Ant Colony Optimization Approaches for the Tree t-Spanner Problem

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Abstract: A tree t-spanner of a given connected graph is a spanning tree T in which the ratio of distance between every pair of vertices is at most t times their distance in the graph, where t is a parameter known as stretch factor of T. The tree t-spanner problem deals with finding a spanning tree in a connected graph whose stretch factor is minimum amongst all spanning trees of the graph. For unweighted graph, this problem is $\mathcal{NP}$-Hard for any fixed $t \geq 4$, whereas for weighted graph, this problem is $\mathcal{NP}$-Hard for any fixed $t > 1$. This paper concerns this problem for connected, undirected, and weighted graph and proposes three variants of ant colony optimization (ACO) approach for this problem. ACO approach is a swarm intelligence technique inspired by the foraging behavior of real ants. All three variants of ACO approach have been tested on a set of randomly generated graph instances. Computational results show the effectiveness of all three variants of ACO approach.

1 INTRODUCTION

Given a connected graph $G(V,E)$, a *tree t-spanner* is a spanning tree (say T) in which the distance between every two vertices (say u and v) in T is at most t times their distance in G. t is a parameter called stretch factor of T and it is determined by the maximum stretch taken over all pairs of vertices in G, where the stretch of a pair of vertices (say u and v) in T is the ratio of the distance between u and v in T to their distance in G. The *tree t-spanner problem* aims to find a spanning tree of G whose stretch factor (t) is minimum amongst all spanning trees of G.

The term t-spanner of a graph G, which was first coined by Peleg and Ullman (Emek and Peleg, 2008), is a spanning subgraph (H) of G in which the distance between every pair of vertices is at most t times their distance in G. This notion (spanner) describes an property about the approximation of pairwise vertex-to-vertex distances in the original graph by spanning subgraph (H). This approximation distance quality is measured by the parameter $t \geq 1$, which is referred to as the *stretch factor* of the t-spanner. Spanners with such distance approximation property make them of practical relevances in various areas such as communication networks, distributed systems, motion planning, network design, and parallel machine architectures (Althófer et al., 1993; Awerbuch et al., 1992; Bhatt et al., 1986; Liestman and Shermer, 1993; Peleg and Ullman, 1987; Peleg and Upfal, 1988). For example, spanners can be used to build synchronizers for transforming synchronous algorithms into asynchronous ones (Emek and Peleg, 2008).

In a connected graph, if a spanning subgraph (say T) is both *t-spanner* and *tree*, then T is referred to as a *tree t-spanner*. Note that a spanning tree of the graph is always a tree spanner, where a tree spanner is a *tree t-spanner* for some $t \geq 1$. Tree spanners also find practical relevance. For example, tree spanners of small stretch factors can be applied in performing multi-source broadcast in a network (Awerbuch et al., 1992), which can greatly simplify the message routing at the cost of only small delay in message delivery. Later, Cai and Corneil (Cai and Corneil, 1995) studied graph theoretic, algorithmic, and complexity issues about tree spanners of weighted and unweighted connected graphs. They proved on edge-weighted graphs that a *tree* 1-*spanner*, if it exists, is a minimum spanning tree and can be found in polynomial time. They also proved on edge-weighted graphs that the problem of finding a *tree t-spanner* is $\mathcal{NP}$-Hard for any fixed $t > 1$. In case of unweighted graphs, the problem of finding a *tree t-spanner* is $\mathcal{NP}$-Hard for any fixed integer $t \geq 4$. For connected, undirected and unweighted graph, this problem is referred to as minimum max-stretch spanning tree problem (Emek and Peleg, 2008; Dragan and Köhler, 2014). In literature, approximation algorithms on some specific types of *unweighted* graph (Cai, 1992; Peleg and Tendler, 2001; Venkatesan et al., 1997) have been developed for the *tree t-spanner problem*; however, such specific types of graph may not be encountered in ev-

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ery situation. One can find various related tree spanner problems in details in (Liebchen and Wünsch, 2008).

