Static Analysis of Conformance Preserving Model Transformation Rules

Fazle Rabbi, Lars Michael Kristensen and Yngve Lamo
Western Norway University of Applied Sciences, Bergen, Norway

Keywords: Formal Methods for Model Driven Software Engineering, Formal Correctness of Model Transformations, Meta-modelling, Domain-specific Modeling Languages.

Abstract: Model transformation is a core element in model driven software engineering and is used for several purposes, including model migration, model synthesis, and code generation. Application of conformance preserving transformation rules guarantee that produced output models will conform to its underlying metamodel. Conformance persevering rules are therefore important in order to ensure the formal correctness of transformations. However, to determine if a rule is conformance preserving requires sophisticated analysis techniques. The contribution of this paper is a new algorithm for checking conformance preserving rules with respect to a set of graph constraints and to prove the soundness of the algorithm. We apply our technique to homogeneous model transformations where input and output models must conform to the same meta-model. The algorithm relies on locality of a constrained graph to reduce the computational cost. We show that the performance of our algorithm depends on the complexity of the graph constraints and model transformation rules, but that it is independent of the size of the input model.

1 INTRODUCTION

Model transformation is the process of transforming a model into another model and plays a key role in model driven software development. A transformation rule describes how a target model can be automatically generated from a source model. Often these models need to conform to the syntax and semantics of a metamodel. There are various applications of model transformations such as model migration, model synthesis, code generation, model simulation, model execution, and model repair. Formal development of transformation rules is an important concern since precisely defined rules can be used to verify that the automated transformations are correct (Varró et al., 2002). Graph transformation is a formal technique to represent model transformation rules enabling reasoning and studying properties of transformation systems. Depending on the source and target language, a transformation can be homogeneous or heterogeneous. In homogeneous model transformation, input models and output models belong to the same language. Heterogeneous model transformation transforms models from one language to another. In general, the result of the application of a model transformation rule may lead to inconsistency, i.e., the target model violating constraints defined in its metamodel. Therefore, the application of a model transformation rule requires conformance checking of the target model which is time consuming. To address this problem, it is of interest to develop techniques to reduce the complexity of conformance checking. Since the application of a conformance preserving transformation rule preserves the conformance of a model, it eliminates the need for conformance checking of target models. This approach is particularly suited for the development of systems where models produced in every step of a model transformation are supposed to be valid i.e., conforming w.r.t a set of constraints.

Current verification approaches for model transformation rules include theorem proving and model checking. Simone et al. proposed a relational and logical approach to graph grammars that allow the analysis of asynchronous distributed systems with infinite state spaces (da Costa and Ribeiro, 2012). They used relational structures to define graph grammars and first-order logic to model graph transformations. They provided a semi-automated process to prove structural properties of reachable graphs using theorem proving. Another theorem proving technique was presented in (Ribeiro et al., 2010) based on translating graph grammars into Event-B specifications preserving its semantics and then using theorem provers available for Event-B for analysis. Automatic verification of model transformation is gaining popularity and several methods have already been proposed.

Baresi et al. (Baresi and Spoletini, 2006) proposed a methodology to analyze graph transformation systems by means of Alloy. Given an initial graph of a system, the method can be used to check the configurations that can be obtained by applying a sequence of transformation rules. In (Wang et al., 2014), Wang et al. investigated the use of the Alloy analyzer for analyzing model transformation systems. A bounded verification approach was used to check if a model transformation system is correct w.r.t conformance by translating a metamodel specification into a relational logic specification in Alloy. The authors in (Troya and Vallecillo, 2010) presented a formal semantics of the ATL model transformation language using rewriting logic and Maude. Through the formalization it was possible to simulate and verify model transformations. Although model checking is an elegant analysis method, it requires building the complete state space. This can easily lead to the state explosion problem thereby limiting its practical applicability. Hackel and Wagner (Heckel and Wagner, 1995) presented an approach that ensures the conformance of graph transformations by automatically adding application conditions to rules. Application conditions are derived by analyzing the constraints individually which can produce an unnecessary large number of application conditions.

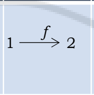
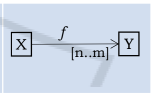
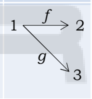
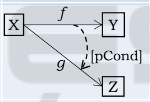
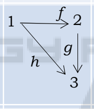
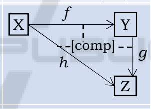
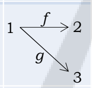
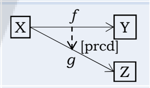
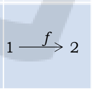
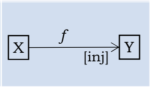
In our approach, we use characteristics of model transformation rules and present an algorithm to check if a transformation rule is conformance preserving w.r.t a given set of constraints. We focus on homogeneous model transformation. We do not automatically modify a rule. We provide an algorithm for checking the conformance preserving property of a transformation rule that can be used to provide feedback to the modeler. The approach is illustrated by an example from the healthcare domain.

The rest of the paper is organized as follows. Section 2 provides background on the theoretical foundation of our approach. Section 3 presents the concept of conformance preserving rules. Section 4 presents our algorithm for checking conformance preserving rules. Section 5 contains a further discussion of related work, and Section 6 concludes the paper with directions for future work. We assume that the reader is familiar with graph transformation systems (Ehrig et al., 2006).

2 MODELLING SPECIFICATION IN DPF

We use Diagrammatic Logic (Diskin and Wolter, 2008) and the Diagram Predicate Framework (DPF) (Rutle, 2010) for the formal development of metamodel specifications. In DPF, a model is represented by a diagrammatic specification $\mathfrak{S} = (S, C^{\mathfrak{S}} : \Sigma)$ consisting of an underlying graph S together with a set of *atomic constraints* $C^{\mathfrak{S}}$ specified by a *predicate signature* Σ. A predicate signature consists of a collection of predicates, each having a name, an arity (shape graph, $\alpha^{\Sigma}(p)$), visualization and semantic interpretation (see Table 1). The underlying graph and arity of predicates specify type graphs with a data algebra as in (Ehrig et al., 2006). A predicate is used to specify a constraint in a model by means of graph homomorphisms. DPF provides a general mechanism of diagrammatic modeling as it supports various kinds of graph structures. DPF provides a formalization of multi level meta-modelling by defining the conformance relation between models at adjacent levels of a meta-modelling hierarchy. DPF has a potentially unbounded number of metalevels.

