Game-theoretic Analysis of Air-cargo Allotment Contract

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Keywords: Air-cargo, Capacity Management, Stochastic Model Applications.

Abstract: Consider an air-cargo carrier and a freight forwarder, which may establish an allotment contract at the start of the season. The allotment size needs to be determined, before their stochastic demands materialize. The forwarder hopes to receive a discount rate, lower than the spot rate. The carrier hopes to increase capacity utilization by handling not only its own direct-ship demand but also the forwarder's demand. We formulate a Stackelberg game, in which the carrier is the leader and offers contract parameters such as the wholesale price and the refund rate for the unused allotment as well as the minimum allotment utilization. Given the carrier's offer, the forwarder decides how much to book as an allotment, in order to maximize its own expected profit. We analyze the game and identify conditions, in which an equilibrium contract coordinates the air-cargo chain. We show that the minimum allotment utilization is needed to construct a coordinating contract. In our numerical examples, we illustrate how to apply our approach to the case study of one of the biggest forwarders in Thailand. The contract can improve both parties' profits, compared to the scenario without any contract, where the forwarder purchases all capacity from the spot market.

1 INTRODUCTION

Air-cargo operations play a crucial role in the modern economy, since they improve efficiency in logistics and increase competitive advantages. Despite the 1% world trade by volume, airfreight represents more than 35% of global trade by value (International Air Transport Association, 2016). Air cargo operations generate almost 10% of the passenger airline revenue, more than twice revenues from the first class. Air cargo consists of various commodity types, e.g., perishable cargo, pharmaceutical products, dry ice, live animals, electronic devices, human remains, and gold bullion. World air-cargo is forecasted to grow over 200% over the next two decades. The largest average annual growth rate is found in Asia (Boeing Company, 2012). The air-cargo growth is partly driven by global liberalization, cross-border e-commerce, and the implementation of supply chain/logistics management strategies, which emphasize on short lead times, e.g., lean management and just-in-time (JIT) production systems. With e-commerce boom, airfreight has become a de facto mode of cross-border transportation, for the customer centric businesses with fast delivery times. A shipper can receive services directly from an air cargo carrier or delegate to a freight forwarder. A large portion of air cargo volume is handled through freight forwarders.

A freight forwarder acts as an intermediary party, who connects a shipper to an airline. It handles various aspects of the shipping process, e.g., pickup and delivery services, customs clearance, import and export documentation, cargo tracking and tracing. Most forwarders do not own an airplane and obtain cargo space on *ad hoc* basis or through a medium- or long-term capacity agreement, also known as the *allotment*. The carrier offers a contract to the forwarder, hoping to increase its capacity utilization. Capacity utilization is one of the top operational problems, faced by the majority of the cargo carriers (Accenture, 2015). The forwarder wants to establish the contract, in order to receive volume discounts or lower freight charges.

Air-cargo spaces are sold in two stages: In the first stage which happens a few months before a season starts, carriers allocate space to forwarders either as part of a binding contract or as part of goodwill (Billings et al., 2003). Each year comprises of two seasons, Winter and Summer schedules, specified by the International Air Transport Association (Slager and Kapteijns, 2004). The allotment allows the forwarder to achieve a more economical rate, compared to the so-called *spot rates* for *ad hoc* shipments. Based on its anticipated volume requirement on a given route, the forwarder pre-books a certain amount

Amaruchkul, K.
Game-theoretic Analysis of Air-cargo Allotment Contract.
DOI: 10.5220/0006551800470058
In *Proceedings of the 7th International Conference on Operations Research and Enterprise Systems (ICORES 2018)*, pages 47-58
ISBN: 978-989-758-285-1

of capacity at a pre-specified rate. The airline allocates cargo capacity on the season's flights as an allotment. The allotment contract is established before the start of each season, whose duration ranges from a few weeks to a year. Typically, about 50-70% of air-cargo space is sold to forwarders through a "hard" block space agreement (BSA) at a negotiated price, a "soft" block permanent booking (PB) or other forms of capacity purchase agreements (Sales, 2013). Carriers in North America typically allocate a small fraction of their capacity, whereas those in Asia Pacific allocate a large fraction (Hendricks and Elliott, 2010). After the forwarder collects and consolidates shipments from its customers, and the actual allotment usage becomes known, the payment is transferred between the carrier and the forwarder. The carrier may impose some cancellation fee for the unused allotment by the forwarder. However, for the airline's most important forwarders, the cancellation clause is rarely enforced; these powerful forwarders pay only for their actual allotment usages. After the unused allotment is released by the forwarder a few weeks before a flight departure, the carrier re-sells the remaining capacity on a *free-sale* or *ad hoc* basis to direct shippers.

Since air-cargo capacity can be sold at different prices to heterogeneous customers but cannot be sold after the flight departure, it is a prime candidate for revenue management (RM) strategies. Overview of RM theory and practice can be found in textbooks, e.g., (Phillips, 2005), (Talluri and van Ryzin, 2004) and (Yeoman and McMahon-Beattie, 2004), (Ingold et al., 2000), and journal articles, e.g., (Netessine and Shumsky, 2002), (Chiang et al., 2007) and (McGill and van Ryzin, 1999). Despite an extensive literature on passenger RM, literature on air-cargo RM is fairly limited. (Kasilingam, 1996) and (Billings et al., 2003) are among the early descriptive papers that provide overview and complexity of the air-cargo RM. (Bazaraa et al., 2001) describes the air-cargo system in the Asia Pacific. (Slager and Kapteijns, 2004) describes the implementation of air-cargo RM system at KLM and highlight key factors that critically affect its performance. Air-cargo RM from business perspective is discussed in (Becker and Dill, 2007) and (DeLain and O'Meara, 2004). The overview and the industry outlook of the air cargo service chain can be found in, e.g., (Boeing Company, 2009) and (International Air Transport Association, 2016).