This paper concerns the *tree t-spanner problem* for connected, undirected and edge-weighted graph $G(V,E,w)$, where V is a set of vertices; E is a set of edges; and for each edge, there exists an edge weight which is a positive rational number. Hereafter, the *tree t-spanner problem* for connected, undirected and edge-weighted graph $G(V,E,w)$ will be referred to as [Tree]_t-SP. Since [Tree]_t-SP is $\mathcal{NP}$-Hard for $t > 1$, metaheuristic techniques are the appropriate approaches that can find high quality solution in a reasonable time. In literature, only genetic algorithm (GA) (Moharam and Morsy, 2017) has been proposed so far for the [Tree]_t-SP. GA (Moharam and Morsy, 2017) is based on generational population model which uses multi-point crossover and mutation genetic operators for generating offsprings.

Ant colony optimization (ACO) approach (Dorigo et al., 1991; Dorigo and Stützle, 2004) is swarm-based metaheuristic technique that has been applied successfully to many combinatorial optimization problems. It is inspired by the foraging behavior of ants. This paper proposes three variants of ACO approach for the [Tree]_t-SP. The proposed three variants of ACO approach have been tested on a set of randomly generated graph instances (see description Section 4.1). On a set of randomly generated graph instances, computational results show the effectiveness of the proposed three variants of ACO approach for this problem.

The rest of this paper is organized as follows: Section 2 and 3, respectively, describes a brief introduction of ACO approach and presents three variants of ACO approach for the [Tree]_t-SP. Computational results are reported in Section 4. Finally, Section 5 contains some concluding remarks.

2 ACO APPROACH

Ant colony optimization (ACO) approach is a swarm-based metaheuristic technique derived from the observation of real ants' foraging behavior. In nature, ants wander randomly in search of food and deposit a chemical substance called pheromones on the paths while wandering. Other ants can smell pheromones and tend to choose their way probabilistically based on paths marked by strong pheromone concentrations, resulting a pheromone-trail. Pheromones on the paths also evaporate with time. While returning to the nest from the food source, ants tend to choose the shortest path marked by strong pheromone concentrations in comparison to longer paths, resulting in further increment on pheromone concentrations on such shortest path. Such strong pheromone trail on this shortest path stimulates more and more ants to follow this shortest path again, which in turn, further reinforces the pheromone trail on this shortest path. After some time, the whole ants in the colony follow that pheromone trail which leads to the shortest path from nest to food source and vice versa. Hence such behavior for searching the shortest path between nest and food source emerges from the cooperation among individuals of the whole colony.

This feature of real ant colonies has been emulated in ant colony optimization (ACO) approach for the solution of hard combinatorial optimization problems. In ACO approach, each artificial ant in the colony builds a solution through incremental procedure that takes the help of heuristic information of the problem under consideration and pheromone trails that are accumulated through search experience. Dorigo *et al.* (Dorigo et al., 1991) developed the first ACO approach called Ant System. Later, different variants of ACO have been proposed in the literature. One can find a good survey of different variants of ACO algorithms and their applications in (Dorigo and Stützle, 2004). Readers can find some recent work on ACO approach (Sundar and Singh, 2013; Sundar et al., 2012; Monteiro et al., 2015)

3 THREE VARIANTS OF ACO APPROACH FOR THE [Tree]_t-SP

This section presents three variants of ACO approach for the [Tree]_t-SP. All components of each variant of ACO approach are the same except the updating rule regarding the pheromone trails component.

Initially, all pairs of shortest paths for a given graph is precomputed. Also, pheromone value at each edge of G is initialized with a high value (here the value is set to 10 which is determined empirically) so that ACO approach performs a wider exploration of the search space during initial iterations before starting biasing the search. Then, at every iteration, each artificial ant in the colony constructs a spanning tree and computes the fitness of its constructed solution; and the update of pheromone trails is performed.