Table 1: Predicates of a signature, Σ.

p	Arity α^{Σ} (p)	Visualization	Semantic interpretation
<mult(n,m)>			*f* must have at least n and at most m instances for each instance of X
<pre-Condition>			For each instance of *f* there exists an instance of *g* with the same source node
<composite>			For each composition of instances *f;g*, there exists an instance of *h such that h = f;g*
<precede>			If there are instances of *f* and *g* with the same source node, then value of Y is less than value of Z
<injective>			Instances of *f* never maps distinct elements of its domain to the same element of its codomain

There are two kinds of conformance: *typed by* and *satisfaction of constraints*. Figure 1 (top) shows a DPF metamodel specification $\mathfrak{S}$ of an Orthopedic department of a hospital. The metamodel specification is constrained by a set of predicates from the signature Σ. Constraints are added into the specifications by graph homomorphisms from the arity (shape graph) of the predicates to the model elements. Below is a list of constraints specified in $\mathfrak{S}$:

- **C1.** *A patient must have exactly one birthdate (specified by <mult(1,1)>)*
- **C2.** *An appointment time-slot (i.e., TS@Dept) allocated to a patient must belong to that patient's assigned doctor (specified by <composite>)*
- **C3.** *An imaging order can only be given to a*

registered patient (specified by <pre-Condition>)

- **C4.** *An exam time-slot (i.e., TS@Lab) can only be allocated to a patient with an imaging order (specified by <pre-Condition>)*
- **C5.** *An appointment time-slot cannot be allocated to more than one patient (specified by <injective>)*
- **C6.** *An exam time-slot cannot be allocated to more than one patient (specified by <injective>)*
- **C7.** *Patient's exam time-slot must be preceded by the appointment time-slot (specified by <precede>)*

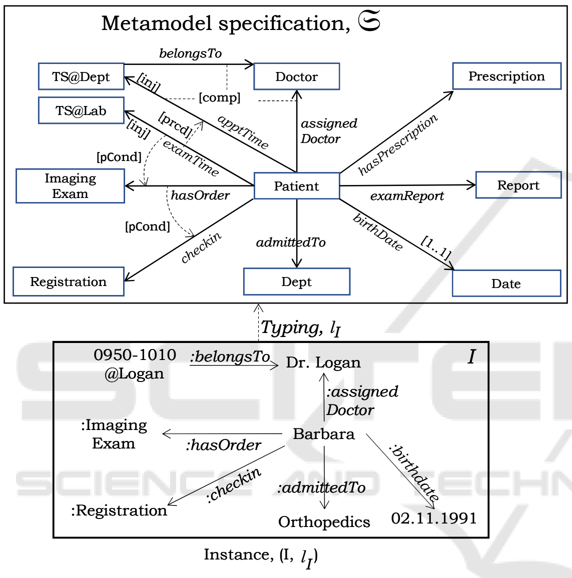


Figure 1: Metamodel specification $\mathfrak{S}$ (top) of an Orthopedic department and an instance of $\mathfrak{S}$ (bottom).

Usually orthopedic doctors need to see patient's X-ray reports while seeing patients. Therefore orthopedic patient's time-slot for the imaging exam must be preceded by the appointment time-slot. Figure 1(bottom) shows an instance (I,ι_I) of the metamodel specification $\mathfrak{S}$. In the instance (I,ι_I), *Barbara* is a patient admitted to the *Orthopedic* department; she is a registered patient and assigned to *Dr. Logan*; an order for radiology exam has been given for *Barbara*. (I,ι_I) is also referred to as a model of $\mathfrak{S}$ and it is typed by $\mathfrak{S}$. Formally, this means that there is a graph homomorphism from the graph I to the graph of $\mathfrak{S}$, denoted as $I : S$ where S is the underlying graph of $\mathfrak{S}$. We use a compact notation for typed attributed graph where data nodes are used for the inscription of graph nodes as depicted in the model (I,ι_I) in Figure 1.

2.1 Coupled Graph Constraints

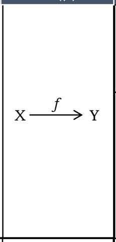
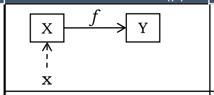
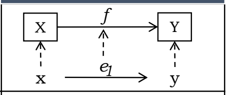
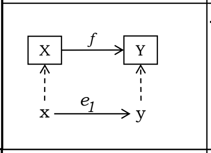
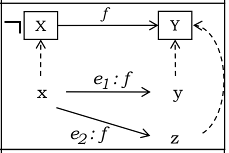
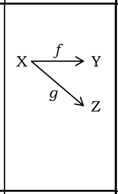
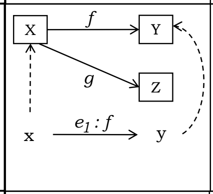
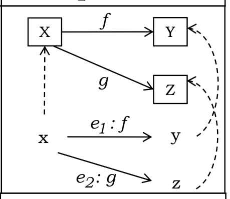
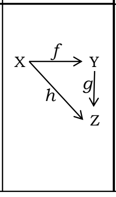
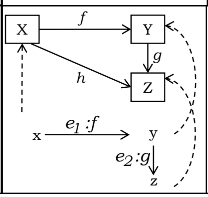
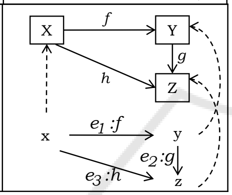
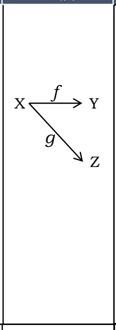
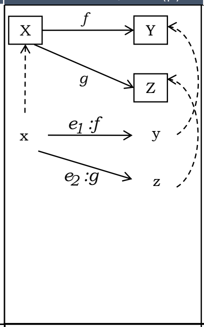
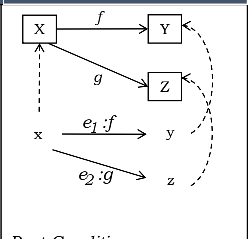
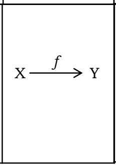
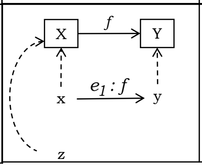
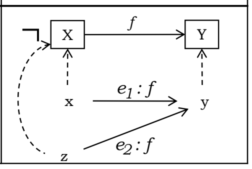
The semantics of a DPF predicate can be specified in various ways. In this paper, we use graph constraints to specify the semantics of the predicates. Typically a graph constraint $N \overset{n}{\leftarrow} L \overset{u}{\rightarrow} R$ consist of three graphs: left L, right R and an application condition N (positive or negative application condition) , and two injective graph homomorphisms n and u where the graphs are typed by the underlying graph of the model (Ehrig et al., 2006). We propose to use graph constraints which conforms to two syntactic formats $\forall L^p \rightarrow \exists R^p$ and $\forall L^p \rightarrow \neg\exists R^p$ where the graphs are typed by the shape graph of the predicates. Therefore we use graph constraints of the following forms where superscript p indicates that the constraint is giving the semantics of a DPF predicate, p. The graph constraints are called coupled graph constraints as they link to predicates.