Key short-term air-cargo operations include booking control (e.g., (Zhang et al., 2017), (Barz and Gartner, 2016), (Levin et al., 2012) and (Amaruchkul et al., 2007)) and shipment routing (e.g., (Prior et al., 2004) and (Yang et al., 2006)), whereas medium- and long-term operations include fleet replacement and capacity contract. (Yeung and He, 2012) gives a brief survey on shipment planning and capacity contracting in the air cargo industry. Articles on an air cargo capacity contract are briefly reviewed below. With the exception of (Barz, 2007) which considers risk-adverse party, the articles below assume that the forwarder is rational, risk-neutral and maximizes its expected profit. (Gupta, 2008) shows that an efficient airline-forwarder service chain can be achieved through two flexible contract schemes. In the first scheme, the airline imposes a fixed upfront payment for reserving an allotment, whereas in the second scheme there is no reservation fee, but the airline chooses the freight rate. (Amaruchkul et al., 2011) considers the airline's allocation problem, in which the forwarder possesses some private information, e.g., its customer demand and operating cost. An optimal allotment, which maximizes the total contribution of the air cargo service chain, is attainable via a contract with an appropriate upfront and cancellation fees. (Hellermann, 2013) proposes an options contract, similar to supply chain contracts in, e.g., electricity generation and semiconductor manufacturing, and investigates the suitability of options contracts in the air cargo industry. Under certain contract parameters and a suitable spot market environment, the options contract outperforms the fixed-commitment contract. (Feng et al., 2015) proposes the tying capacity allocation mechanism, in which multiple routes with different capacity utilization are included in the contract. (Tao et al., 2017) studies an option contract to mitigate the carrier's capacity utilization risk. These papers employ a mechanism design approach and propose a contract scheme to improve the air cargo service chain. Ours contributes to this literature: We propose a different scheme, which includes the wholesale price and the penalty cost associated with the unused allotment usage as well as the required allotment utilization.

Our mathematical model captures important aspects of the allotment problem, commonly found in practice. In our model, the size of the allotment depends on the forwarder's anticipated customer demand. Contract terms, e.g., a freight charge and a cancellation fee, may also affect the forwarder's decision: A deep discount or a small cancellation fee makes the allotment more attractive, from the forwarder's viewpoint. On the other hand, if the penalty fee associated with the unused allotment is very large, or the wholesale contract price is still very high, compared to the spot rate, the forwarder may not want a large allotment or may not want to establish a contract at all. The allotment affects the profits of both forwarder and carrier. The carrier's revenue depends on both the contract payment from the forwarder and the revenue from direct shippers. At the time that the contract is established, the direct-ship demand and the forwarder's demand remain unknown. The contract parameter and the allotment need to be determined, before demands materialize. To this end, we formulate a Stackelberg game, in which the carrier is the leader and offers a set of contract parameters to the forwarder. Based on its demand distribution and the contract parameter, the forwarder determines the best allotment size, which maximizes its own expected profit. Our article analyzes this sequential game of the air-cargo allotment contract.

The rest of this article is organized as follows. Section 2 presents the Stackelberg game, and it is analyzed in Section 3. For a given contract parameter, we solve for the forwarder's allotment, which maximizes its own expected profit. After obtaining the forwarder's best response, we solve for the equilibrium of the game, i.e, the optimal contract parameter offered by the carrier. Section 4 provides some numerical examples to illustrate our approach, and the benefit from the allotment contract is quantified numerically. Finally, Section 5 gives a summary and a few extensions.

2 FORMULATION

Consider an air-cargo service chain, which consists of a freight forwarder and an air-cargo carrier endowed with cargo capacity of κ. In the strategic level, the air-cargo capacity is assumed to be a one-dimensional quantity; in practice, the allotment agreement is given in terms of weight. (In the operational level, the forwarder is charged based on the *dimensional weight*, which the volume is converted to a "volume weight." If the cargo measurements are in centimeters, the volume weight is obtained by dividing the cubic centimeters by 6000. The chargeable weight is the higher of the gross weight (in kilograms) and the volume weight. This article concerns with the forwarder's strategic decision making, not operational.) The freight forwarder wants to pre-book capacity in bulk with an airline to achieve a discount rate, which is less than or equal to the spot rate, denoted by v. The carrier wants to block some space as an allotment for the forwarder, to achieve better cargo utilization, because its own direct-ship demand may not be enough to fill the cargo hold.

Let x be the size of the allotment, D_f the random customer demand to the forwarder, D_a the direct-ship demand to the carrier, on the given route. The sequence of events is as follows: Prior to the start of the selling season, the carrier offers a wholesale price w to the forwarder, and the forwarder determines the size of the allotment, x. When the allotment agreement is made, the random demands to both forwarder and carrier are still unknown. Assume that the forwarder's demand materializes before the carrier's demand. (This is typically observed in practice, since the forwarder spends much more effort to attract demand as early as possible.) A few weeks before departure, the forwarder knows its actual requirement. The actual allotment usage is $\min(D_f, x)$, and the unused allotment is $(x - D_f)^+$. If demand is less than the allotment, then the forwarder can return the unwanted space, and the airline sells the remaining capacity on free-sale basis. The carrier may impose a penalty cost for the unused allotment if the forwarder releases the space after the cutoff time, usually 48 hours before departure time. On the other hand, suppose that the actual demand is greater than the allotment. The forwarder books the excess demand, $(D_f - x)^+$ at the spot rate, v.

Let p_f be the forwarder's per-unit price charged to its customer. The forwarder's revenue from accepting its own demand D_f is given as $p_f D_f$. Let h be the unit penalty cost associated with the unused allotment, $(x - D)^+$. The unit cost could be the allotment reservation fee minus the refund. The rate of the refund might depend on how easily the carrier could sell remaining space in the spot market. It can also depend on the timing when the unused allotment is released. The penalty cost could also include loss of goodwill; if the allotment utilizations by the forwarder are consistently low on most flights, the carrier might reduce the allotment size or terminate the allotment contract in the next season. By contrast, in some cases in which the forwarder is much more powerful than the carrier, the unused allotment could be fully refunded, and the unit cost would be negligible. The forwarder's expected contribution for an allotment x is defined as

$$\begin{aligned}\pi(x) &= E[p_f D_f - w\min(D_f, x) \qquad (1)\\ &\qquad - v(D_f - x)^+ - h(x - D_f)^+]\\ &= E[(p_f - w)\min(D_f, x) \qquad (2)\\ &\qquad + (p_f - v)(D_f - x)^+ - h(x - D_f)^+]\end{aligned}$$

where (2) follows from the identity $\min(D_f, x) = D_f - (D_f - x)^+$. In (1), the first term is the forwarder's revenue, the second term the wholesale payment for the actual allotment usage, the third term the forwarder's spot purchase for the excess demand, and the fourth term the penalty cost associated with the unused allotment. Note that from (2), the forwarder's *contribution margin* per unit allotment is $(p_f - w)$, and that associated with the spot capacity is $(p_f - v)$. The forwarder's margin is defined as the price charged to the customer minus the *incremental cost*, which is the difference between the total cost the forwarder would experience if it makes the commitment to its customer and the total cost it would experience if it does not (Phillips, 2005). For instance, the forwarder's incremental cost might include fuel and security surcharges, terminal handling fees, and customs clearance, which would be incurred from handling one unit of cargo, as well as any commissions or fees the forwarder would incur, assuming that the forwarder do not pass any of these costs to its customers. (If the forwarder directly passes on, say the fuel surcharge to its customer, this particular cost would not be included in the incremental cost.)

