

Bifocal Parallel Coordinates Plot for Multivariate Data Visualization

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Abstract: Visualization of multivariate data using parallel coordinates plot (PCP) becomes overwhelming as the number of dimensions/variables increases beyond one dozen or so. Here we propose bifocal parallel coordinates plot (BPCP) based on the focus + context approach. BPCP splits vertically the overall rendering into the focus and context regions whose sizes can be adjusted to optimize the use of the available space. The focus area maps a few selected dimensions of interest, referred to as priority axes, at sufficiently wide spacing. The remaining dimensions are represented in the context area in a compact way so as to retain useful information and provide the data continuity. The focus display can be further enhanced with various options, such as axes overlays, scatterplot, and nested juxtaposed PCPs. In order to accommodate an arbitrarily large number of dimensions, the context display supports multi-level stacked view, each PCP level mapping a subset of the context axes. With flexible interactivity, BPCP can manage the priority axes and data rendering with respect to the corresponding dimensions to support exploratory visualization while providing useful context on the same visualization display.

1 INTRODUCTION

Parallel coordinates plot (PCP) is a popular technique for multivariate data visualization (Avidan and Avidan 1999; Few 2006; Inselberg, 2009; Heinrich and Weiskof, 2013). It maps data points in a multidimensional space to a 2D display surface by laying out all dimensions/variables/attributes as parallel vertical axes at uniform spacing. Each data item is rendered as a polygonal line with its vertices on these axes. PCP visualization helps us quickly reveal patterns, trends, relationships, anomalies in the multivariate data. Generally, static visualization is not much of practical use because the data lines quickly fill up the display space often resulting in visual clutter. To generate insights into the multivariate information requires appropriate ways of interacting with the data samples and dimensions (Siirtola and Raiha, 2006; Inselberg, 2009). Once regions of interest are identified, the interactivity helps perform more focused analysis.

PCP is expected to accommodate arbitrarily large numbers of dimensions and data lines in a finite display area. More dimensions require adding more axes in a linear order. Such tightly packed axes degrade visual resolution and make navigating the data space difficult. Each data line simply consists of

many segments which are short so it is difficult for the user to read the data lines. From a practical viewpoint the user is not able to visually comprehend all dimensions at a time or the user may not be even interested to analyse all dimensions on equal footing. It thus makes sense that only a subset of the dimensions be better examined at a time. For example, only five variables for 25-dimensional automobile dataset (Figure 1) might be of the user's current interest. If we render the data with respect to the five selected dimensions only, we will have widely placed parallel axes. Visual clarity improves considerably and the corresponding data segments become fairly long and are easy to read.

The problem is that the data information with respect to all other dimensions that are not mapped into the current PCP is completely lost. One may toggle between all-axes plot (an overview) and the selected-axes plot (a detailed view) or use a miniature PCP as overview and a regular plot as detail together (Gruendl et al., 2016). Another option is to support a regular plot (main view) and maintain an axis repository to hold axes currently of less importance (Riehmman et al., 2012). Such overview-detail or detail-on-demand approaches suffer from a (time) disconnect issue between the two views tending to divide the user's attention.

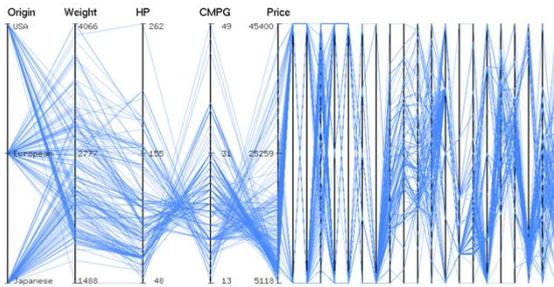


Figure 1: Parallel coordinates plot of 25-dimensional automobile data. Five axes representing *origin*, *weight*, *horsepower*, *city mpg* and *price* are widely spaced in the left half space (focus display) and the remaining axes are squeezed in the right half space (context display).

