Assignment-based MIP Modeling for Solving a Selling Firm *m*TSP with Time Limit Constraints

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Abstract: This paper presents a version of the multiple traveling salesmen problem with service time limit constraints and travel times. The time required to provide merchandising service at any outlet location is predetermined based on the customer type. The objective is to minimize the number of salesmen hired by a selling firm while visiting and providing services to all customers without exceeding salesmen's allowed working times. The paper proposes an assignment-based mixed integer programming model for solving the salesmen problem of a selling firm that applies a sub-tour elimination restriction. A case example is presented for illustrating the applicability and suitability of the proposed approach for solving the problem tackled in this work.

1 INTRODUCTION

Salesmen play an important role in the distribution channel of goods to have an access to the customer and market as primary means of selling and distributing products and achieving selling firm's strategic objectives. Salesman allocation and routing are critical tasks in sales and the distribution management. Selling firms that distribute goods to large number of customers must perform detailed analysis of distribution routes and salesmen allocation to maintain successful salesforce and reduce the salesmen cost plus the costs for the tours.

In transportation and distribution management, traveling salesman is a key routing function. This function involves determining the geographic tour a salesman will travel to visit a set of locations and serve a number of customers. Distribution and sales planners often face the problem of determining the efficient way of touring individual salesmen in order to minimize the number of salesmen who can travel to reach all sales locations in which a set of customers is to be accessed and served. In reality, sales firms must determine and hire multiple salesmen to serve their customers, as well as determining the sequence in which customers are to be visited in each tour to minimize the total distance traveled.

The multiple traveling salesman problem (*m*TSP) is a generalization of the traveling salesman problem (TSP) in which more than one salesman is used (Bektas, 2006). Given a set of customer locations, one main office location (where *m* salesmen are located), traveling times between different locations, and time limit constraints during which the customer location must be visited and served. The objective of the *m*TSP is to determine a set of tours or routes for visiting and serving all customers so as to minimize the number of salesmen. The requirements on the set of tours are: all of the tours must start and end at the same main office location, and each customer location must be visited exactly once by a single salesman.

The importance of multiple traveling salesmen problem is shown in its huge savings when the marketing and distribution costs are reduced by minimizing the number of salesmen used.

Therefore, in the context of sales and distribution in supply chains, traveling salesmen is one of the important problems in distribution and supply center management. The multiple salesmen problem is extended in various ways; the purpose of this paper is to propose and apply an approach for highly relevant extensions of the classical traveling salesmen problem in the context of sales and distribution management. This research is concerned with salesmen problem of a selling firm where its

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customers are located in cities within a certain region. The aim is to solve this problem such that the number of salesmen is minimized while visiting and serving all customers and satisfying salesman touring and working time constraints.

We primarily focus on the development of general model to determine a touring network a salesman would travel through multiple locations to reach and serve set of customers in an attempt to achieve the goal of minimizing total salesmen cost plus the costs for the tours through minimizing the number of salesmen used. Nevertheless, as far as the author is aware, no published research has addressed this problem, or proposed an approach that optimizes the number of salesmen with respect to their work time availability and service time limit constraints, and incorporates a net flow formula for solving such problem. The number of salesmen is to be determined by the optimal solution but bounded by a given upper bound of the number salesmen.

2 LITERATURE SURVEY

Extended literature is available about various approaches on traveling salesman and multiple traveling salesmen problems. These approaches usually depend on the areas of applications.

The TSP arises in main real-world applications including the drilling problem of printed circuit boards (PCBs) in actual production environment introduced by Grötschel et al. (1991), overhauling gas turbine engines of aircrafts reported by Plante et al. (1987), analysis of the structure of crystals presented by Bland and Shallcross (1989), connection of components on a computer board and vehicle routing reported by Lenstra and Kan (1974), and order-picking and material handling in warehouses proposed by Ratliff and Rosenthal (1983).

mTSP has numerous real-life applications. Macharis and Bontekoning (2004), Wang and Regan, (2002), and Basu et al. (2000) reported comprehensive review on various application of mTSP. The main applications of mTSP include production scheduling presented by Gorenstein (1970), Carter and Ragsdale (2002) and Tang et al. (2000), school bus routing reported by Angel et al. (1972), crew scheduling described by Svestka and Huckfeldt (1973), and Lenstra and Kan (1975), mission planning presented by Brummit and Stentz (1998), designing system networks suggested by Saleh and Chelouah (2004), security service investigated by Calvo and Cordone (2003) and Kim and Park (2004), vehicle routing (VRP) discussed by Mole et al. (1983); Laptore et al. (1985), Ralphs, 2003), and Mitrović-Minić et al. (2004).

In the context of mathematical formulation of TSP and mTSP, many formulations are available in literature. Orman and Williams (2006) and O¨ncan et al. (2009) have provided surveys on several formulation of the problem. Among these, the formulations proposed by Dantzig et al. (1954), Applegate et al. (2003), Christofides et al. (1981), Svestka and Huckfeldt (1973), Kulkarni and Bhave (1985), and Laporte and Nobert (1980).