- $\forall(L^p : \alpha^\Sigma(p)) \rightarrow \exists(R^p : \alpha^\Sigma(p))$, read as "for all matches of the condition pattern L^p (typed by $\alpha^\Sigma(p)$) in a model, there exists a match of the required pattern R^p (typed by $\alpha^\Sigma(p)$) in the model"
- $\forall(L^p : \alpha^\Sigma(p)) \rightarrow \neg\exists(R^p : \alpha^\Sigma(p))$, read as "for all matches of the condition pattern L^p (typed by $\alpha^\Sigma(p)$) in a model, there does not exist a match of the forbidden pattern R^p (typed by $\alpha^\Sigma(p)$) in the model"

Here L^p and R^p are typed attributed graphs over the arity of the predicate p and there exists an inclusion attributed graph morphism $m_c : L^p \hookrightarrow R^p$. A coupled graph constraint gc may have a post-condition $PC(gc)$ imposed on R^p. Table 2 shows the semantics of predicates from signature Σ in terms of graph constraints. The semantic of the *<mult(1,1)>* predicate is given by two graph constraints where the patterns are typed by α^Σ(*<mult(1,1)>*), i.e., the arity of the *<mult(1,1)>* predicate. The graph constraint giving semantic to the *<precede>* predicate has a post-condition which ensures that the time specified in y must precede the time specified in z.

Let $gc \in GC(p)$ be a graph constraint linked to a predicate p. A match (δ, m_L) of the condition pattern $(L^p : \alpha^\Sigma(p))$ for the graph constraint gc in a model (I,ι_I) is given by an atomic constraint $\delta : \alpha^\Sigma(p) \rightarrow S$ and an injective morphism m_L such that constraint δ and the injective graph homomorphism m_L together with the typing morphisms $\iota_c : (L^p \cup R^p) \rightarrow \alpha^\Sigma(p)$ and $\iota_I : I \rightarrow S$ constitute a commuting square: $\iota_c;\delta = m_L;\iota_I$ as shown in Figure 2(a). If gc has a required pattern $(R^p : \alpha^\Sigma(p))$, then for any match (δ, m_L) of the condition pattern in (I,ι_I), a match (δ, m_R) of the required pattern must exist, which is given by the commuting diagram in Figure 2(b). If gc has a forbidden pattern $(R^p : \alpha^\Sigma(p))$, then for any match (δ, m_L) of the

Table 2: A set of graph constraints giving semantics to the predicates in Σ.

p	Arity $\alpha^{\Sigma}(p)$	Semantic in Graph Constraint	
		$L^p : \alpha^{\Sigma}(p)$	$R^p : \alpha^{\Sigma}(p)$
<mult(1,1)>			
			
<pre-Condition>			
<composite>			
<precede>			 *Post Condition : `startTime(y) < startTime(z) ∧ endTime(y) < endTime(z)'*
<injective>			

condition pattern in (I, ι_I), a match (δ, m_R) of the forbidden pattern must not exist such that it constitutes a commuting diagram as shown in Figure 2(c). A valid model is typed by its metamodel specification and conforms to the constraints specified in its metamodel specification. Formally, it states that a valid model (I, ι_I) satisfies all the constraints defined in $\mathfrak{S}$, which is written as $I \models \mathfrak{S}$.

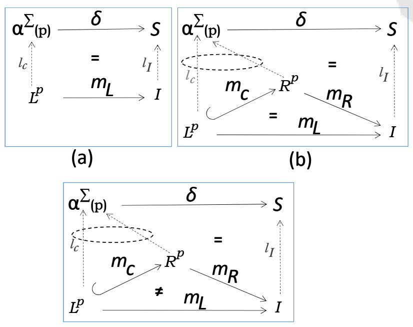


Figure 2: (a) match of a condition pattern ; (b) match of a required pattern; (c) satisfaction of a forbidden pattern.

3 CONFORMANCE PRESERVING RULES

DPF provides functionality to specify graph-based model transformations (Rutle et al., 2012). We use the standard double-pushout (DPO) approach (Ehrig et al., 2006) for defining transformation rules. A model transformation rule $(r : N \xleftarrow{n} L \xleftarrow{m_l} K \xrightarrow{m_r} R)$ has a matching pattern (L), a gluing graph (K), a replacement pattern (R) and an optional negative application condition, $NAC(n : L \rightarrow N)$ where L, K, R, N are typed by $\mathfrak{S}$ and m_l, m_r, n are injective graph morphisms. We use a transformation approach where transformation rules have a set of negative application conditions as proposed by Lambers et al., in (Lambers et al., 2008).

$$\begin{tikzcd}
L \arrow[d, "m"'] & K \arrow[l, "m_l"'] \arrow[r, "m_r"] \arrow[d] & R \arrow[d, "m^*"] \\
I & D \arrow[l] \arrow[r] & I^*
\end{tikzcd}$$

Given a model (I, ι_I), a model transformation $I \overset{r,m}{\Longrightarrow} I^*$ via a transformation rule $r : L \leftarrow K \rightarrow R$ with a set of negative application conditions NAC_r and a match $m : L \rightarrow I$ consists of the double pushout as shown in the diagram above. Here, the injective morphism m satisfies each NAC in NAC_r, written $m \models NAC_r$. When a rule is applied, some elements from the source model are deleted and some elements are added to the target model. The rest of the source model remain unchanged in the target model. A rule is applied as long as it satisfies its negative application conditions. Negative application conditions are typically used in graph transformation to prohibit an infinite number of rule applications. Figure 3 shows a model transformation rule for allocating resources (i.e., appointment time-slot, exam time-slot) to patients in a model of the metamodel specification from Figure 1. We use the concept of '*time-slot*' for an appointment or an examination to represent the time assigned for a scheduled appointment. The transformation rule r_1 encodes the following instructions:

- Allocate appointment time-slot t_1 and exam time-slot t_2 to patient pt_1 if t_1 belongs to the doctor whom pt_1 is assigned to and satisfies the following conditions:
  - Allocate t_1 to pt_1 if t_1 is not allocated to any other patient pt_2;
  - Allocate t_2 to pt_1 if t_2 is not allocated to any other patient pt_3;

The typing information of a modelling element in r_1 appears after a colon (:). The green color (thick arrow) is used to represent elements that the rule is going to produce. The rule r_1 has two negative application conditions to make sure that the patients are assigned using available resources.

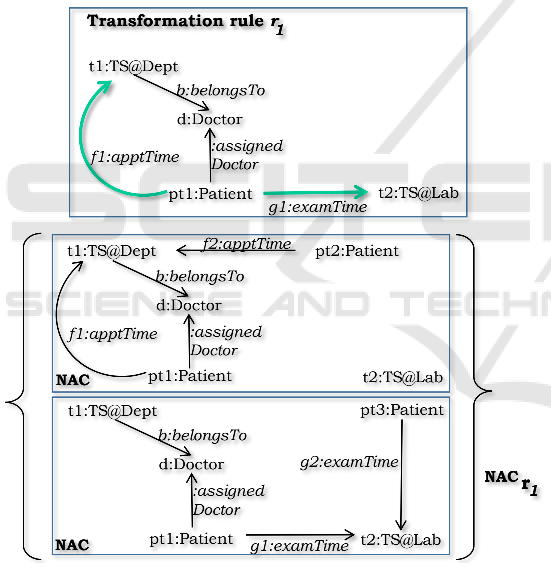


Figure 3: Transformation rule r_1 for individual resource allocation of patients.