In (1), the contract parameter w is interpreted as the per-unit price for the actual allotment usage, and h the per-unit penalty cost for the unused portion of the allotment. They can be interpreted differently as follows: The transfer payment from the forwarder to the carrier can be written as

$$\begin{aligned} & w\min(D_f, x) + h(x - D_f)^+ \\ & \qquad = wx + (h-w)(x-D_f)^+ \qquad (3) \\ & \qquad = wx - (w-h)(x-D_f)^+ \qquad (4) \end{aligned}$$

In (3) and (4), we can interpret w as the wholesale price for the entire allotment x, paid upfront by the forwarder. If $h = w$, then the forwarder pays for the entire allotment x at the wholesale price w upfront, and there are no additional monetary transfers. If $h > w$, then the forwarder pays for the allotment x upfront at the wholesale price of w, and after its demand is realized, the penalty rate of $(h-w)$ is charged for the unused portion of the allotment; see (3). If $h < w$, then the forwarder pays for the allotment x upfront at the wholesale price of w as before, but after its demand is realized, the refund rate of $(w-h)$ for the unused portion of the allotment is returned from the carrier to the forwarder; see (4). Since the air-cargo selling season is so short that we do not need to account for monetary discount; thus, the expected profit is not affected by the timing in which the payment is collected. Our formulation subsumes both refund and penalty rates for the unused portion of the allotment. After the forwarder's demand D_f materializes, the forwarder pays the penalty cost to the carrier for the unused portion of the allotment if $h > w$; on the other hand, if $h < w$, the carrier returns the refund to the forwarder.

In practice, the contract terms may include the wholesale price and the penalty cost (or the refund rate) as well as the required allotment utilization. Define the allotment utilization as the ratio of the expected actual allotment usage to the allotment size x:

$$u(x) = \frac{1}{x} E[\min(D_f, x)]. \qquad (5)$$

If the utilization is too low, in the next season, the airline might decrease the allotment, or increase the contract rate, or impose a minimum volume requirement, or terminate the allotment contract. Let $u_r \in (0,1)$ be the required allotment utilization. In practice, the forwarder generally needs to maintain the utilization to be at least 60-70%.

$$\max\{\pi(x) : u(x) \geq u_r)\}. \qquad (6)$$

Let $\Omega = (w, h, u_r)$ be the contract parameter. After the forwarder releases its unused allotment, the carrier re-sells this to direct shippers. Let p_a be the carrier's price charged to the direct customers. The carrier's expected profit is defined as:

$$\begin{aligned} \psi(x, \Omega) = E[& p_a \min(D_a, \kappa - \min(D_f, x)) \\ & + w\min(x, D_f) + h(x - D_f)^+] \qquad (7) \end{aligned}$$

In (7), the first term is the carrier's revenue from selling the remaining cargo space, $\kappa - \min(D_f, x)$, to its own direct-ship customers, the second term the forwarder's payment for the actual allotment usage, and the third term the penalty paid by the forwarder to the carrier for the unused allotment.

Note that the carrier's expected profit depends on both the contract parameter Ω and the forwarder's decision on the allotment x. In our Stackelberg game, we assume that the carrier is the leader which offers a contract parameter Ω, and the forwarder is the follower, which decides on the allotment x. Let $x_f^*(\Omega)$ denote the forwarder's best response to the contract parameter Ω; i.e.,

$$x_f^*(\Omega) = \text{argmax}\{\pi(x) : 0 \leq x \leq u_r\}.$$

At the equilibrium solution, the carrier anticipates the forwarder's best response $x_f^*(\Omega)$, and the carrier chooses the best contract parameter Ω, which maximizes its own expected profit:

$$\begin{aligned} \max_{\Omega}\ & E[p_a \min(D_a, \kappa - \min(x_f^*(\Omega), D_f)) \\ & \quad + w\min(D_f, x_f^*(w)) + h(x_f^*(w) - D_f)^+] \qquad (8) \\ \text{subject to:} & \qquad x_f^*(\Omega) \leq \kappa \qquad (9) \end{aligned}$$

In Section 3, we will determine the equilibrium of the game. This is sometimes referred to as the *decentralized chain*, in which each party maximizes its own expected profit.

Finally, we consider the entire air-cargo service chain: Suppose that the forwarder and the carrier are owned by the same firm, called the *integrator*.

This is referred to as the *centralized chain*. The total chain's expected profit is defined as the sum of the forwarder's expected profit and that of the carrier:

$$\begin{aligned}\tau(x) &= \pi(x) + \psi(x,\Omega)\\ &= E[p_f D_f + p_a \min(D_a, \kappa - \min(D_f, x))\\ &\quad - v(D_f - x)^+].\end{aligned} \tag{10}$$

In (10), the first term is the revenue from the customers, which need dedicated services as offered by the forwarder, the second term the revenue from direct-ship customers, and the third term is the cost from purchasing spot capacity for the excess demand. In practice, $p_f \geq p_a$ because the forwarder offers value-added service, e.g., customs clearance and door-to-door service. Eq. (10) assumes that the integrator accepts all the forwarder's demand, D_f. The integrator handles x units of the forwarder's demand using its own capacity and reserves the remaining capacity $\kappa - \min(x, D_f)$ for the direct-ship customers. The forwarder's excess demand is handled through the spot market. Note that in the integrator's profit, there are no payment terms, because we assume that the forwarder and the carrier belong to the same firm, and their payments cancel out when we analyze the entire service chain.

The service chain is said to be *efficient* if the total expected profit of the chain (the integrator's expected profit) is equal to the sum of the profits of the two parties. The contract which allows the efficiency to occur is said to *coordinate* the service chain (Cachon, 2003). The coordinating contract is a desirable feature, since there would be no efficiency loss from entering into the contract, and the service chain risk is shared appropriately. In the analysis, we will find an equilibrium coordinating contract, if exists.

3 ANALYSIS

For each $i \in \{f, a\}$, assume demand D_i is a nonnegative continuous random variable. Let F_i be the distribution function of D_i, $\bar{F}_i$ the complementary cumulative distribution function, F_i^{-1} the quantile function, and ξ_i the density function.