Beside the mathematical formulation approaches, some authors have introduced heuristic techniques to solve the TSP and mTSP problem. Balas and Toth (1985), Laporte (1992), and Fischetti et al. (2002) have presented surveys of algorithms for the problem. A number of well-known heuristic approaches have been developed to solve this problem, which include the algorithms presented by Dell'Amico and Toth (2000), Carpaneto et al. (1995), and Fischetti and Toth (1992). However, the best available algorithm for the symmetric TSP was developed by Applegate et al. (2006), which is the culmination of a line of research including Padberg and Hong (1980), Padberg and Grötschel (1985), Padberg and Rinaldi (1991), and Grötschel and Holland (1991).

Furthermore, Bektas (2006) listed a number of variations on the *m*TSP, instead of one depot, the multi-depot *m*TSP has a set of depots, with *a set of* salesmen at each depot . In one version, a salesman returns to the same depot from which he started. In another version, a salesman does not need to return to the same depot from which he started but the same number of salesmen must return as started from a particular depot. Bektas (2006) listed another variation that gives specifications on the number of salesmen such as having the number of salesmen to be a fixed number or it may be determined by the solution but bounded by an upper bound. Bektas also listed one more variation when the number of salesmen is not fixed; he assumes that there may be a fixed cost associated with activating a salesman. In the *fixed charge* version of the *m*TSP, the overall cost to minimize includes the fixed charges for the salesmen plus the costs for the tours. There is a variation of the *m*TSP with time limit constraints associated with each node during which the node must be visited by a tour.

3 PROPOSED APPROACH – ASSIGNMENT-BASED MIP MODEL FOR *m*TSP

We have developed an assignment-based mixed integer programming formulation for the mTSP (AMIP-mTSP).

The proposed AMIP-mTSP formulation is based on a graph $G=(I,A)$, where I is the set of $|I|$ nodes, and A is the set of $½*|I|*|I-1|$ bidirectional arcs. There is traveling time T_{ij} associated with each arc i-j and visiting node i requires service time S_j where both i and $j \in I$. We assume that the main office location is node 1 and there are m salesmen at the main office. We define a binary variable x_{kij} that takes the value 1 if salesman k is traversing arc i-j and x_{kij} takes the value 0 otherwise.

Prior to presenting the AMIP-mTSP formulation in detail, we introduce the notation given in Table 1.

Table 1: MIP-mTSP Model Notations.

	Description
I	is the set of all locations/points visited by salesmen
I'	is a subset of locations/points visited by salesmen excluding their main office (location point 1)
K	is the set of all salesmen
x_{kij}	is equal to 1 if salesman k travels form location i to j, and 0 otherwise$^{(DV)*}$
y_k	is equal to 1 if salesman k is assigned to visit any customer location, and 0 otherwise$^{(DV)*}$
M	is arbitrarily big number
T_{ij}	is the traveling time between location i and $j^{(IP)*}$
S_j	is the average service time at location $j^{(IP)*}$
H	is the maximum available working time for each salesman$^{(IP)*}$
O_i	is the relative position of location i in set I *(ex. set [a,b,c,],* $O_a=1, O_b=2$, *and* $O_c=3)^{(IP)*}$
N	is the maximum number of locations that can be visited by any salesman as number of elements/locations in a set I.

IP = Input Parameter* * DV = Decision Variable*

The objective function and functional constraints of the proposed AMIP-mTSP model are detailed as follows.

Objective Function:

$$Min \sum_{k=1}^{K} y_k \qquad (1)$$

Subject to:

$$\sum_{i=1}^{I}\sum_{j=1}^{I} x_{kij} \le My_k \;\; \forall \; k \in K \qquad (2)$$

$$\sum_{i=1}^{I}\sum_{k=1}^{K} x_{kij} = 1 \;\; \forall \; j \in I' \qquad (3)$$

$$\sum_{j=1}^{I}\sum_{k=1}^{K} x_{kij} = 1 \;\; \forall \; i \in I' \qquad (4)$$

$$\sum_{j=2}^{I} x_{k1j} = y_k \;\; \forall \; k \in K \qquad (5)$$

$$\sum_{i=2}^{I} x_{ki1} = y_k \;\; \forall \; k \in K \qquad (6)$$

$$\sum_{j=1}^{I}\sum_{k=1}^{K} x_{kji} = \sum_{j=1}^{I}\sum_{k=1}^{K} x_{kij} \;\; \forall \; i \in I \qquad (7)$$

$$\sum_{i=1}^{I}\sum_{j=1}^{I} (T_{ij} + S_j) x_{kij} \le H_k \;\; \forall \; k \in K \qquad (8)$$

$$O_i - O_j + N \sum_{k=1}^{K} x_{kij} \le N - 1 \;\; \forall \; i \ne j \in I' \qquad (9)$$