One problem with this version of the transformation rule is that it does not guarantee the conformance of constraint **C4** (an imaging order must be given for a patient before exam time-slots). The application of the rule may allocate a radiology exam time-slot to a patient who does not have an imaging order. Also the rule does not guarantee the conformance of constraint **C7** (Patient's exam time-slot must be preceded by the appointment time-slot). To illustrate this, Figure 4 shows a model (I,ι_I) of $\mathfrak{S}$. The rule r_1 of Figure 3 can be applied over (I,ι_I) in four different ways since there are four matches. (I^*,ι_{I^*}) in Figure 4 shows a result of the application of rule r_1 giving a resource allocation where Barbara is assigned the exam time-slot $0945-1000@Lab$ at the radiology department and the appointment time-slot $0950-1010@Logan$ at the orthopedics department with Dr. Logan. Here (I^*,ι_{I^*}) is not conforming to the constraint expressed by the <*precede*> predicate. A time-slot $t2$ which include information about the start-time and end-time of a scheduled appointment is preceded by a time-slot $t1$ if the start-time and end-time of $t2$ are less than the start-time and end-time of $t1$, respectively. The startTime and endTime are two time-functions that specify the start-time and end-time of an appointment time-slot. To check if a model conforms to a graph constraint which has a post-condition, we use a replacement operator $/.$ (pronounced "slash-dot") that replaces the variables of an expression with the image of a match. Figure 4 illustrates how we check the satisfaction of the atomic constraint $(<precede>,\delta_1)$ over model (I^*,ι_{I^*}) by its graph constraint. The graph constraint used to give the semantics of the <*precede*> predicate has a post-condition given by an expression. To check if the post-condition is satisfied, the variables of the expression are replaced with elements from (I^*,ι_{I^*}) via a match $m_R : R^p \to I^*$. This resource distribution is not valid as (I^*,ι_{I^*}) is not conforming to the resource constraint **C7**: Barbara's exam time-slot $0945-1000@Lab$ must be preceded by her appointment time-slot $0950-1010@Logan$. Even if the rule is applied on a valid model, it does not guarantee that the result will be a valid model conforming to the metamodel specification. The portion of the model that is not conforming to the constraints are highlighted in red (thick arrow) in the figure.

The rule r_1 can be enhanced so that while matching with a model it makes sure that the result will be a valid model. Since the addition of new instances of *examTime* and *apptTime* in a valid model of $\mathfrak{S}$ can possibly violate atomic constraints **C2**, **C4**, **C5**, **C6** and **C7**, we enhance rule r_1 with an additional matching condition to make sure that when applied on a valid model of $\mathfrak{S}$, the addition of new elements does not violate any of the above mentioned constraints. The enhanced rule is equipped with an additional matching condition: '$startTime(t2) < startTime(t1) \wedge endTime(t2) < endTime(t1)$' which makes sure that the constraint **C7** is not violated. Therefore the application of the enhanced rule will not require any further conformance checking. Formally, the application of a transformation rule that is equipped with a set of additional matching conditions is defined below:

Definition 1.(Application of Transformation Rule) *Let* (I,ι) *be a model of a metamodel specification* $\mathfrak{S} = (S,C^{\mathfrak{S}})$ *and* $r : L \leftarrow K \rightarrow R$ *a rule with a set*

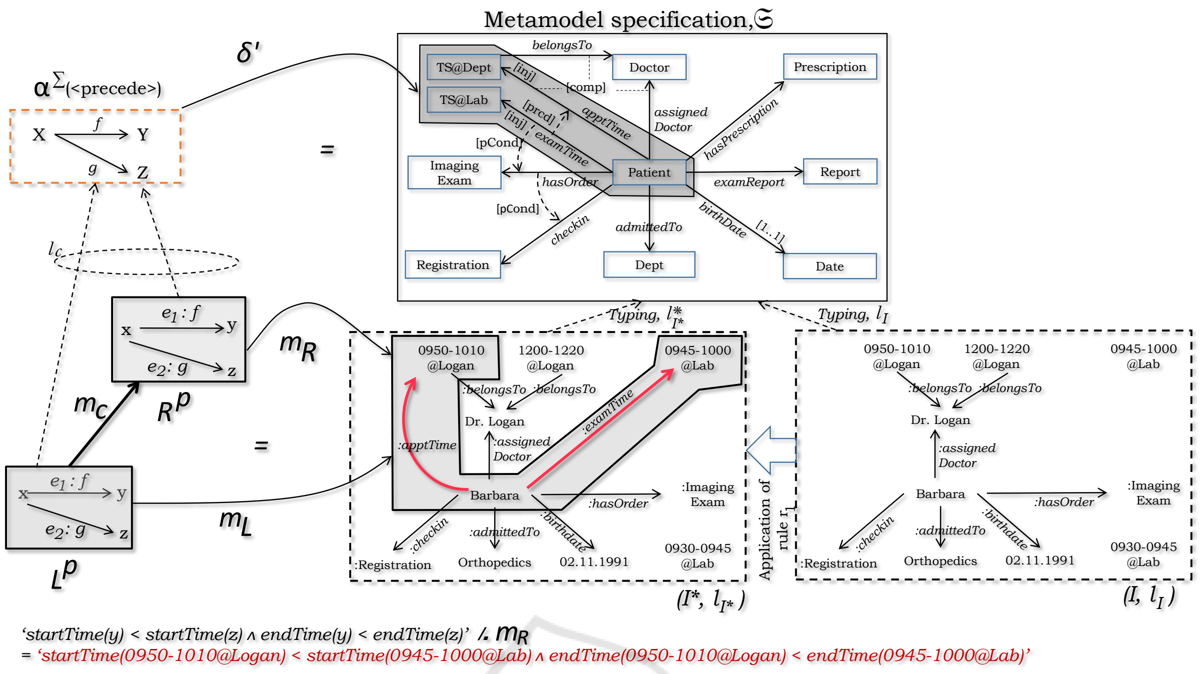


Figure 4: Application of rule r_1 over a valid model (I, ι_I) and the checking for the satisfaction of a graph constraint.

of negative application conditions NAC_r and a set of additional matching conditions MC_r. The rule r is applicable on (I, ι) if there exists a match $m : L \rightarrow I$ that constitute a double pushout diagram and the match m satisfies all the negative application conditions and additional matching conditions.