In this paper, we propose bifocal parallel coordinates plot (BPCP) based on the focus + context approach (Spence and Apperley, 2013). BPCP represents the parallel axes corresponding to a few priority dimensions at sufficiently wide spacing and maps the remaining dimensions in a compact way (Figure 1). It renders data line segments with respect to the selected dimensions at high visual resolution (providing a focus view which can be further enhanced using available extra space) while retaining at the same time the information with respect to all other dimensions for context. The aim is to produce a good visualization by effectively using a finite display without much losing valuable information. The focus + context approach was previously applied in parallel coordinates for highlighting certain axes groups (Brodbeck and Girardin, 2003) and data clusters (Novotny and Hauser, 2006). Our work is expected to serve as a systematic bifocal presentation of parallel coordinates. We present a novel design of the proposed BPCP method to effectively split the total parallel coordinates plot area into two parts. We then explore various ways of enriching data visualization in the “focus” part and also the ways of simplifying rendering in the “context” part.

2 RELATED WORK

Parallel coordinates plot has been a subject of extensive investigation due to its applications in visualization of multivariate data and high-dimensional geometry. PCP helps discover the multivariate relations by transforming the problem into 2D pattern recognition problem involving 2^n subsets, and also has relative merits for tasks like clustering and outlier detection (Inselberg, 1997; Zhou et al., 2008). Due to the visual clutter caused

by over-plotting of data polylines and closely spaced coordinate axes, PCP visualization may be confusing and even intimidating at first, but with interactivity it can actually be very approachable.

When the number of dimensions (k) increases, the axes arrangement is crucial for finding and understanding complex multivariate relations. To compare different variables side-by-side requires the reorder of axes and the trial of multiple arrangements. For instance, the parallel coordinates matrix plot (Heinrich et al., 2012) shows all layouts simultaneously. A good axes order can be found using the contribution- and similarity-based reordering methods (Lu et al., 2016; Peltonen and Lin, 2017). If k is large, only a subset of important axes can be included in the main PCP view (Riehmman et al., 2012; Gruendl et al., 2016).

Dimension spacing which is the gap between adjacent axes is also important. The default spacing is chosen to be uniform. The dimensions are not equivalent to each other, and one way of conveying this information is to vary dimension spacing (Yang et al., 2003). For instance, similar dimensions are mapped closer than the unrelated dimensions. Extra space between the adjacent axes allows the user to explore the pattern in detail. Horizontal zooming in/out and panning or distortion can be used to adjust the spacing of the concerned axes or to even collapse a group of axes (Brodbeck and Girardin, 2003). Specifying such local changes is difficult because of narrow axial spacing when k becomes large.

One or more polylines to emphasize the selected data samples can be highlighted with the rest still in the background. Data lines can be pinched from the above and below so that they are picked up (Inselberg, 2009). Alternatively, a subset of data items is selected by means of brush. Axis-aligned brush picks a range on an axis corresponding to an interval on the respective dimension in the data domain (Turkey et al., 2011). Visual clutter can be reduced by data filtering or clustering so as to reveal patterns and anomalies (Fua et al., 1999; Peng et al., 2004; Zhou et al., 2008).

With increasing k , various tasks related to dimension management and interacting with data samples become impractical at some point both effectiveness- and performance-wise. Inselberg (2009) has questioned the number of dimensions PCP can handle. The answer is not “many” on a single display. It is not possible to map a large number of dimensions at the same time without cluttering the display. Reduction techniques like principal component analysis and multidimensional scaling condense many dimensions to a few

dimensions (Jolliffe, 1986; Mead, 1992). Similar dimensions may be grouped together and mapped as closely-spaced axes or even as a single representative axis. A more direct solution is dimension filtering, which is to eliminate the repetitive variables or remove unimportant axes (Yang et al., 2003).

