black lines represent the median for each box plot. The multiple comparisons for the multiway ANOVA show a significant effect of the factor *Data* on the vector "Log Error". The two grouping variables are *visualization method* and *data*. Data points beyond the whiskers are displayed using +.

Post Hoc Comparisons – Visu

		Prior Odds	Posterior Odds	$BF_{10,U}$	error %
ParaBub	ParaCoord	0.587	0.061	0.103	2.696e −5
	ParaSet	0.587	0.086	0.146	8.581e −5
ParaCoord	ParaSet	0.587	0.086	0.146	1.113e −4

(a) *Factor: Visualization method*

Post Hoc Comparisons – Data

		Prior Odds	Posterior Odds	$BF_{10,U}$	error %
1	2	0.587	0.161	0.275	3.340e −5
	3	0.587	1.275e +12	2.170e +12	2.226e −18
2	3	0.587	1.080e +22	1.838e +22	6.454e −28

(b) *Factor: data*

Figure 10: The Post Hoc tests performed on the results of the task 2.

Regarding the factor "data", we clearly identified two clusters: one is composed of the data with no correlation and a medium correlation, and the other cluster is composed of the strongly correlated data.

We can conclude that adding a visual encoding of frequency improves performance of the user in a frequency task consisting in evaluating the most represented category. It does neither improve the performance of the user in frequency tasks consisting in evaluating the proportion of samples belonging to a given category, which is somewhat surprising, nor for a similarity task consisting in assessing the level of correlation.

For the task T3, the better performance of Parallel Sets in comparison with *Parallel Bubbles* might be caused by the difference of accuracy with which

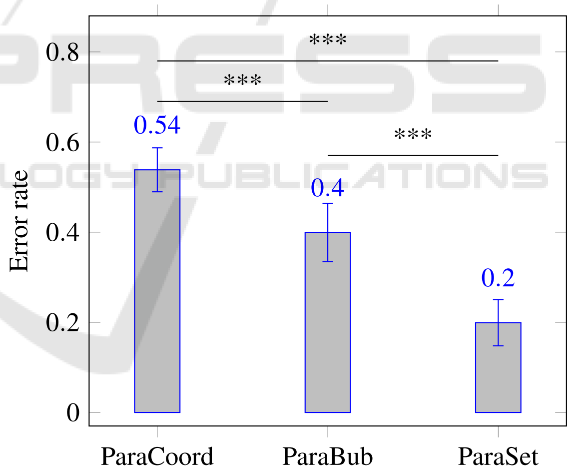


Figure 11: Average error rate for T3 (frequency) on all datasets. Error bars show 95% CI. The ANOVA indicates that the means of all three visualization methods are pairwise significantly different.

we perceive the channels used. *Parallel Bubbles* use an area encoding: the area of bubbles is linearly proportional to the number of items, the radius of a bubble being equal to the square root of the frequency, multiplied by two. For Parallel Sets, we use a length encoding: the height of the boxes was defined as linearly proportional to the frequency. The difference in performance when using these two encodings can be explained by Steven's Psychophysical Power Law (Stevens, 1975a): this law states that the appar-

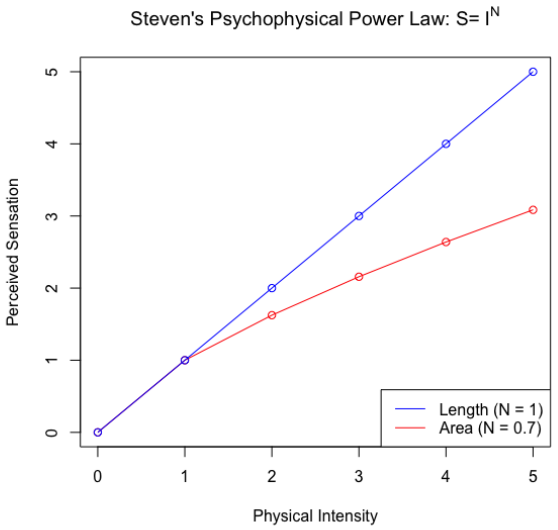


Figure 12: The psychophysical power law of Stevens (Stevens, 1975a) for the Area and Length encodings. "The apparent magnitude of all sensory channels follows a power function based on the stimulus intensity."

ent magnitude of an area is perceptually compressed (by a factor of 0.7), while the perception of length is very close to the true value (see figure 12). Based on these results, in the conclusion we propose a few approaches to improve the *Parallel Bubbles*.

4 CONCLUSION AND FUTURE WORKS

Our study attempted to find out which visualization method gives the best performance for typical data analysis tasks. Participants were asked to perform three types of tasks: in the first task (T1) participants had to assess the level of correlation between the continuous and the categorical dimension. In the second task (T2), they had to estimate the percentage of samples belonging to a given category. In the third task (T3), they had to assess which of the four categories was the most represented. Our hypothesis is invalidated for the tasks T1 and T2: for these two tasks, adding a visual encoding of frequency does not empower the user with significantly better data analysis capabilities. However, the results of task T3 confirm one of our sub-hypotheses: a visual encoding of frequency in the form of bubbles (*Parallel Bubbles)* or lines (Parallel Sets) results in a significant difference in performance between all three visualization methods for a frequency tasks consisting in identifying the most represented categorical value.

Parallel Sets delivered the best performance in all tasks, but *Parallel Bubbles* present two main advantages over Parallel Sets: first, they are easier to implement, requiring the simple add of a circle on each categorical value of a Parallel Coordinates plot. Second, they are best suited for continuous values. Thus we recommend the use of *Parallel Bubbles* for basic data analysis tasks performed on mixed categorical and continuous datasets.

As future works, it would be interesting to further extend the *Parallel Bubbles* method, implement it in a functional system and use it in a real context. It would be pertinent to test it in a use case. Having explicitly verifiable tasks in the beginning of the survey allowed us to filter out non-expert users. However, in a future study, the tutorial could be improved by signaling more clearly to users that their answers would be scrutinized, as suggested by Kittur et al. (2008). This would likely increase the time spent on tasks, and the quality of the answers.

One way to improve the *Parallel Bubbles* would be to replace the circles, that are evenly spaced, but have variable areas, with vertically stacked bars scaled proportionally to the frequency of the categorical value they represent. The user would assess the length of segments, instead of areas. This would counterbalance the psychophysical effects related to area representation defined by Stevens (1975b). The effectiveness of this encoding would approach that of Parallel Sets, and its performance in similarity and frequency tasks would probably increase. Next studies will continue to focus on the categorical axis, comparing the area encoding of frequency (bubbles) with alternative encodings such as density functions, histograms and dot plots.

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