Define $\upsilon^{-1} : (0,1) \to (0,\infty)$ as the inverse function of the utilization function; i.e., $u(x) = t$ if and only if $\upsilon^{-1}(t) = x$. In words, $\upsilon^{-1}(t)$ corresponds to the allotment at which the utilization is exactly equal to t.

Lemma 1.

1. *The forwarder's constrained maximization*
$$\max\{\pi(x) : u(x) \geq u_r\}$$
is equivalent to
$$\max\{\pi(x) : x \leq \upsilon^{-1}(u_r)\}.$$
2. *In the carrier's problem (8), constraint (9)*
$$x_f^*(\Omega) \leq \kappa$$
holds, if
$$u_r \geq u(\kappa).$$

Proof. The first derivative of the expected utilization $u(x) = E[\min(D_f, x)]/x$ with respect to the allotment x is

$$u'(x) = -\frac{1}{x^2}\int_0^x t\xi_f(t)dt < 0. \tag{11}$$

Eq. (11) follows from

$$E[\min(D_f, x)] = \int_0^x t\xi_f(t)dt + x\bar{F}_f(x) = \int_0^x \bar{F}_f(t)dt.$$

Since $u'(x) < 0$, the utilization is decreasing in x. For all $x \leq \upsilon^{-1}(u_r)$, $u(x) \geq u_r$. In particular, $u(x) \geq u_r \geq u(\kappa)$ implies that $x \leq \kappa$ since u is decreasing. □

It follows from Lemma 1 that the utilization constraint imposes the upper bound $\upsilon^{-1}(u_r)$ on the allotment size. The utilization constraint involving the expected value becomes a simple linear constraint $x \leq \upsilon^{-1}(u_r)$.

Theorem 1 characterizes the expected profit function and derives its maximum point. The forwarder's best response function $x_f^*(\Omega)$ is given analytically.

Theorem 1.

1. *If $w \geq v$, the forwarder's expected profit is decreasing and maximized at $x_f^*(\Omega) = 0$.*
2. *If $w < v$ and $h = 0$, the forwarder's expected profit given an allotment $\pi(x)$ is increasing and maximized at $x_f^*(\Omega) = \upsilon^{-1}(u_r)$.*
3. *If $w < v$ and $h > 0$, the forwarder's expected profit is concave, unimodal and maximized at*
$$x_f^*(\Omega) = \min\{F_f^{-1}\left(\frac{v-w}{v-w+h}\right), \upsilon^{-1}(u_r)\}. \tag{12}$$

Proof. Using identity $\min(D_f, x) = D_f - (D_f - x)^+$, the forwarder's expected profit can be written as

$$E[(p_f - w)D_f - (v - w)(D_f - w)^+ - h(x - D_f)^+].$$

For shorthand, let $r = v - w$. If $r \leq 0$, the second term $r(D - x)^+$ is increasing, and the forwarder's expected profit function $\pi(x)$ is decreasing; thus, it is maximized at $x_f^* = 0$.

Note that the first term $E[(p_f - w)D_f]$ does not depend on the allotment. Maximizing the forwarder's expected profit is equivalent to minimizing the "expected total cost"

$$\eta(x) = E[r(D_f - x)^+ + h(x - D_f)^+].$$

Its first derivative with respect to x is

$$\begin{aligned}\eta'(x) &= -r\bar{F}_f(x) + hF_f(x)\\ &= -r + (r+h)F_f(x).\end{aligned}$$

Note that at $\eta'(0) = -r < 0$, $\eta(x)$ is decreasing when $x = 0$. Also, $\lim_{x\to\infty}\eta'(x) = h > 0$, $\eta(x)$ is increasing when x is large. Hence, $\eta(x)$ and the expected profit $\pi(x)$ are unimodal. Furthermore, $\eta''(x) = (r+h)\xi_f(x) \geq 0$, $\eta(x)$ is convex, and the forwarder's expected profit $\pi(x)$ is concave. Setting the first derivative to zero and invoking the first part of Lemma 1, we obtain the expression for the optimal solution. □

The first part of Theorem 1 asserts that if the wholesale price is greater than the spot price, the forwarder would not pre-book any allotment at all. On the other hand, suppose that the wholesale price does not exceed the spot price. If the carrier imposes no penalty cost associated with the unused allotment (i.e., $h = 0$), the forwarders expected profit is increasing, and the forwarder would choose the largest allotment that the carrier would allow, i.e., the upper bound $\upsilon^{-1}(u_r)$ given by the required allotment utilization u_r. The last part of Theorem 1 asserts that if there is a positive penalty cost (i.e., $h > 0$), the forwarder should pre-book the allotment, which balances the cost associated with the unused allotment and the opportunity cost from not having enough allotment. In the newsvendor (single-period) inventory model, the first is referred to as the *overage*, and the latter is referred to as the *underage*; an optimal order quantity that minimizes the expected total cost $E[c_u(D-q)^+ + c_o(q-D)^+]$ is $q^* = F^{-1}(c_u/(c_u+c_o))$ where c_u (resp., c_o) is the unit underage (resp., overage) cost from ordering less (resp., more) than demand, and F is the distribution of demand D. See a standard textbook in operations management for the newsvendor model; e.g., chapter 5 in (Nahmias, 2009). An optimal allotment in Theorem 1 bears a striking resemblance to the optimal order quantity in the newsvendor model.

In (12), the quantity $(v-w)/(v-w+h) = 1/(1+\theta)$ is called the *critical ratio*, where $\theta = h/(v-w)$. The penalty cost h is equal to the constant θ times the margin difference $r = (v-w)$. It plays an important role in the forwarder's allotment decision, x_f^*: The larger the critical ratio or the smaller the constant θ, the larger the allotment size, since the quantile F_f^{-1} is an increasing function. In particular, if the critical ratio is 0.5 or the constant $\theta = 1$, then an optimal allotment is exactly equal to the median demand. If the critical ratio is greater (resp., less) than 0.5 or the constant θ is less (resp., greater) than 1, then an optimal allotment is greater (resp., less) than the median demand. If the penalty cost associated with the unused allotment, h, increases, the forwarder should decrease the allotment size. On the other hand, the forwarder should increase the allotment, if the margin difference $r = (v-w)$ increases. These sensitivity analyses make economic sense.

In Theorem 2, we analyze the carrier's problem when the penalty cost $h > 0$ is given *a priori*. In Theorem 3, we provide the analysis when there is no penalty cost, $h = 0$. Depending on the market power of the two parties, some of the contract parameters might be pre-determined *a priori*. For instance, on the high-demand route, the carrier might be more powerful than the forwarder, and all contract parameters $(w,h,u_r) = \Omega$ can be determined by the carrier. On the low-demand route, the forwarder might be more powerful, and the carrier is able to choose only one parameter, says the wholesale price w; the rest of the contract parameters, e.g., (h,u_r), are pre-specified *a priori*.