Our proposed bifocal PCP technique exploits several of the above-mentioned ideas for its design and effectiveness for multivariate data visualization. Both “focus” and “context” display areas can support various ways of interacting with the axes and data samples. Our tests used the automobile dataset (25 variables and 200 observations) and the cardiac arrhythmia medical dataset (280 variables of which are 130 used here, and 452 records) available from the UCI machine learning repository (<http://archive.ics.uci.edu/ml>).

3 DESIGN OF BIFOCAL PCP

In parallel coordinates plot, all dimensions (variables) are laid out as vertical axes at uniform spacing. If k dimensions are mapped on the display surface of width X and height Y , the axial spacing is given by $\Delta X = X/(k-1)$. If k increases, the axes are packed more compactly so as to fit all of them within a given finite area. In the proposed bifocal parallel coordinates plot (BPCP), the overall display is vertically split into two regions corresponding to “focus” and “context”, which use different axial spacing (Figure 1). It thus applies a one-dimensional focus + context across the axis dimension (i.e., in the horizontal direction). The focus region maps a few axes of interest at wider interval than the average ΔX enabling a detailed view of the data with respect to the corresponding dimensions. On the other hand, the context region accommodates all remaining axes by packing them tightly to retain full information about the data as much as possible. To explain the design of the proposed BPCP, we consider three parameters as follows:

3.1 Focus Area

The total display area spanned by parallel coordinates and data lines is usually extended horizontally more than vertically, i.e., $X > Y$. For a large number of dimensions, it makes sense to consider the total parallel coordinates plot width of $X = 2Y$ (Figure 2). The 2:1 display area can use the horizontal spread of the computer screen fully for

PCP while leaving extra space in the vertical direction for displaying axes labels, user controls, and other features. When the display region is divided into the focus and context parts, the questions arise about their sizes and locations. The default option is to have each part as a square (Figure 2, middle) such that $X_F = Y$ (focus width) and $X_C = Y$ (context width). To accommodate more dimensions in the focus area, we need to increase its width. There is a limitation because the remaining axes must be mapped as well. We limit the horizontal spread of the focus area to the three-fourth of total display width ($X_F = 1.5Y$) so the minimum context area width is $0.5Y$.

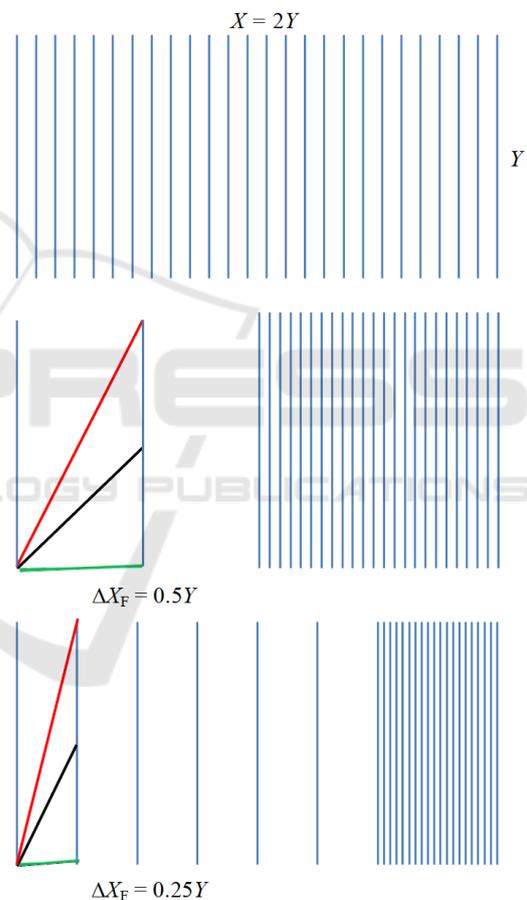


Figure 2: Axes layout of BPCP in a display area of width X and height Y . *Top*: All k axes are placed at equal spacing. *Middle*: The plot consists of two equal parts: the focus part showing three axes and the context part showing remaining $k-3$ axes. *Bottom*: The focus area maps seven axes. Three orientation cases (low, mid and high tilt angles with the horizontal direction) of data line segment between the adjacent axes are shown.