3.1 Construction of Solution

Each artificial ant, say k, constructs a spanning tree (solution), say T, on $G(V,E,w)$ with the help of heuristic information and pheromone value. Initially,

T is an empty set (Φ). S is another set which is initially empty (Φ). A vertex, also called *root* vertex (say r) of T to be grown, is selected randomly from V and is added to S. For each vertex x in V, the attribute $d_T(r,x)$ that is the distance from the *root* r of T to x is maintained. For the *root* vertex r, $d_T(r,r)$ is set to 0. Hereafter, similar to Prim's algorithm (Prim, 1957), iteratively, ant k, at each step, selects an edge connecting a vertex, say u, in S to a vertex, say v, in $V \backslash S$; however, instead of selecting an edge with minimum cost, an edge is selected according to a probabilistic action choice rule based on heuristic information and pheromone values. The probability of selecting an edge ($prob^k_{e_{uv}}$) by ant k is determined as follows:

$$prob^k_{e_{uv}} = \frac{[\tau_{e_{uv}}]^\alpha [\eta_{e_{uv}}]^\beta}{\sum\limits_{v_1 \in S} \sum\limits_{v_2 \in V \backslash S} [\tau_{e_{v_1 v_2}}]^\alpha [\eta_{e_{v_1 v_2}}]^\beta} \quad (1)$$

where $\tau_{e_{v_1 v_2}}$ is the pheromone concentration on the edge $e_{v_1 v_2}$; $\eta_{e_{v_1 v_2}}$ is the heuristic value that is available a priori, and that is equal to the weight on edge $e_{v_1 v_2}$; and α and β are two parameters which determine the relative influence of the pheromone trail and the heuristic information respectively. By this probabilistic action choice rule, the probability of selecting a particular edge (u, v) increases with the value of associated pheromone trail $\tau_{e_{uv}}$ and of the heuristic information value $\eta_{e_{uv}}$. The parameters α and β together help in searching high quality solution in the search space. Hence, being biased by higher heuristic and pheromone value, this probabilistic action choice rule plays an important role in exploring various good (in terms of fitness) spanning trees in the search space on a graph instance. The selected edge e_{uv} is added to T and v is added to S. Update $d_T(r,v)$ through "$d_T(r,v) = d_T(r,u) + w(u,v)$". This process is repeated until $V \backslash S$ becomes empty. At this juncture, a spanning tree T is constructed.

3.2 Fitness Computation

Once the spanning tree T that is also a tree spanner is constructed, its fitness, in terms of its stretch factor t, is computed. Before computing fitness, lowest common ancestors for pairs of vertices in T with the root vertex r, are preprocessed by least common ancestor algorithm (Harel and Tarjan, 1984), where lowest common ancestor for any two vertices, say u and v, in T is the common ancestor of vertices u and v that is located farthest from the root, and is referred to as LCA(u,v). In doing so, one can get the lowest common ancestor query for any pair of vertices say u and v in T, i.e., LCA(u, v) in constant-time. Distance $d_T(u,v)$ between any two vertices u and v in T can be computed as

$$d_T(u,v) = d_T(r,u) + d_T(r,v) - 2 \times (d_T(r, LCA(u,v))) \quad (2)$$

One can easily determine the maximum stretch factor t of T taken over all pairs of vertices x, y in G, i.e.,

$$\frac{d_T(u,v)}{d_G(u,v)} \quad \forall (u,v) \in G \quad (3)$$

3.3 Update of Pheromone Trails

Once all ants have constructed their solutions at the current iteration (say *it*) of ACO approach, the pheromone trails are updated at the end of the current iteration, i.e. *it*. We follow three ways for the update of pheromone trails based on *local pheromone update*, *global pheromone update*, and *mixed pheromone update* that also define three variants of ACO approach for the [Tree]_t-SP.

- □ *Local Pheromone Update:* First, decrease the pheromone value on each edge, say e_{uv}, of G by a constant factor, and then augment the pheromone on those edges of G that are selected by the iteration's best solution (S^{ib}) or that are selected by the iteration's best ant. The pheromone trails which are updated by the iteration's best ant on the edges of G are as follows:

$$\tau_{e_{uv}}(it+1) = \rho \; \tau_{e_{uv}}(it) + \Delta\tau^{ib}_{e_{uv}} \quad (4)$$

where $0 < \rho \leq 1$ is the persistence rate, and $\Delta\tau^{ib}_v$ is the amount of pheromone that the iteration's best ant deposits on the edges of the iteration's best solution (S^{ib}). The parameter ρ is used to avoid unlimited accumulation of pheromone trails as the ACO approach progresses, and simultaneously it also helps the ACO approach in forgetting bad decisions taken in past. In addition, if an edge is not selected by the ants, the pheromone value of this edge decreases exponentially in the number of iterations. $\Delta\tau^{ib}_{e_{uv}}$ is defined as follows:

$$\Delta\tau^{ib}_{e_{uv}} = \begin{cases} p_{aug1}, & \text{if } e_{uv} \in S^{ib}; \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

where p_{aug1} is a parameter to be determined empirically. This equation explains that the better the solution S^{ib} in terms of fitness is, the more pheromone the edges that are part of this solution receive. Hence edges that are part of solution with higher fitness are more likely to be selected by ants in future iterations of the approach, thereby receiving more reinforcement. Hereafter, this variant of ACO approach will be referred to as *ACO_Local*.

- *Global Pheromone Update:* It is similar to *local pheromone update* except augmenting pheromone. In this variant of ACO approach, augment pheromone on those edges of G that are present in the *best-so-far* solution (say S^{gb}) found by *best-so-far* ant since the start of ACO approach. The pheromone trails which are updated by the *best-so-far* ant on the edges of G are as follows:

$$\tau_{e_{uv}}(it+1) = \rho\ \tau_{e_{uv}}(it) + \Delta\tau^{gb}_{e_{uv}} \quad (6)$$

where $\Delta\tau^{gb}_{v}$ is the amount of pheromone *best-so-far* ant deposit on the edges of *best-so-far* solution (S^{gb}). $\Delta\tau^{gb}_{e_{uv}}$ is defined as follows:

$$\Delta\tau^{gb}_{e_{uv}} = \begin{cases} p_{aug2}, & \text{if } e_{uv} \in S^{gb}; \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

where p_{aug2} is a parameter to be determined empirically. Hereafter, this variant of ACO approach will be referred to as *ACO_Global*.

- *Mixed Pheromone Update:* It is similar to *local pheromone update* except augmenting the pheromone. In this variant of ACO approach, two ways in mixed way are applied in augmenting the pheromone: first augment the pheromone on those edges of G that are present in the iteration's best solution (S^{ib}) or that are selected by iteration's best ant; and second, at every P_{gb} generations, augment the pheromone on those edges of G that are present in the *best-so-far* solution (say S^{gb}) found by *best-so-far* ant since the start of ACO approach, where P_{gb} is a parameter to be determined empirically. The update of pheromone trails in this approach is as follows:

$$\tau_{e_{uv}}(it+1) = \rho\ \tau_{e_{uv}}(it) + \Delta\tau^{mixed} \quad (8)$$

where $\Delta\tau^{mixed}$ is the sum of amount of pheromone ($\Delta\tau^{ib}_{e_{uv}}$, i.e. p_{aug1}) augmenting on the edges of S^{ib} and amount of pheromone ($\Delta\tau^{gb}_{e_{uv}}$, i.e. p_{aug1}) augmenting on the edges of S^{gb} at every *iter*, where $\Delta\tau^{ib}_{e_{uv}}$ is similar to equation 5 and $\Delta\tau^{gb}_{e_{uv}}$, which is applied at every P_{gb} generations, is similar to equation 7. Note that here $\Delta\tau^{gb}_{e_{uv}}$ at every P_{gb} generations is additional reinforcement. Hereafter, this variant of ACO approach will be referred to as *ACO_Mixed*.

Algorithm 1: Pseudo-code of ACO approach for the [Tree]_t-SP.

Compute all pairs of shortest paths of G;
Initialize pheromone value at each edge of G;
while *Termination criteria is not satisfied* **do**
    for $k \leftarrow 1$ **to** pop_{ant} **do**
        Each ant k constructs a spanning tree of G with the help of heuristic information and pheromone value;
    Perform update of pheromone trails;

The lower pheromone trail limit, i.e. τ_{min} is explicitly set for the proposed three variants of ACO approach. The pseudo-code of ACO approach for the [Tree]_t-SP is given by Algorithm 1, where pop_{ant} is the size of colony (population) of ants.