Theorem 2. *Assume that the penalty cost is fixed and strictly positive, $h > 0$ and that the required utilization is $u_r = u(\kappa)$. The carrier's problem (8) can be reformulated as the allotment being the decision variable:*

$$\begin{aligned}\max_{x\leq\kappa} E[&p_a \min(D_a, \kappa - \min(x,D_f))\\ &+ \omega(x)\min(x,D_f) + h(x-D_f)^+] \quad (13)\end{aligned}$$

where

$$\omega(x) = v - \frac{F_f(x)}{\bar{F}_f(x)}h. \quad (14)$$

Further assume that the two demands D_a and D_f are independent. Suppose that the ratio of the spot to the carrier's price is $v/p_a > F_a(\kappa)$ and that the penalty cost is

$$0 < h < [(p_a - \omega(\kappa)](1/F_f(\kappa) - 1). \quad (15)$$

Then, the carrier's expected profit is maximized at x_a^* which is the root of:

$$\begin{aligned}\bar{F}_f(x)[&\omega(x) - h - p_aF_a(\kappa - x)]\\ &+ E[\min(x,D_f)]\omega'_f(x) + h. \quad (16)\end{aligned}$$

Proof. Suppose that $h > 0$. For $w \geq v$, the forwarder's best response is zero allotment. For $w < v$, $\omega(x)$ is implicitly as

$$F_f(x) = \frac{v-\omega}{v-\omega+h}.$$

After re-arranging and collecting terms, we obtain the wholesale price as in (14).

Let $\zeta(x)$ be the objective function in (13). Note that

$$\begin{aligned}\zeta'(x) &= (\omega(x)-h)\bar{F}_f(x)+E[\min(x,D_f)]\omega'(x)\\ &\quad +h-p_aP(D_a>\kappa-x,D_f>x)\\ &=\bar{F}_f(x)[\omega(x)-h-p_a\bar{F}_a(\kappa-x)]\\ &\quad +E[\min(x,D_f)]\omega'(x)+h\end{aligned}$$

where the last equation follows from the assumption that D_a and D_f are independent, so $P(D_f>s,D_a>t)=\bar{F}_f(s)\bar{F}_a(t)$. Note that $\omega(x)$ is differentiable (since the distribution is continuous), so $\omega'(x)$ is continuous, and the first derivative $\zeta'(x)$ is continuous. Recall that the carrier requires that the allotment $0<x\leq\kappa$. Note that at the two end points,

$$\zeta'(0)=v-p_a\bar{F}_a(\kappa)$$

and that

$$\zeta'(\kappa)=\bar{F}_f(\kappa)[\omega(\kappa)-h-p_a]+E[\min(x,D_f)]\omega'(x)+h.$$

If $v/p_a>F_a(\kappa)$, then $\zeta'(0)>0$. Also $\zeta'(\kappa)<0$ from (15). Finally, if follows the intermediate value theorem that there exists x_a^* which solves $\zeta'(x)=0$. Note that the carrier's constraint $x\leq\kappa$ holds, since $u_r=u(\kappa)$; see Lemma 1. □

Unless the cumulative distributions $F_i; i=1,2$ are very simple, a closed-form solution to (16) does not exist, and the equilibrium of the game needs to be found numerically; see numerical examples in Section 4.

Theorem 3. *Suppose that $h=0$ and*

$$w=v-\varepsilon$$

where $\varepsilon>0$ is a very small positive number. Then, the carrier's problem can be re-formulated as the required utilization being the decision variable:

$$\begin{aligned}\max_{u_r\geq u(\kappa)} & E[p_a\min(D_a,\kappa-\min(\upsilon^{-1}(u_r),D_f)\\ &+(v-\varepsilon)\min(\upsilon^{-1}(u_r),D_f)]\end{aligned} \tag{17}$$

Proof. It follows from Theorem 1 that if $w<v$ and $h=0$, then $\pi(x)$ is increasing and maximized at the upper bound on the allotment, $\upsilon^{-1}(u_r)$. Constraint $u_r\geq u(\kappa)$ ensures that $x_f^*(\Omega)\leq\kappa$; see Lemma 1. □

Theorem 3 states that if the penalty cost is $h=0$ and that the wholesale price is just below the spot price, then the forwarder chooses the largest allotment which satisfies the required utilization. In the carrier's problem, the required utilization becomes the only decision variable.

In practice, it is uncommon to find that an airline provides a full refund for the unused allotment (thus, $h=0$). To ensure its high customer service level, the forwarder may ask for a very large allotment (much greater than its anticipated customer demand) and release the unwanted allotment so late that the airline does not have enough time to re-sell it in the spot market. To prevent the forwarder to pre-book a large allotment, Theorem 3 suggests that the carrier needs to impose the minimum utilization requirement $[u(x)\geq u_r]$ where $u_r\geq u(\kappa)$. This bound u_r might be tighter than $u(\kappa)$ [specifically, $u_r>u(\kappa)$], since in practice the carrier might not want to allocate the entire capacity κ as the allotment and might want to reserve some space for the direct-ship customers.

Finally, we analyze the integrator's problem. Let $x_0=\text{argmax}\{\tau(x):0\leq x\leq\kappa\}$

Theorem 4.

1. *If $v>p_a$, then $x_0=\kappa$.*
2. *If $v<p_a$, then x_0 is a root of*

$$v\bar{F}_f(x)-p_aP(D_f>x,D_a>\kappa-x). \tag{18}$$

In particular, if the two demands D_a and D_f are independent, then

$$x_0=\left[\kappa-F_a^{-1}\left(1-\frac{v}{p_a}\right)\right]^+.$$

Proof. Using $(D_f-x)^+=D_f-\min(x,D_f)$, the integrator's expected profit (10) becomes

$$\tau(x)=(p_f-v)E[D_f]+\phi(x) \tag{19}$$

where

$$\phi(x)=E[v\min(D_f,x)+p_a\min(D_a,\kappa-\min(D_f,x))]. \tag{20}$$

In (19), the first term is constant; thus, maximizing $\tau(x)$ is equivalent to maximizing $\phi(x)$. Note that $\phi(x)$ is identical to the expected profit in the well-known two-class capacity allocation in the airline passenger revenue management context; see e.g., Chapter 2 in (Talluri and van Ryzin, 2004) and (Amaruchkul et al., 2011). Recall in the two-class model, we need to determine the booking limit x for the discount (class-2) customers, which arrive first, and the remaining capacity is reserved for the full-fare (class-1) customers, which arrive last; the airline's expected profit is

$$E[p_2\min(x,D_2)+p_1\min(D_1,\kappa-\min(x,D_2))] \tag{21}$$

where for $i=1,2$, the parameter p_i and the random variable D_i are price and demand for the class-i customers, respectively. By comparing (20) to (21), we clearly see that the forwarder's demand D_f is equivalent to the discount-fare customer D_2, and the carrier's direct-ship demand D_a is equivalent to the full-fare customer D_1. If $v>p_a$, we would accept D_f as much as possible, so $x_0=\kappa$. On the other hand, if