The left position of the focus area with the context area on the right as shown in Figures 1 and 2

perhaps works well. Another option is to focus symmetrically about the centre so that its left and right sides together provide the context display. Moreover, the focus region can be allowed to glide along the horizontal direction to any position in an interactive manner, and the affected axes and data lines need to be redrawn accordingly (Brodbeck and Girardin, 2003).

3.2 Priority Axes

The dimensions which are represented in the focus display with wider spacing than the average spacing are referred to as priority (focus) axes. We need, at least, two priority axes to focus on so that the lines joining the data values on the corresponding variables can be drawn with better visual resolution and the relationships can be explored in detail. Practically it makes more sense to have three priority axes (Figure 2, middle). We can explore the relationships of a central axis with two other axes, one on the left and one on the right and then provide a flexible option of making one of three axes the central axis. The maximum number of the priority axes, however, can vary depending on the focus display and the total number of dimensions, but it must be kept small. The priority axes can be selected manually by the user as important dimensions/variables/attributes. Or, they can be automatically found based on some measures, for instance, as highly correlated dimensions using Pearson correlation. The priority axes can be manipulated interactively by adding to and deleting the axis from the focus display.

3.3 Axes Spacing

It is important that the parallel axes be laid out in the focus region sufficiently wide from each other. The data polygonal lines consist of segments between successive adjacent axes. The visual impression of these data lines depends on various angles the component segments make while moving from left to right. Generally, a unity slope (that is, angle 45° with the horizontal direction) of a line is considered to give the best visual representation (von Huhn, 1931). Obviously, the unity slope is unachievable for all data line segments. The lines connecting the opposite ends of the adjacent axes make tilt angles of $\pm 45^\circ$ and cross each other at 90° if the horizontal gap between the axes is equal to the axis length (i.e., the vertical plot extent). We have $\Delta X_F = Y$, which should be taken as the widest gap as it is the case with focus width of Y for two priority axes.

However, it makes more sense to consider 45° tilt angle for average situations where the difference between the data marks on two adjacent axes is equal to the half of the axis length (Figure 2, middle). We have $\Delta X_F = 0.5Y$, which is the case with focus width of Y for three priority axes. The data lines between two successive axes make tilt angles in the range 0° (when the lines connect the data marks at the same height) to $\pm 63.4^\circ$ (when the lines connect the opposite ends of the axes).

One issue still is that the line segments for small differences in the data marks between two adjacent axes appear almost horizontal. Further decreasing the dimension spacing can increase such low-angle tilts. For $\Delta X_F = 0.25Y$, the lines connecting the opposite ends of the axes make $\pm 76^\circ$ so the angle of extreme line crossing becomes 28° , which is visually discernable. Assuming $0.25Y$ as the minimum axial spacing for the largest focus display width of $X_F = 1.5Y$ (Figure 2, bottom), the maximum number of the priority axes we should allow is given by $k_F = 1 + X_F/\Delta X_F = 7$. The maximum seven dimensions to focus on make sense with the general notion that parallel coordinates plot is the most effective for the datasets with fewer than one dozen dimensions (Inselberg, 1997; Few 2006).

The focus axial spacing (ΔX_F) is the main parameter controlling the design of the proposed BPCP. As discussed above, we recommend that the gap between adjacent focus axes be between $0.25Y$ and Y , where Y is the vertical display extent (height) set for the overall plot. A spacing value outside this range either results in very closely packed axes or very wide focus coverage. Our design strictly imposes the focus axial spacing range. It then constrains the number of the priority dimensions (k_F) between 2 and 7. Finally, it adjusts the focus display width (X_F) between $0.25Y$ (when $\Delta X_F = Y$ for two priority axes) to $1.5Y$ (when $\Delta X_F = 0.25Y$ for seven priority axes) for the 2:1 display. If the number of dimensions is very large, we need a wider plot. We set $X = 3Y$ when k is greater than 31, but the average spacing is still less than one tenth of the vertical extent. For the minimum focus axes spacing ($0.25Y$), we can now map up to nine priority axes supporting the widest focus display of $2Y$.