4 COMPUTATIONAL RESULTS

The proposed three variants of ACO approach (*ACO_Local*, *ACO_Global*, and *ACO_Mixed*) for the [Tree]_t-SP have been implemented in C and tested on a set of randomly generated graph instances. All experiments have been performed on a Linux with the configuration of 3.2 GHz $\times$ 4 Intel Core *i5* processor with 4 GB RAM. Each variant of ACO approach has been executed 10 independent runs on each instance. We have allowed each variant of ACO approach to execute for 1000 generations.

Subsequent subsections discuss about the generation of graph instances and the parameter tuning. In addition, a comparison study of all three variants of ACO approach on graph instances is also discussed.

4.1 Graph Instances

Since instances used in GA (Moharam and Morsy, 2017) for the [Tree]_t-SP are not available, and it should be also noted that authors (Moharam and Morsy, 2017) tested the effectiveness of their GA on only two instances (i.e. first instance is a graph of 50 vertices and second one is a graph of 100 vertices (one can see Table 5 of (Moharam and Morsy, 2017)). To test our proposed ACO approaches, two set of graph instances $G(V,E,w)$ with V = 50, 100 are generated randomly in 500×500 plane. A graph instance is generated as follows: each point representing a vertex in G is selected randomly in 500×500 plane. The Euclidean distance between two vertices (v_1, v_2) is its edge weight. For each set, three different complete graph instances are generated, and with the help of each generated graph $G_i(V_i,E_i)$, further three sparse graphs with different edge density, i.e., $|E_{i1}|$, $|E_{i2}|$, $|E_{i3}|$ are generated, where $|E_{i1}| = 0.8 \times |E_i|$; $|E_{i2}| = 0.6 \times |E_i|$; and $|E_{i3}| = 0.4 \times |E_i|$. Note that $0.8 \times |E_i|$ ($0.6 \times |E_i|$ and $0.4 \times |E_i|$) means 20% (40% and 60%) random edges of E_i are not considered into E_{i1} (E_{i2} and E_{i3}). All graph instances are represented in *A1_A2_A3* (see Table 2, where *A*1 is the total number of vertices of that graph (see column *Vertex* of Table 2); *A*2 is *X*% (see column *Edge* of Table 2) random edges of total number of edges in its corresponding complete graph with *A*1 vertices are not considered in current graph instance; and *A*3 presents different graph instance (in terms of edge density) with the same number of vertices (*A*1). In Table 2, consider *50_0.0_1* instance which is a complete graph with $A1 = 50$, $A2 = 0.0$ and $A3 = 1$. Three different instances with different edge-density have been generated from this graph by doing changes in *A*2, i.e. $A2 = \{0.2, 0.4, 0.6\}$. The generated graph instances corresponding to *50_0.0_1* are *50_0.2_1*, *50_0.4_1* and *50_0.6_1*.

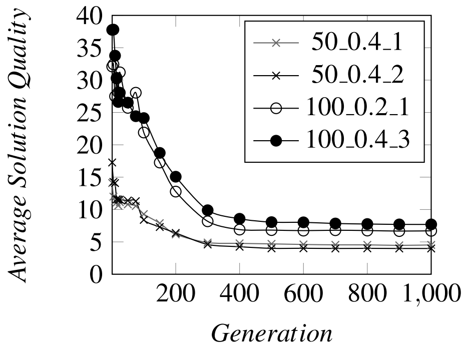
Hence, 24 graph instances have been generated from the two sets of graph instances with $|V| \in \{50, 100\}$.

Table 1: Possible values of each parameter used in ACO approaches for the [Tree]_t-SP.

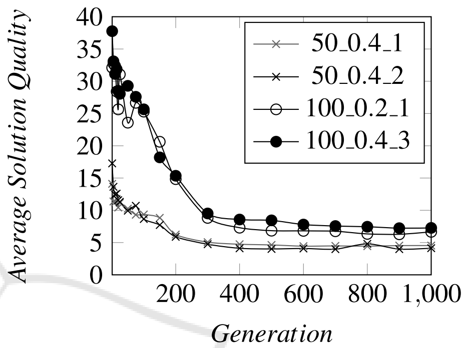
Parameter	Description	Characteristics
ant_pop	The population of ants	{10, 20, 30, 40}
τ_{min}	Value of τ_{min}	{0.001, 0.05, 0.01, 0.005}
α	A parameter	{0, 1, 2}
β	A parameter	{0, 1, 2}
P	A parameter	{0.05, 0.1,0.2}
ρ	A parameter	{0.975, 0.98, 0.985, 0.99}
P_{gb}	A parameter	{10, 20, 30}