Definition 2.(Conformance Preserving Rule) *Given a metamodel specification $\mathfrak{S} = (S, C^{\mathfrak{S}} : \Sigma)$. A transformation rule r is conformance preserving w.r.t a set of atomic constraints from $C^{\mathfrak{S}}$ if the application of r on any valid model $(I, \iota_I) \models \mathfrak{S}$ always results in a valid model of $\mathfrak{S}$.*

4 ANALYSIS FOR CHECKING CONFORMANCE PRESERVING RULES

In this section, we present an algorithm to automatically check if a transformation rule is conformance preserving w.r.t a set of constraints specified in a metamodel. Automatic check of conformance preserving rules requires that the constraints specified in a metamodel is processed. To develop an efficient method for determining if a rule is conformance preserving or not, we need to analyze the possibility of the rule to make changes that may violate a given constraint. If a rule makes changes to only the unconstrained portion of a graph, then we can claim that the rule will preserve conformance by its application. If a rule makes changes to the constrained portion of a graph, it is possible that the rule will preserve conformance by its application. We present an algorithm with the aid of a set of patterns to make sure that consistency preserving rules exhibit certain desirable structures.

4.1 A Sufficient Condition for Conformance

Here we present three conditions to determine if a transformation rule $r : L \leftarrow K \rightarrow R$ can make changes to the constrained portion of a graph i.e., if r can possibly affect an atomic constraint (p, δ):

- **Cond 1:** r creates an element x of type X where X is constrained by a predicate p and X is mapped by the condition pattern of a graph constraint $gc \in GC(p)$ via the typing morphism of L^p and the atomic constraint (p, δ), i.e., $X \in \iota_c; \delta(L^p)$;
- **Cond 2:** r deletes an element y of type Y where Y is constrained by a predicate p and Y is mapped by the elements from $(required\ pattern,\ R^p \setminus condition\ pattern,\ L^p)$ via the typing of $(R^p \cup L^p)$ and the atomic constraint (p, δ), i.e., $Y \in \iota_c; \delta(R^p \setminus L^p)$;
- **Cond 3:** r creates an element x of type X where X is constrained by a predicate p and X is mapped by the elements from $(forbidden\ pattern,\ R^p \setminus$

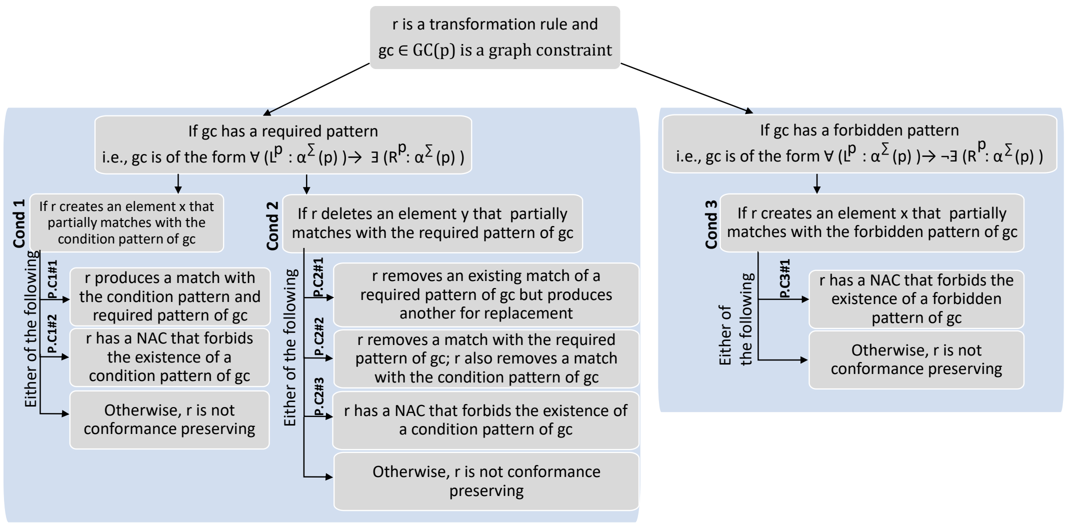

r is a transformation rule and gc ∈ GC(p) is a graph constraint
If gc has a required pattern i.e., gc is of the form ∀ (L^p : α^Σ (p))→ ∃ (R^p: α^Σ (p))
If gc has a forbidden pattern i.e., gc is of the form ∀ (L^p : α^Σ (p))→ ¬∃ (R^p: α^Σ (p))
Cond 1
If r creates an element x that partially matches with the condition pattern of gc
Either of the following
P.C1#1
r produces a match with the condition pattern and required pattern of gc
P.C1#2
r has a NAC that forbids the existence of a condition pattern of gc
Otherwise, r is not conformance preserving
Cond 2
If r deletes an element y that partially matches with the required pattern of gc
Either of the following
P.C2#1
r removes an existing match of a required pattern of gc but produces another for replacement
P.C2#2
r removes a match with the required pattern of gc; r also removes a match with the condition pattern of gc
P.C2#3
r has a NAC that forbids the existence of a condition pattern of gc
Otherwise, r is not conformance preserving
Cond 3
If r creates an element x that partially matches with the forbidden pattern of gc
Either of the following
P.C3#1
r has a NAC that forbids the existence of a forbidden pattern of gc
Otherwise, r is not conformance preserving

Figure 5: Informal description of the algorithm illustrating the intuition.

condition pattern, L^p) via the typing of $(R^p \cup L^p)$ and the atomic constraint (p, δ), i.e., $X \in \iota_c; \delta(R^p \setminus L^p)$;

Intuitively, **Cond 1**, **2**, and **3** checks if a rule can create a new match with the condition pattern, delete an existing match of a required pattern, or create a new match with the forbidden pattern of a graph constraint, respectively.

Lemma 1. *Given a metamodel specification $\mathfrak{S}$ with a set of constraints $C^{\mathfrak{S}}$. A transformation rule r is conformance preserving if it does not satisfy any of* ***Cond 1-3****.*

Proof. Let (I, ι) be a valid instance of $\mathfrak{S}$ and the application of r on (I, ι) produces an instance (I^*, ι^*). There are three ways (I^*, ι^*) may violate a constraint from $C^{\mathfrak{S}}$: (i) r produces a new match with the condition pattern L^p of a graph constraint where the corresponding required pattern is missing; (ii) r deletes an existing match of a required pattern; (iii) r produces a new match with the forbidden pattern. However, it can be seen that if r does not satisfy any of **Cond 1-3**, then it does not affect any constraint from $C^{\mathfrak{S}}$ because of the following reasons:

- r does not satisfy **Cond 1**; therefore, it does not produce any new match with the condition pattern L^p of a graph constraint.
- r does not satisfy **Cond 2**; therefore, it does not delete any existing match of a required pattern.
- r does not satisfy **Cond 3**; therefore, it does not produce any new match with the forbidden patterns.