The above design is such that bifocal parallel coordinates plot is ineffective when k is less than four for the 2:1 display. For k between four and nine, the focus area with only axial spacing greater than the average ΔX can be supported. For $k > 9$, the average axial spacing is smaller than $0.25Y$ and the full range of ΔX_F values can be exploited. This is the situation with the number of dimensions greater than 13 for the 3:1 display.

4 ENHANCING FOCUS DISPLAY

The focus display can be further enriched with additional rendering and analysis options to enable an in-depth, interactive visualization of multivariate data with respect to the priority dimensions.

4.1 Axes Management

Understanding data dimensions in PCP involves the manipulation of the corresponding parallel axes, which enable us to read off the values and ranges of data samples. To facilitate the visual perception of data distributions on the respective dimensions, the axes can encode additional information using overlays like circle and box plots (Figure 3, top). This can be helpful in deciding the priority axes.

Axes reordering option enables the user to detect patterns within the data and pay more attention to important dimensions (Johansson et al., 2008; Lu et al., 2016; Peltonen and Lin, 2017). The pair-wise relationships among the dimensions are easier to interpret when the corresponding axes are adjacent to each other. To explore the relationships of a priority axis with all other priority axes, the concerned axis is first brought to the central part with the data lines redrawn. The central axis represents *CMPG* with its adjacent axes representing *weight* and *HMPG* in Figure 3. City mpg is negatively correlated with weight and positively correlated with highway mpg. To relate the central axis to non-adjacent axes (on its left and right), the data lines are drawn directly connecting to the respective coordinates while suppressing or even removing the intermediate axes and line segments (Figure 3). This helps examine the relationships of city mpg with four more variables including origin, horse power, price, and the number of cylinders. Only the focus axes and the corresponding data segments are affected so these operations are fast.

4.2 Data Presentation

A large dataset can be better understood by breaking it into subsets/groups and then performing inter- and intra-group analyses. The user can choose a priority axis (referred to as active axis drawn first) to map the dataset into multiple subsets. If the active axis represents categorical variable, the data samples corresponding to each coordinate value form a subset. For instance, the automobile dataset has three values for the *origin* variable which result in three subsets: American, European, and Japanese cars (shown by red, green and blue in Figure 4). For

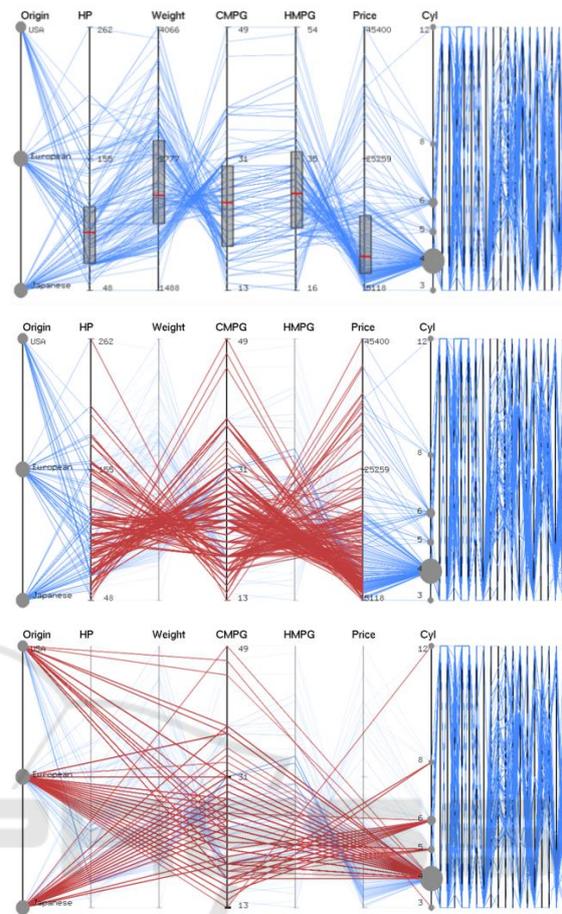


Figure 3: Focus PCP ($2Y/3$ wide) of 7 priority axes on left and context PCP ($Y/3$ wide) on right. Circle plot for categorical variable and boxplots for continuous variables are shown. The data lines are directly drawn (red) from the central axis to next-nearest neighbours (middle) and next-nearest neighbours (bottom).