$v < p_a$, we need to balance between accepting D_f now or waiting for D_a which arrives later. The optimality condition is derived in e.g., Chapter 2 in (Talluri and van Ryzin, 2004) and (Amaruchkul et al., 2011). □

Recall that in the integrator's problem (10), we accept all of the forwarder's demand. If the integrator's capacity is not enough, then the spot market is used to handle the excess demand. Thus, if the spot price is very expensive, then the integrator would allocate all capacity for the forwarder's demand in order to minimize the spot purchase, and no space is protected for the direct-ship customers. On the other hand, if the spot market is not that large, then the integrator would protect $\kappa - x_0$ for the direct-ship customer which arrives after the forwarder's demand.

It follows from the optimality equation in Theorem 4 that the integrator's optimal allotment increases when the spot price v increases, or the direct-ship per-unit price p_a decreases, or that the direct-ship demand becomes smaller in the usual stochastic ordering sense. These directional changes make economic sense.

Corollary 1. *Suppose that $h = 0$ and $w = v - \varepsilon$. At the equilibrium, the carrier imposes the required utilization $u_r = x_e(\varepsilon)$ where*

$$x_e(\varepsilon) = \left[\kappa - F_a^{-1}\left(1 - \frac{v-\varepsilon}{p_a}\right)\right]^+$$

is the equilibrium allotment. For $\varepsilon > 0$, $x_e(\varepsilon) < x_0$, and

$$\lim_{\varepsilon \to 0} x_e(\varepsilon) = x_0. \tag{22}$$

Proof. The expression for the equilibrium allotment follows immediately by observing the similarity between the carrier's problem (17) and the integrator's problem (20). In the integrator's problem (20), the unit contribution from the allotment usage is v, whereas in the game (17), the corresponding unit contribution is $v - \varepsilon$. Since $w = v - \varepsilon$ where ε is a small positive number, we have that $x_e(\varepsilon) < x_0$. □

It follows from (22) that a contract with no penalty cost can be made as close as possible to the coordinating contract. The air cargo service chain becomes efficient, in the limiting sense, when the carrier imposes no penalty cost.

Suppose that the carrier imposes a positive penalty cost $h > 0$. When the service chain is efficient, the optimality conditions in Theorem 1 and Theorem 4 coincide. If the integrator's optimal allotment x_0 solves the carrier's problem (13), or it is the root of (14), then there exists the equilibrium coordinating contract in the decentralized chain, and the wholesale price satisfies

$$w_0 = v - \gamma h \tag{23}$$

where $\gamma = F_f(x_0)/\bar{F}_f(x_0)$. If $\gamma h < v < p_a$, then the wholesale (23) is positive, and the allotment does not exceed the carrier's capacity κ. As in Theorem 2, in most cases, the equilibrium coordinating contract needs to be found numerically.

4 NUMERICAL EXAMPLES

We provide a numerical example to illustrate our approach of finding an equilibrium in the air-cargo contract game. The forwarder's demand is taken from one of the leading logistics providers in Thailand. During 2014, the forwarder did not pre-book any allotments on the route from BKK airport (Bangkok International Airport, Thailand) to DUB airport (Dublin Airport, Ireland). In the past, around 89% of the forwarder's customer demands were handled by one of the largest Persian Gulf carriers, and the rest by the other airlines in the spot market. Assume that the per-unit price the forwarder charges its customer is $p_f = 63$ THB/kg, the per-unit price the carrier charges its direct shippers $p_a = 60$ THB/kg, and the spot price $v = 58$ THB/kg. We need to estimate the demand distribution from the historical data. Some classical descriptive statistics are as follows: Minimum is 2.72 (kg); maximum 1142.32, median 294.88; mean 340.13; standard deviation 211.68; skewness 1.41; kurtosis 5.69. The coefficient of variation is 0.62; in words, the standard deviation is more than half of the mean. The mean is greater than the median, and skewness is positive. We fit the gamma distribution with the (shape, rate) parameter, $(\alpha_f, \beta_f) = (2.6031, 0.0077)$. The corresponding p-value is 0.3390. There is insufficient evidence at 0.01 level of significance to reject the null hypothesis that the forwarder's demand follows the gamma distribution. Assume that the direct-ship demand also follows the gamma distribution with the (shape, rate) parameter $(\alpha_a, \beta_a) = (5.76, 0.01)$, so the expected direct-ship demand is 576 kg, and the standard deviation is 240 kg. The carrier's cargo capacity is assumed to be $\kappa = 1000$.

In Examples 1–3, we assume that the required utilization $u_r = u(\kappa)$:

$$u_r = u(\kappa) = \frac{E[\min(D_f, \kappa)]}{\kappa} = 336.48/1000 = 33.65\%.$$

In Examples 1–2, we assume that the penalty cost is fixed at $h = 56$ THB/kg.

Example 1 (Forwarder's problem)**.** We study the effect of the wholesale price to the forwarder's allotment decision. We consider two cases: In the first case, the forwarder's expected profit is concave and unimodal and the forwarder's decision is given in Theorem 1, whereas in the second case, it is decreasing and the forwarder's decision is zero allotment.