□

4.2 Desired Patterns for Conformance

A rule r can be conformance preserving if it satisfies some conditions from **Cond 1-3** and complies with desired patterns described below. Figure 5 illustrates a diagram representing the intuition of the proposed method where **P.C1#1, P.C1#2,...** indicates a pattern number.

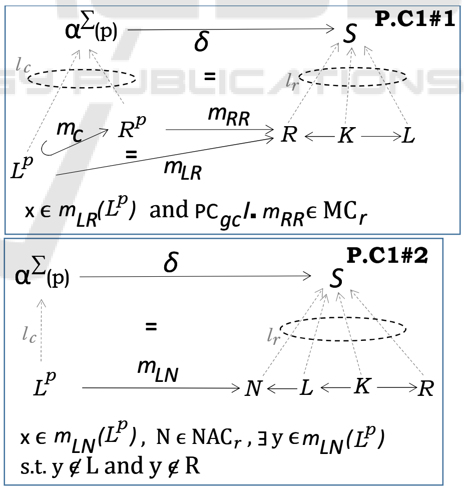


Figure 6: Patterns for a conformance preserving rule r that satisfies **Cond 1**.

In our approach, if a rule satisfies **Cond 1** for a graph constraint gc, it has to comply with the patterns specified in Figure 6. Patterns specified in the figure makes sure that if the creation of an element produces a new match with the condition pattern of a graph constraint, the required pattern must exist (**P.C1#1**); otherwise a new match with the condition pattern is not produced by the application of rule r (**P.C1#2**). Note

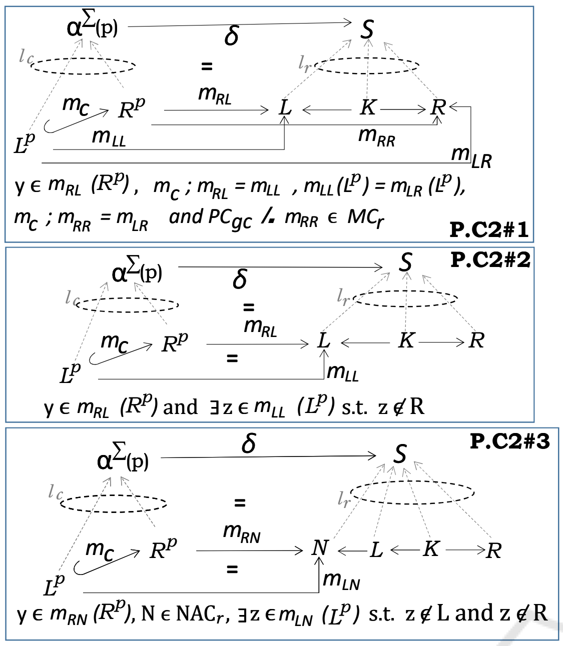

P.C2#1
y ∈ m_RL (R^p), m_c ; m_RL = m_LL , m_LL(L^p) = m_LR (L^p),
m_c ; m_RR = m_LR and PC_gc /. m_RR ∈ MC_r
P.C2#2
y ∈ m_RL (R^p) and ∃z ∈ m_LL (L^p) s.t. z ∉ R
P.C2#3
y ∈ m_RN (R^p), N ∈ NAC_r, ∃z ∈ m_LN (L^p) s.t. z ∉ L and z ∉ R

Figure 7: Patterns for a conformance preserving rule r that satisfies **Cond 2**.

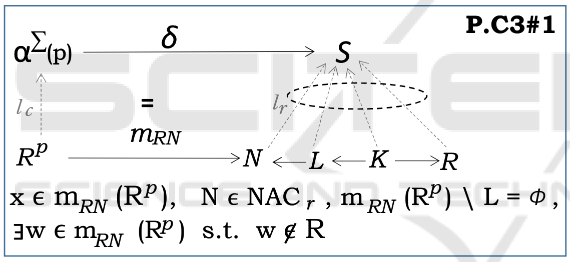

P.C3#1
x ∈ m_RN (R^p), N ∈ NAC_r , m_RN (R^p) \ L = Φ ,
∃w ∈ m_RN (R^p) s.t. w ∉ R

Figure 8: Pattern for a conformance preserving rule r that satisfies **Cond 3**.

that in the graph patterns, solid arrows are representing injective graph homomorphisms. Pattern **P.C1#1** ensures that if a graph constraint has a post-condition (PC_{gc}), a compliant rule r includes the post-condition into the set of matching conditions MC_r by replacing PC_{gc} with the matching elements from r.

The patterns presented in Figure 7 makes sure that if the deletion of an element removes an existing match of a required pattern of a graph constraint, then either the match of the condition pattern is also removed (**P.C2#2**) or another match of a required pattern is produced (**P.C2#1**); otherwise (**P.C2#3**) makes sure that the deletion of an existing element does not remove an existing match of a required pattern.

The pattern **P.C3#1** presented in Figure 8 makes sure that the creation of an element does not produce a match with the forbidden pattern of a graph constraint. **P.C3#1** also ensures that the forbidden pattern of gc is included in the NACs of the rule via the condition $m_{RN}(R^p) \setminus L = \phi$.

4.3 Algorithm for Checking Conformance Preserving Rule

Algorithm 1 provides a method for checking the conformance preserving property of a rule w.r.t a set of graph constraints.

Theorem 1.(Soundness of Algorithm. 1) *Let $\mathfrak{S} = (S, C^{\mathfrak{S}} : \Sigma)$ be a metamodel specification and r (typed by S) a transformation rule which is determined to be conformance preserving w.r.t $C^{\mathfrak{S}}$ by Algorithm 1. If r is applied on a valid model $(I, \iota_I) \models \mathfrak{S}$ then the result (I^*, ι_{I^*}) will be a valid model of $\mathfrak{S}$.*

Proof. Let GC be a set of constraints giving semantics to the set of constraints $C^{\mathfrak{S}}$. To prove the theorem by contradiction, it is sufficient to show that there exists a $gc \in GC$ such that (I^*, ι_{I^*}) does not satisfy gc. There are three ways in which it is possible for (I^*, ι_{I^*}) to violate the graph constraint:

i gc is of the form $\forall(L^p : \alpha^{\Sigma}(p)) \rightarrow \exists(R^p : \alpha^{\Sigma}(p))$ and $\exists\, A \subseteq (I^* \setminus I)$ such that a new match (δ, m_{LI^*}) is produced from $(L^p : \alpha^{\Sigma}(p))$ to I^* but a corresponding match from $(R^p : \alpha^{\Sigma}(p))$ to I^* is missing.

ii gc is of the form $\forall(L^p : \alpha^{\Sigma}(p)) \rightarrow \exists(R^p : \alpha^{\Sigma}(p))$ and $B \subseteq (I \setminus I^*)$ such that a required match from $(R^p : \alpha^{\Sigma}(p))$ to I is removed but a match from $(L^p : \alpha^{\Sigma}(p))$ to I still remains in I^*.

iii gc is of the form $\forall(L^p : \alpha^{\Sigma}(p)) \rightarrow \neg\exists(R^p : \alpha^{\Sigma}(p))$ and $\exists\, A \subseteq (I^* \setminus I)$ such that a new match is produced from the forbidden pattern $(R^p : \alpha^{\Sigma}(p))$ to I^*.