a continuous variable, data groups correspond to different, non-overlapping ranges of data values. For instance, three groups can represent low $1/4$, mid $1/2$ and high $1/4$ intervals on the axis representing, say *price* variable. Different clustering techniques (Fua et al., 1999; Zhou et al., 2008) can be used to identify groups for a multiple-set mapping of a given dataset.

Scatterplot is effective in correlation perception and similarity detection (Huamin et al., 2012; Kanjanabose et al., 2015). We add scatterplot for all priority axes pair directly below their respective PCP (Figure 4). The width and height of the scattered plot is adjusted based on the focus axial gap ΔX_F . In each scatter plot, the vertical axis is the same as the priority axis just above it and the horizontal axis is the right adjacent priority axis. The data lines in the

PCP and data points in the scatterplot can be brushed in a consistent way so that two plots can complement each other. The focus display can be also appended with a data table instead to show the actual data values on the priority axes.

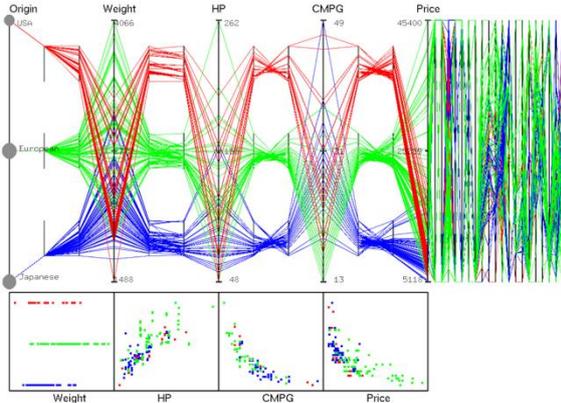


Figure 4: Focus region showing three nested PCPs, one for each subset of the automobile data: American (red), European (green), and Japanese (blue). The nested plots are symmetrically placed in between adjacent priority axes. The context region shows a normal compact PCP. Scatter plots are for the adjacent priority axes pairs.

4.3 Nested PCP

The nested parallel coordinates plot method has recently been proposed to perform comparative visualization of two or more datasets (Wang, 2016). It helps in exploring both intra-set and inter-set correlations among different variables/parameters from a single visualization when multiple datasets need to be analyzed together. We adopt this method to visualize multiple subsets (groups/clusters) of the data with respect to the priority axes (Figure 4). We treat these subsets as if they were different datasets and visualize them as different PCPs by embedding juxtaposed plots within in the normal plot. The original priority axes are globally scaled and the data lines for different groups are superimposed (overlapped) in the regions near the axes thereby enabling direct intergroup comparison. The nested axes are locally scaled and laid out in the central region between the adjacent original axes pairs thereby enabling visualization of different data subsets in distinct PCPs. The left and right axes in each nested plot are the same as the original priority axes on its left and right.

The default width of a nested PCP is set at one-third of the focus axial spacing (ΔX_F) symmetrically about the middle line between the original adjacent axes pair. The horizontal spread can be adjusted by

calculating the positions of two axes in the i^{th} nested pair as $(i - 0.5)\Delta X_F \pm \Delta x$, where $i = 1, 2, \dots, k_F - 1$ (from the left to the right), and Δx can vary between $0.1\Delta X_F$ and $0.4\Delta X_F$. The vertical extents and positions of the embedded plots are determined by uniformly dividing the original display height Y . If n_s is the number of data subsets/groups ($n_s = 3$ in Figure 4), the end positions of the j^{th} nested axes ($j = 1, 2, \dots, n_s$ counting from the bottom) are given by $(j - 0.5)Y/n_s \pm \Delta y$, where Δy can vary between $0.2Y/n_s$ and $0.5Y/n_s$. Our design assures that the nested axes never overlap with each other horizontally or vertically. The nested plot count should be kept small, not more than five. Explicit encodings, such as bundling and distorting can further aid the visual perception of the data lines (Wang et al., 2016).