Suppose that the wholesale price does not exceed the spot price: $w = 45 \leq 58 = v$. Recall the forwarder's expected profit (1)

$$E[p_f D_f - w\min(D_f,x) - v(D_f - x)^+ - h(x - D_f)^+].$$

For the first term in (1), the forwarder's mean demand $E[D_f]$ is the expected value of the gamma distribution:

$$E[D_f] = \frac{\beta_f^{\alpha_f}}{\Gamma(\alpha_f)} x^{\alpha_f - 1} e^{\beta_f x}$$

where $\Gamma(\alpha)$ is the gamma function, defined by

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx.$$

For the second term in (1), we evaluate the expected allotment usage $E[\min(D_f,x)]$ using the limited expected value (LEV) function. The LEV function for gamma random variable, Y, with the shape parameter α and the scale parameter θ is

$$E[\min(Y,x)] = \alpha\theta\Gamma(\alpha+1;x/\theta) + x[1 - \Gamma(\alpha;x/\theta]$$

where $\Gamma(\alpha;x)$ is the incomplete gamma function defined by

$$\Gamma(\alpha;x) = \frac{1}{\Gamma(\alpha)} \int_0^x t^{\alpha-1} e^{-t} dt.$$

(For the gamma distribution, the scale parameter is equal to the reciprocal of the rate parameter; for instance, the scale parameter of D_f is $1/\beta_f = 1/0.0077 = 129.8701$.) For the last two terms in (1), we use the following

$$E[(D_f - x)^+] = E[D_f] - E[\min(D_f,x)]$$
$$E[(x - D_f)^+] = x - E[\min(D_f,x)]$$

where the expected allotment usage is calculated previously using the LEV function. Figure 1 shows the forwarder's expected profit as a function of the allotment, given that the carrier offers the wholesale price of $w = 45$.

The case $w = 45 \leq 58 = v$ corresponds to the last case in Theorem 1: The forwarder's expected profit function is concave, unimodal and maximized at $x_f^*(45) = 156.39 \approx 156$. When the carrier offers the per-unit price of $w = 45$ for the actual allotment usage and charges the per-unit penalty of $h = 56$ for the unused allotment, the forwarder chooses the allotment of $x_f^*(45) = 156$, which maximizes its own expected profit.

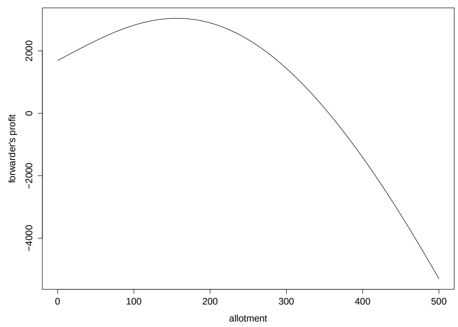

Figure 1: Forwarder's expected profit as a function of the allotment, given that the carrier offers the wholesale price of $w = 45 \leq 58 = v$.

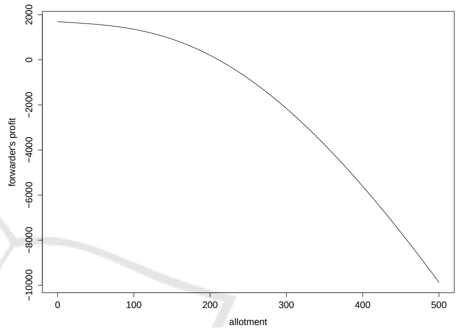

Figure 2: Forwarder's expected profit as a function of the allotment, given that the carrier offers the wholesale price of $w = 60 > 58 = v$.

On the other hand, suppose that the carrier offers the wholesale price greater than the spot price, $w = 60 > 58 = v$. Then, the forwarder's expected profit is decreasing in the allotment; see Figure 2.

When the spot price is smaller than the wholesale price, there is no need for the forwarder to pre-book any space; the forwarder can purchase capacity at the spot market for the consolidated cargo. This corresponds to the first case in Theorem 1. □

Example 2 (Carrier's problem)**.** In Example 1, we consider two cases, $(w,x_f^*) = 45,156)$ and $(w,x_f^*) = (60,0)$. In this example, the wholesale prices are varied from 0 to 70, and we plot the forwarder's best response, i.e., the allotment which maximizes the forwarder profit function for a given wholesale price.

Figure 3 shows the forwarder's best response as a function of the wholesale price. The wholesale price function $\omega(x)$ is the inverse of the forwarder's best response function.

After the wholesale price function $\omega(x)$ is readily found, we are now in the position to solve for the equi-

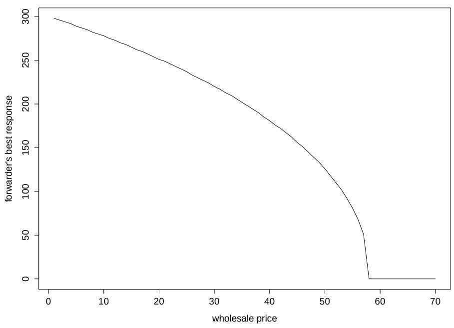

Figure 3: Forwarder's best response to the carrier's offered wholesale price.

librium of the Stackelberg game, x_a^* which maximizes the carrier's expected profit (8) given the forwarder's best response:

$$\max_{x\in(0,\kappa]} E[p_a \min(D_a, \kappa - \min(x, D_f)) + \omega(x)\min(x, D_f) + h(x - D_f)^+] \quad (24)$$

To calculate the first term in (24), we again make use of the LEV. For all $x \le \kappa$, we have that

$$\begin{aligned} &E[\min(D_a, \kappa - \min(D_f, x)] \\ &= \int_0^\infty E[\min(D_a, \kappa - \min(t, x)]\xi_f(t)dt \\ &= \int_0^x E[\min(D_a, \kappa - t)]\xi_f(t)dt \\ &\quad + \int_x^\infty E[\min(D_a, \kappa - x)]\xi_f(t)dt \\ &= \int_0^x E[\min(D_a, \kappa - t)]\xi_f(t)dt \\ &\quad + E[\min(D_a, \kappa - x)]\bar{F}_f(x). \end{aligned}$$

The carrier's profit is maximized at $x_a^* = 181$. From the wholesale price function $\omega(x_a^*) = \omega(181) = 40$. At the equilibrium, the carrier first offers the wholesale price of $w^* = 40$, and the forwarder responses with the allotment of $x_f^*(w^*) = 181$, which maximizes its expected profit. Recall that the penalty cost $h = 56$. At an equilibrium, the contract parameter can also be interpreted as the per-unit upfront payment $w^* = 40$ for the entire allotment, $x_f^*(w^*) = 181$, and a penalty rate of $h - w = 16$ for the unused portion of the allotment.

Recall that in 2014, the forwarder did not prebook any allotments on the BKK-DUB route. When it uses only the spot market, the forwarder's expected profit is $(p_f - v)E[D_f] = (63 - 58)(338.0649) = 1690$ THB per flight. With the allotment contract, the forwarder's profit is improved by 126.39%. Table 1 summarizes the expected profits of both parties and the total profit in the service chain.

Table 1: Benefit from allotment contract with $(w^*, x_f^*(w^*)) = (40, 181)$.