4.2 Parameter Tuning

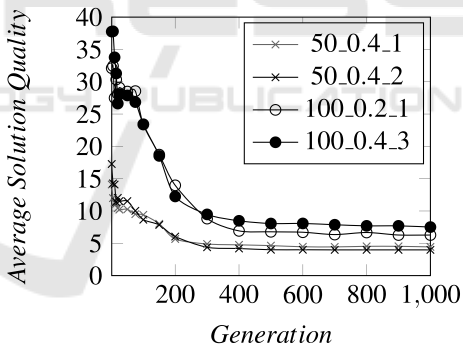
ACO approaches are stochastic approaches, and parameters used in our proposed three variants of ACO approach for the [Tree]_t-SP play significant roles in finding high quality solutions. It is to be noted that determining a perfect tuning of various parameters used is a difficult task; however, it is always possible to determine approximate parameter values that provide good results overall. pop_{ant}, α, β, ρ, τ_{min}, and $P \in \{p_{aug1}, p_{aug2}\}$ (based on *ACO_Local*, *ACO_Global*, and *ACO_Mixed*) are the parameters that have been used in the three variants of ACO approach for this problem. Various possible values related to each parameter have been considered from our initial experiments and available literatures (one can see Table 1). On investigation, $pop_{ant} = 30$, $\alpha = 1$, $\beta = 2$, $\rho = 0.975$, $\tau_{min} = 0.001$, $P = 0.05$, and $P_{gb} = 20$ are the values of parameters that approximate high quality solutions overall; however, these values are in no way optimal



(a) *ACO_Local*



(b) *ACO_Global*



(c) *ACO_Mixed*

Figure 1: Improvement of average solution quality over successive generations.

parameter values for all instances. One may study (Birattari, 2005; Eiben and Smith, 2011) for a detailed coverage of parameter tuning in metaheuristic techniques in general. It should be noted that all values of parameters are same for each variant of ACO approach, as it was observed experimentally that each variant of ACO approach produces high quality solution overall.

Table 2: Results of three variants of ACO approach on different graph instances for the [Tree]_t-SP.