5 SIMPLIFYING CONTEXT DISPLAY

The proposed bifocal PCP packs all non-priority axes (that is, context axes) much more closely than in the normal plot. As long as they do not overlap with each other, individual dimensions should be readable. However, the data lines depending on their count can clutter the display to varying degree. The goal is to retain the relevant information in the context display and maintain the data continuity. For instance, when brushing is applied, the user should see the effects on the selected data samples not only in *focus* but also in *context* (Figure 5). We can add appropriate axes overlays for showing aggregates or distributions of the data samples along the respective dimensions while removing the data lines (if needed) to minimize visual clutter. It supports interactive ways of translating and reversing the axes.

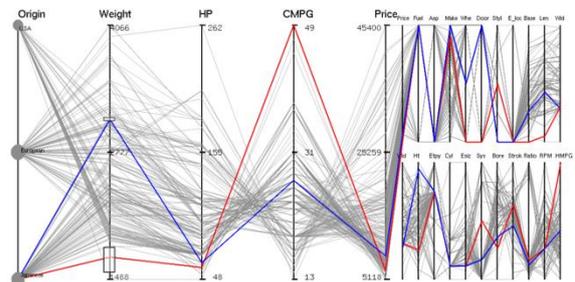


Figure 5: Focus PCP ($2Y/3$ wide) of 5 priority axes and context PCP ($Y/3$ wide) with a two-level stacking of 20 context axes for the automobile data. The last priority axis (*price*) is repeated in the first level. The last context axis (*width*) in the first level is repeated at the beginning of the second level. Two data items are highlighted in red (light car) and blue (heavy car).

To visually discern the dimensions requires that a minimum gap be maintained between the adjacent axes in the context display. This gap depends on the screen resolution and zooming level. The user can set a minimum gap in the number of the pixels such that it can be, say, three times wider than the pixel width of the axes. For total PCP display of the aspect ratio 2:1, such minimum axial spacing is achievable even for the worst situation where 28 axes (out of total maximum 31 axes allowed) are packed in the context display of width $0.5Y$ (with the widest focus area containing only three priority axes). The 3:1 display does not impose an upper bound on the total number of dimensions so the axial spacing in the context area can be arbitrarily small.

To avoid the spacing problem with an arbitrarily large number of dimensions, we propose a multi-level parallel coordinates plot. A similar approach has been previously proposed in the case of the star plot technique (Sangli et al., 2016). The context axes are divided into multiple groups and the context plot area is horizontally partitioned into the equal number of parts or levels. Each axes group is then mapped to a different level subarea which is vertically compressed. It thus represents a vertical stacking of context axes. The width of each subarea is the same as before the split so the axial spacing increases but the axes get shorter. The number of levels (m) in the stacked view can be adjusted interactively but should be kept small. The multi-level axial spacing

(when $m > 1$) is constrained to be smaller than $0.25Y/m^2$ (that is, the minimum spacing allowed in the focus display divided by m^2), where Y is the focus display height. Only two-level plot is allowed for the 2:1 display (Figure 5) but more levels are allowed for the 3:1 display. For example, if the context width $X_C = Y$ for the data containing total 130 dimensions (Figure 6), $m = 3$ is allowed. Nine are the priority axes and the remaining 121 axes are distributed among three PCP levels. The context axial spacing $\Delta X_C = X_C/41 = 0.025Y$, being smaller than $0.25Y/9$. Assuming that the context display width is 500 pixels, we have $\Delta X_C = 12$ pixels (that also includes axial width) for a three-level plot. For 400 axes, $m = 4$ is allowed. For this four-level stacked plot, the axial spacing is 5 pixels wide if $X_C = 500$ pixels. In Figure 6, the data lines taking low *weight* values tell that the heart rate varies considerably among small children (7 to 11 years old). In the context area, these lines mostly remain close, but they are quite scattered on some axes.