$$x_{kij} = 0 \text{ or } 1 \;\; \forall \; i \& j \in I, \text{ and } k \in K \qquad (10)$$

$$y_k = 0 \text{ or } 1 \;\; \forall \; k \in K \qquad (11)$$

The objective function (1) minimizes the number of salesmen hired to visit and perform merchandising services at all customer locations. Constraint (2) ensures that if a salesman has at least one customer location to visit, he must be hired. Constraint (3) and (4) state that each customer must be visited once by a salesman coming from the company or another customer location. Constraint (5) confirms that if a salesman is hired, he must visit a customer location from the main location. Constraint (6) ensures that if a salesman is hired, he must return to the main location from the customer location visited. Constraint (7) imposes the net flow rule that salesman entering each customer location must leave it. This constraint also ensures that all salesmen who leave the main location must return back to it. Constraint (8) guarantees that the total time of traveling and serving customers by each salesman does not exceed the available working time. Constraint (9) applies the elimination of sub tours proposed by *Miller-Tucker-Zemlin* (1960). Constraint (10) and (11) declare that the decision variables x_{kij} and y_k are binary.

4 ILLUSTRATIVE EXAMPLE

We illustrate the proposed AMIP-mTSP approach by solving the multiple traveling salesmen problem shown in Figure 1. In this problem, customer locations CL1 through CL4 are to be visited for merchandizing services given that ML is the main location where *m* salesmen are located.

The average service time in each location including the time spent in the main office are shown in Table 2 while the traveling times between the five locations are given in Tables 3, and for avoiding reflexive traveling to the same location, the traveling times between the same locations are given a large number, 999. In this example, we assumed that the average available working time for each salesman is 60 time units.

Table 2: Service times in Location *j*.

		Locations		
ML	**CL1**	**CL2**	**CL3**	**CL4**
5	10	15	20	25

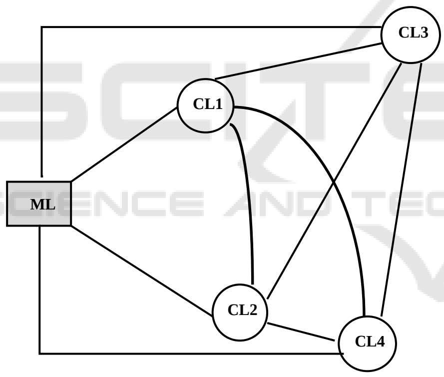

Figure 1: Illustrative Example Problem Representation.

Table 3: Traveling times between location *i* and *j*.

	j				
i	**ML**	**CL1**	**CL2**	**CL3**	**CL4**
ML	999	6	8	4	3
CL1	6	999	9	2	5
CL2	8	9	999	4	7
CL3	4	2	4	999	4
CL4	3	5	7	4	999

The optimal salesman tours obtained as results of solving the AMIP-mTSP optimization model of are reported in Table 4. Only two salesmen are required to visit all customer locations, each salesman *k* served 2 customer locations such that each customer is served by precisely one salesman.

Table 4: Optimal Solution for Illustrative Example.

Salesman ID	**Salesman Tour**	**Total Service time**
1	*ML→CL1→CL4→ML*	54
2	*ML→CL2→CL3→ML*	56

The above illustrative example was solved using GAMS 22.6 using the CPLEX solver. The system used to solve the proposed model is Dell Inspiron 15 3000 Series laptop with Windows 10 and Intel(R) Core(TM) i3 6006U CPU at 2.0 GHz processor, 4GB of RAM.

The minimum number of salesmen and their tours for visiting all customer locations obtained as proven optimal solution reflect the accuracy of the proposed model formulation. AMIP-mTSP model solved the illustrative example in approximately 114 CPU milliseconds with absolute and relative gaps of zero. For the proposed model, the number of constraints is $2|K|*[1+|I|)]+|I|+|I'|*[2+|I'-1|]$ and the number of variables is $|K|*[(|I|*|I-1|)+1]$, where $|\bullet|$ represents the cardinality of a set. For instance, for $|I|=5$, $|K|=4$, the number of constraints and variables are 85 and 84 respectively. It is anticipated that even if for moderate number of customer locations/nodes, the proposed AMIP-mTSP model can still yield optimal solutions in reasonable computer CPU time and memory.

5 CONCLUSIONS

In this paper, we described the development of mathematical formulation of an assignment-based MIP optimization model for the mTSP with time limit constraints. The objective of the proposed AMIP-mTSP model is to optimize the allocation and touring of individual salesmen for minimizing the number of salesmen assigned to visit all sales points and provide merchandizing service to all customers while satisfying all the touring and allowed working time constraints for individual salesman.

The AMIP-mTSP model provided promising solutions; the results reveal the applicability and suitability of the proposed AMIP-mTSP approach for solving multiple salesmen problem with time limit constraints for selling firms. The touring of salesmen of a selling firm is only one application example of a problem that can be modeled as assignment-based mTSP with the aim of minimizing the number of salesmen.

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