Case (i): r satisfies **Cond 1** since a new match for the condition pattern is produced. According to Algorithm 1, r must comply with either **P.C1#1** or **P.C1#2**. The pattern in **P.C1#2** has a *NAC* that prevents the existence of pattern that matches with $(L^p : \alpha^{\Sigma}(p))$. Since a new match with $(L^p : \alpha^{\Sigma}(p))$ is produced in (i), r must comply with **P.C1#1**. Therefore, when the rule is applied, a corresponding match (δ, m_{RI^*}) from $(R^p : \alpha^{\Sigma}(p))$ to I^* for the match (δ, m_{LI^*}) must exist. Therefore (I^*, ι_{I^*}) satisfies the graph constraint gc. Hence we reach to a contradiction.

Case (ii): This case is explained by considering three matches:

- $(\delta, m_{LI}) : (L^p : \alpha^{\Sigma}(p)) \rightarrow (I : S)$,
- $(\delta, m_{RI}) : (R^p : \alpha^{\Sigma}(p)) \rightarrow (I : S)$,

Algorithm 1: Check for conformance preserving rule.

```
Require: a coupled transformation rule $r$, a set of graph constraints GC
  C := R \ L of $r$     //set of elements created by r
  D := L \ R of $r$     //set of elements deleted by r
  for each x in C do
    for each gc ∈ GC do
      if gc has a required pattern $R^p$
        and $r$ satisfies Cond 1 for x and gc then
        if $r$ does not comply with P.C1#1 or P.C1#2 for x then
          return ``may not be conformance preserving''
      if gc has a forbidden pattern $R^p$
        and $r$ satisfies Cond 3 for x and gc then
        if $r$ does not comply with P.C3#1 for x then
          return ``may not be conformance preserving''
  for each y in D do
    for each gc ∈ GC do
      if gc has a required pattern $R^p$
        and $r$ satisfies Cond 2 for y and gc then
        if $r$ does not comply with P.C2#1 or P.C2#2 or P.C2#3
        for y then
          return ``may not be conformance preserving''
  return ``conformance preserving''
```

- $(\delta, m_{LI^*}) : (L^p : \alpha^\Sigma(p)) \to (I^* : S)$

where $m_c; m_{RI} = m_{LI}$, $m_{LI}(L^p) = m_{LI^*}(L^p)$ and there does not exist a corresponding match $(\delta, m_{RI^*}) : (R^p : \alpha^\Sigma(p)) \to (I^* : S)$ such that $m_c; m_{RI^*} = m_{LI^*}$. Therefore $\exists\, y \in B$ such that $y \in (m_{RI}(R^p) \setminus m_{LI}(L^p))$ from which we obtain $Y \in \iota_c; \delta(R^p \setminus L^p)$ where Y is the type of y. Hence, r satisfies **Cond 2** and according to Algorithm 1, r must comply with either **P.C2#1**, **P.C2#2** or **P.C2#3**. Pattern in **P.C2#3** has a *NAC* that prevents the existence of pattern that matches with $(L^p : \alpha^\Sigma(p))$. The pattern in **P.C2#2** makes sure that the existence of pattern that matches with $(L^p : \alpha^\Sigma(p))$ is removed. Since in case (ii), the matching with the condition pattern remains, the rule r must comply with **P.C2#1**. However, pattern **P.C2#1** makes sure that a corresponding match for the required pattern is produced which contradicts with the second case.

Case (iii): r satisfies **Cond 3** since a new match for the forbidden pattern is produced. According to Algorithm 1, r must comply with **P.C3#1**. Pattern **P.C3#1** has a *NAC* that prevents the existence of pattern that matches with $(R^p : \alpha^\Sigma(p))$. Therefore we reach to a contradiction.

In all three cases we have shown that if r is applied on a valid model $I \models \mathfrak{S}$ then the result (I^*, ι_{I^*}) cannot violate the constraints specified in $\mathfrak{S}$. □

Complexity of Algorithm 1: The complexity of the algorithm depends on two factors: (i) the size of graph patterns of the graph constraints and (ii) the size of graph patterns in transformation rules. The size of a graph pattern refers to the number of vertices of the graph. The performance of the algorithm depends on injective matching. Finding an injective match from an n-vertex graph (G) to a m-vertex graph (H) has complexity $2^{O(n \log m)}$ as finding all possible vertex subsets of H of size at most n is $m^{O(n)}$ and for each subset we need to try all possible mappings from G. The algorithm avoids processing the models of a system, therefore it is expected to analyze the transformation rules fast because in a typical situation, the size of graph patterns in graph constraints and transformation rules would be very small compared to the size of models.

Theorem 2. *Given a metamodel specification $\mathfrak{S} = (S, C^{\mathfrak{S}} : \Sigma)$ and a set of conformance preserving rules $\mathcal{R} = \{r_1, ...r_n\}$ w.r.t a set of atomic constraints $C^{\mathfrak{S}}$. If the rules are applied on a valid model of $\mathfrak{S}$ a finite number of times, the result will be a valid model of $\mathfrak{S}$.*

Proof. The theorem can be proved by induction over the number of application of the transformation rules as the results produced in each step are valid models of $\mathfrak{S}$.

□

5 RELATED WORK

There has been a great deal of research related to the formal analysis of termination, confluence, functional behaviour of model transformation systems (Hermann et al., 2010) (Bruggink et al., 2014) (Heckel et al., 2002) (Plump, 2010) and tool support (Arendt et al., 2010) (Taentzer, 2003). One important difference between our approach and existing approaches is that our approach rest on diagrammatic logic. Our approach is closely related to the work of Heckel and Wagner (Heckel and Wagner, 1995). They ensured consistency of graph transformations by automatically adding application conditions to single pushout (SPO) rules. They propose a technique for deriving application conditions from SPO rules of the form $L \xrightarrow{r} R$ and constraints. Constraints are specified in the form $P \xrightarrow{c} Q$ where P and Q are directed graphs and c is an injective morphism. In their approach, a post-condition (i.e., an application condition over the right hand side of a rule) is constructed as a set of all right-sided constraints by generating all possible gluings of the premise P and the graph R. The post-condition is then used to construct a left-sided constraint (i.e, an application condition over L) by inverse decomposition of pushout diagrams. One issue with this approach is that a post-condition induced by a constraint may include a large number of right-sided constraints. A simple technique was presented in (Heckel and Wagner, 1995) to reduce the number of right-sided constraints from a post-condition. The idea of the reduction is based on the removal of right-sided constraints that is obtained from a gluing $R \xrightarrow{s} S \xleftarrow{p} P$ where the image of P in S does not depend on elements generated by rule r i.e., $p(P) \cap s(R - r(L)) = \emptyset$. This reduction technique however cannot handle situations where a rule deletes an element that matches with the required pattern Q of a constraint $c : P \to Q$ (see **Cond 2** of Figure 5). To illustrate this issue, consider an input graph G with $c; m_q = m_p$ where $m_p : P \to G$ and $m_q : Q \to G$ are two injective morphism. Now consider a rule $L \xrightarrow{r} R$ where $p(P) \cap s(R - r(L)) = \emptyset$ which means that the reduction will disregard the constraint c and no left-sided constraint will be constructed. But it is possible for the rule to remove an element x from $m_q(Q)$ which results in an output not conforming to its metamodel.