6 CONCLUSIONS

The parallel coordinates plot becomes less effective when the dimensionality of the data becomes too high. To overcome this problem, we have proposed bifocal parallel coordinates plot (BPCP) to provide

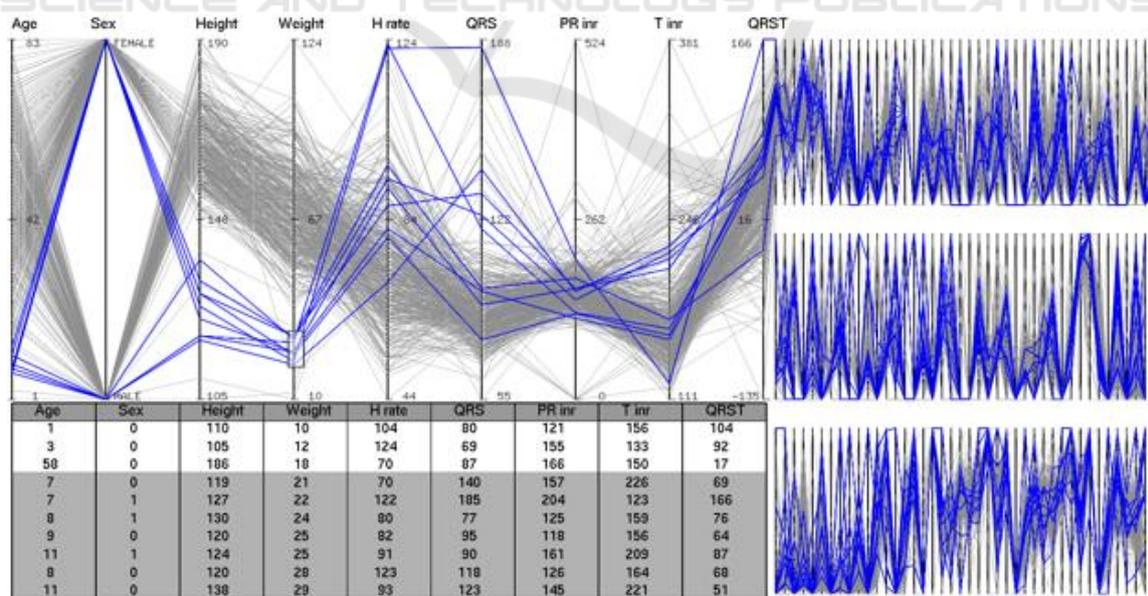


Figure 6: The focus region ($2Y$ wide) showing 9 priority axes and the context region (Y wide) showing three-level PCPs for the medical dataset consisting of 130 variables. The overall PCP display aspect ratio is 3:1. Each level accommodates 41 context axes. The last priority axis also appears as the first context axis. The first axis in each level is the last axis of the level above it. The data items using a low-weight brush are highlighted in the PCPs and data table.

“focus” and “context” views on a single visualization display by partitioning the overall rendering into two regions with flexible widths. The focus PCP renders the data with respect to priority dimensions (whose number is kept small, below 10) so that the corresponding axes are widely spaced. The display can be enriched by adding ancillary visualizations including axes overlays, embedded parallel coordinates, and scatter plots. The context PCP renders the same data with respect to all remaining axes, which are tightly packed in a single plot or a multi-level stacked layout. By experimenting on two datasets consisting of 25 and 130 dimensions, we have demonstrated the potential effectiveness of BPCP in visually exploring high/ultra-high dimensional multivariate data, which are on a rise in today’s big data world.

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