	Profit		
	Forwarder	Carrier	Total
W/O contract	1690	34025	35715
W/ contract	3826	40568	44393
% Δ	126%	19%	24%

In this numerical example, we see that the allotment contract makes both parties better off. Moreover, the carrier's cargo load factor with the contract is

$$\frac{1}{\kappa}\{E[\min(D_f, x_a^*)] + E[\min(D_a, \kappa - \min(D_f, x_a^*))]\} = 71.72\%$$

whereas that without the contract (the allotment $x = 0$) is

$$\frac{1}{\kappa}E[\min(D_a, \kappa)] = 60.88\%.$$

With the allotment contract, the carrier improves its cargo load factor by 17.8%. $\square$

Example 3 (Coordinating contract with positive penalty cost)**.** In the above example, the integrator's optimal allotment is

$$x_0 = F_a^{-1}(1 - v/p_a) = 778.79$$

and the optimal integrator's expected profit is $\tau(x_0) = 50395.14$. Note that $v - \gamma h = 58 - (24.2338)(56) = -1299.093 < 0$. The coordinating contract does not exists in this route. The chain efficiency is $44393/50395.1 = 88.09\%$.

Suppose that on another route, the forwarder's demand is gamma with the parameter $(\alpha_f, \beta_f) = (6.32, 0.0077)$; the forwarder's shape parameter increases, so the expected demand increases from 338 to 821 kg. This route is more popular, so we assume that prices are higher; specifically, $p_f = 190$ and $p_a = 180$. The integrator's optimal allotment is $x_0 = 343$, and the integrator's optimal profit is 220468.2.

Suppose the penalty cost remains $h = 56$. From (23), $w = v - \gamma h = 55.7968 \approx 56$ where $\gamma = 0.03934$. However, this coordinating contract $(w, x) = (56, 343)$ is not the equilibrium solution of the game, which is $(w, x) = (56, 335)$. At the equilibrium, the profits of the forwarder and carrier are 108891.6 and 111569.5, respectively, and the allotment is smaller than the optimal allotment in the integrated chain. Although the equilibrium contract does not coordinate the chain, but the service chain efficiency

$$\frac{\pi(335) + \psi(335)}{\tau(343)} = \frac{108891.6 + 111569.5}{220468.2} = 99.997\%$$

Table 2: Coordinating contracts with zero penalty cost.

Wholesale Price	Profit		
	Forwarder	Carrier	Total
30	10977	39418	50395
40	7660	42735	50395
56	2354	48042	50395

is nearly 100%. The total chain profit under the equilibrium contract is slightly smaller than the optimal integrated profit.

Suppose that the penalty cost is $h = 25.5$. From (23), $w = v - \gamma h = 56.99676 = 57$. This contract $(w, x) = (57, 343)$ is also the equilibrium of the game. Hence, the chain is coordinated, and the efficiency is exactly 100%. From practical viewpoint, the two contracts may be selected by the carrier, since their profits are nearly identical. In the first contract, the upfront payment of $(56)(335) = 18760$ is collected upfront, and there are no additional payments. In the second contract, the upfront payment of $(57)(343) = 19551$ is collected upfront, and the refund rate of $57 - 25.5 = 31.5$ for the unused portion of the allotment is paid from the carrier to the forwarder, after it releases the unwanted space. □

Example 4 (Coordinating contract with no penalty cost)**.** In the limiting sense, a set of coordinating contracts can be constructed with zero penalty cost and the minimum utilization requirement. From Examples 1 and 3, the integrator's optimal profit is maximized at $x_0 = 778.79$. The forwarder's expected utilization at x_0 is $E[\min(D_f, x_0)]/x_0 = 331.65/778.79 = 0.4258 = 42.58\%$. Suppose that the carrier requires that the forwarder to maintain the expected utilization of $u_r = 0.4258$. For any wholesale prices less than the spot price $v = 56$, the forwarder's best response is $\upsilon^{-1}(u_r) = x_0$; see Theorem 1. Table 2 shows the profits of both parties and the total profit of the chain.

All of these contracts (with no penalty cost $h = 0$ and wholesale prices smaller than the spot price, $w < v = 56$) coordinate the chain. This may shred some light on why contracts, commonly found in practice, sometimes impose no penalty for the unused portion of the allotment but do impose the allotment utilization. The profit division between the two parties depends on the wholesale price. If the carrier is very powerful, it can offer a wholesale price just below the spot price and takes almost all of the chain's profit. On the other hand, if the forwarder is more powerful, then the wholesale price becomes smaller, and the forwarder gains the larger portion of the the chain's profit.

□

5 CONCLUDING REMARK

We consider the carrier and the forwarder, which may enter into an allotment contract before the selling season starts. The forwarder pre-books an allotment at some reservation fees. After the forwarder's customer demand materializes, the forwarder returns the unwanted space, and some cancellation fees may be applied. We formulate the contract design problem as the Stackelberg game, in which the carrier is the leader and offers a set of contract parameters. The forwarder responses by choosing the best allotment, which maximizes its expected profit, given the contract parameter the carrier proposed first. We determine the closed-form solution for the forwarder's best response and give a sufficient condition for the game to possess an equilibrium. In particular, we show that the contract, commonly found in practice, with no penalty cost but with the utilization requirement is efficient (in the limiting sense). Our numerical example qualifies the benefit from the allotment contract.

A few extensions are as follows: Like many consumer goods, volume discounts are commonly found in the air cargo industry; consequently, the margin may depend on the actual usage, and the penalty cost may depend on the amount of the returns. The discount may be applied to all the units, actually used by the forwarder, or it may be applied only to the additional units beyond the breakpoint. The all-units case is more common in Thailand. The contract payment may consist of the minimum payment plus the variable payment depends on the volume; for instance, for the first 500 kilograms, the forwarder charges a fixed payment of 51000 THB, and the variable payment is 60 THB/kilogram for each additional kilogram beyond the breakpoint, 500. The variable payment is sometimes discounted. The penalty cost may depend on not only the amount of returns but also the timing that the unused allotment usage is released to the carrier. It would be of practical interest to find an optimal allotment, when dynamic pricing is in placed. Another extension is to consider multiple freight forwarders. Our approach could be used to construct a heuristic solution to the case of multiple forwarders; D_f would represent the sum of all forwarders' demand, and all other cost parameters would be the averages of all forwarders (e.g., p_f would be the average of all forwarders' price to their customers). Moreover, our approach could also be applied for the passenger airline allotment problem. In the passenger airline industry, the airline usually blocks a pre-specified number of seats to a wholesaler/agent or other airline, which has established contracts such as interline or codeshare agreement prior to the start of the selling season. We hope to pursue these or related problems.

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