Later on, this approach for ensuring consistency was adapted for a double pushout approaches and generalized for high level transformation systems (Ehrig et al., 2006). The approach was further enhanced for nested constraints in (Habel and Pennemann, 2009). Although the approach presented in (Ehrig et al., 2006; Habel and Pennemann, 2009) can deal with situations where a rule add/delete elements, the construction of application conditions do not include any reduction technique. This results in a large number of application conditions. In our approach, we rely on the modeller to develop transformation rules and automatically check conformance using our algorithm. The proposed algorithm filters out trivially conformance preserving rules as described in section 4.1 before checking the existence of the desired patterns in section 4.2 for optimal performance.

Becker et al. (Becker et al., 2006) developed a verification technique for structural safety property of a transformation system which is very similar to our approach in the sense that their technique is based on checking the locality of transformation rules against a set of safety properties. In their approach, the authors checked if the application of transformation rules can violate any safety property given as a set of forbidden graph patterns. Dyck and Giese (Dyck and Giese, 2015) improved the technique for the automated verification of structural invariants for graph transformation systems by extending the expressive power. They provided support for negative application conditions in constraints and support for application conditions in transformation rules. However, both techniques only check against forbidden patterns while in our approach we support checking the conformance property of transformation rules against both required and forbidden patterns. Making sure that the application of a transformation rule does not violate any required pattern is more complex than checking against a set of forbidden patterns as it involves more scenarios to cover for the checking algorithm.

6 CONCLUSION

In this paper, we presented a static analysis technique for checking the conformance property of transformation rules. The static analysis technique processes the semantics of graph constraints and analyzes if a transformation rule exhibits certain structure in order to be conformance preserving rule w.r.t a set of constraints. We presented the idea in the context of DPF which provides a formal framework for metamodelling. Future work includes the implementation of the algorithm and undertaking performance evaluation tests. Also, as part of future work, we plan to adapt the algorithm to more expressive constraint language such as nested graph constraints.

REFERENCES

Arendt, T., Biermann, E., Jurack, S., Krause, C., and Taentzer, G. (2010). Henshin: Advanced Concepts and Tools for In-place EMF Model Transformations. In *Proceedings of the 13th International Conference on Model Driven Engineering Languages and Systems: Part I*, MODELS'10, pages 121–135. Springer-Verlag.

Baresi, L. and Spoletini, P. (2006). *On the Use of Alloy to Analyze Graph Transformation Systems*, pages 306–320. Springer.

Becker, B., Beyer, D., Giese, H., Klein, F., and Schilling, D. (2006). Symbolic invariant verification for systems with dynamic structural adaptation. In *Proceedings of the 28th International Conference on Software Engineering*, ICSE '06, pages 72–81, New York, NY, USA. ACM.

Bruggink, H. J. S., König, B., and Zantema, H. (2014). *Termination Analysis for Graph Transformation Systems*, pages 179–194. Springer.

da Costa, S. A. and Ribeiro, L. (2012). Verification of graph grammars using a logical approach. *Science of Computer Programming*, 77(4):480 – 504.

Diskin, Z. and Wolter, U. (2008). A diagrammatic logic for object-oriented visual modeling. *Electronic Notes in Theoretical Computer Science*, 203(6):19 – 41. Proceedings of the 2nd Workshop on Applied and Computational Category Theory (ACCAT 2007).

Dyck, J. and Giese, H. (2015). *Inductive Invariant Checking with Partial Negative Application Conditions*, pages 237–253. Springer International Publishing, Cham.

Ehrig, H., Ehrig, K., Prange, U., and Taentzer, G. (2006). *Fundamentals of Algebraic Graph Transformation*. Monographs in Theoretical Computer Science. Springer.

Habel, A. and Pennemann, K.-h. (2009). Correctness of high-level transformation systems relative to nested conditions. *Mathematical. Structures in Comp. Sci.*, 19(2):245–296.

Heckel, R., Küster, J. M., and Taentzer, G. (2002). *Confluence of Typed Attributed Graph Transformation Systems*, pages 161–176. Springer.

Heckel, R. and Wagner, A. (1995). Ensuring Consistency of Conditional Graph Grammars - A Constructive Approach -. *ENTCS*, 2(C):118–126.

Hermann, F., Ehrig, H., Orejas, F., and Golas, U. (2010). *Formal Analysis of Functional Behaviour for Model Transformations Based on Triple Graph Grammars*, pages 155–170. Springer.

Lambers, L., Ehrig, H., Prange, U., and Orejas, F. (2008). *Embedding and Confluence of Graph Transformations with Negative Application Conditions*, pages 162–177. Springer.

Plump, D. (2010). Checking graph-transformation systems for confluence. *ECEASST*, 26.

Ribeiro, L., Dotti, F. L., da Costa, S. A., and Dillenburg, F. C. (2010). Towards theorem proving graph grammars using event-b. *ECEASST*, 30.

Rutle, A. (2010). *Diagram Predicate Framework: A Formal Approach to MDE*. PhD thesis, Department of Informatics, University of Bergen, Norway.

Rutle, A., Rossini, A., Lamo, Y., and Wolter, U. (2012). A formal approach to the specification and transformation of constraints in mde. *Journal of Logic and Algebraic Programming*, 81(4):422–457.

Taentzer, G. (2003). AGG: A graph transformation environment for modeling and validation of software. In Pfaltz, J. L., Nagl, M., and Böhlen, B., editors, *Applications of Graph Transformations with Industrial Relevance, Second International Workshop, AGTIVE 2003, Charlottesville, VA, USA, September 27 - October 1, 2003, Revised Selected and Invited Papers*, volume 3062 of *Lecture Notes in Computer Science*, pages 446–453. Springer.

Troya, J. and Vallecillo, A. (2010). *Towards a Rewriting Logic Semantics for ATL*, pages 230–244. Springer.

Varró, D., Varró, G., and Pataricza, A. (2002). Designing the automatic transformation of visual languages. *Sci. Comput. Program.*, 44(2):205–227.

Wang, X., Büttner, F., and Lamo, Y. (2014). Verification of graph-based model transformations using alloy. *ECEASST*, 67.