Instance	Characteristics		*ACO_Local*				*ACO_Global*				*ACO_Mixed*			
	Vertex	Edge	Best	Avg	SD	ATET	Best	Avg	SD	ATET	Best	Avg	SD	ATET
50_0.0_1	50	0%	4.92	4.97	0.04	6.53	4.73	5.19	0.29	6.74	4.92	4.97	0.05	6.54
50_0.2_1	50	20%	4.49	4.58	0.08	8.15	4.51	4.63	0.10	8.40	4.44	4.56	0.06	8.16
50_0.4_1	50	40%	4.53	4.57	0.04	6.66	4.53	4.77	0.25	6.88	4.44	4.54	0.06	6.66
50_0.6_1	50	60%	4.64	4.83	0.21	4.65	4.74	5.08	0.21	4.81	4.57	4.71	0.13	4.66
50_0.0_2	50	0%	5.02	5.19	0.17	6.52	5.11	5.32	0.20	6.75	5.02	5.27	0.13	6.53
50_0.2_2	50	20%	4.69	4.79	0.11	7.93	4.70	4.96	0.20	8.24	4.69	4.82	0.16	7.93
50_0.4_2	50	40%	3.98	4.04	0‘06	6.61	4.13	4.47	0.24	6.85	3.98	4.19	0.23	6.62
50_0.6_2	50	60%	4.05	4.07	0.03	4.57	4.04	4.42	0.27	4.73	4.05	4.06	0.02	4.59
50_0.0_3	50	0%	5.45	5.63	0.09	6.61	5.45	5.91	0.39	6.77	5.45	5.69	0.13	6.56
50_0.2_3	50	20%	4.87	4.91	0.02	8.22	4.85	5.01	0.15	8.47	4.87	4.91	0.02	8.22
50_0.4_3	50	40%	4.87	4.91	0.02	8.22	4.85	5.01	0.15	8.49	4.87	4.91	0.02	8.22
50_0.6_3	50	60%	4.38	4.52	0.08	4.64	4.43	4.70	0.23	4.79	4.26	4.50	0.14	4.64
100_0.0_1	100	0%	6.54	6.91	0.22	45.37	6.71	7.04	0.24	45.55	6.67	6.83	0.17	45.28
100_0.2_1	100	20%	6.71	6.83	0.08	62.40	6.66	6.92	0.24	64.66	6.28	6.66	0.18	62.36
100_0.4_1	100	40%	6.49	6.83	0.22	51.20	6.24	6.92	0.46	52.81	6.42	6.68	0.18	51.00
100_0.6_1	100	60%	6.80	7.40	0.33	34.51	6.64	7.09	0.29	35.65	6.77	7.22	0.37	34.55
100_0.0_2	100	0%	6.66	6.76	0.08	45.50	6.66	6.99	0.29	45.47	6.39	6.55	0.13	45.48
100_0.2_2	100	20%	6.23	6.47	0.19	62.83	6.04	6.80	0.40	62.76	6.18	6.39	0.11	62.82
100_0.4_2	100	40%	6.06	6.19	0.16	50.84	6.10	6.61	0.49	50.92	6.09	6.20	0.15	50.95
100_0.6_2	100	60%	5.97	6.12	0.16	34.51	6.21	6.65	0.33	35.74	5.98	6.19	0.18	34.51
100_0.0_3	100	0%	8.07	8.73	0.15	45.69	7.51	8.12	0.41	45.52	7.60	8.08	0.21	45.59
100_0.2_3	100	20%	7.71	7.93	0.20	63.16	7.49	8.83	0.61	63.30	7.58	7.89	0.17	63.14
100_0.4_3	100	40%	7.70	7.89	0.18	51.19	7.27	7.90	0.46	52.98	7.51	7.78	0.17	51.04
100_0.6_3	100	60%	7.53	7.93	0.17	34.38	7.25	7.53	0.28	35.67	7.38	7.73	0.23	34.39

4.3 Performance Evaluation

Table 2 reports the results of each variant of ACO approach (i.e., *ACO_Local*, *ACO_Global*, and *ACO_Mixed*) for the [Tree]_t-SP on a set of graph instances. In Table 2, column *Instance* denotes the name of each graph instance in *A1_A2_A3* format; columns *Vertex* and *Edge* respectively denote the total number of vertices and edges of the graph; and columns *Best*, *Avg* and *SD* and *ATET*, respectively denote the best value, the average value, the standard deviation and the average total execution obtained by *ACO_Local*, *ACO_Global*, and *ACO_Mixed* over 10 runs.

One can also observe in Table 2 that among all three variants of ACO approach, *ACO_Local*, in terms of *Best*, is better on 3, is equal on 4 and is worse on 17 graph instances in comparison to other variants of ACO approach; *ACO_Local*, in terms of *Avg*, is better on 6, is equal on 3 and is worse on 15 graph instances in comparison to other variants of ACO approach; *ACO_Global*, in terms of *Best*, is better on 11, is equal on 1 and is worse on 12 graph instances in comparison to other variants of ACO approach; *ACO_Global*, in terms of *Avg*, is better on 2 and is worse on 22 graph instances in comparison to other variants of ACO approach; *ACO_Mixed*, in terms of *Best*, is better on 6, is equal on 4 and is worse on 14 graph instances in comparison to other variants of ACO approach; and *ACO_Mixed*, in terms of *Avg*, is better on 13, is equal on 3 and is worse on 8 graph instances in comparison to other variants of ACO approach. Based on experimental observation, *ACO_Global* is better in terms of *Best* and in terms of *Avg ACO_Mixed* is better; however one can also notice that the results obtained by all three variant of ACO approaches are close to each other.

4.4 Convergence Analysis

To examine the effectiveness of the proposed three variants of ACO approach for the [Tree]_t-SP, we have performed experiments based on convergence of each proposed variant of ACO approach. For this, four graph instances i.e. *50_0.4_1*, *50_0.4_2*, *100_0.2_1* and *100_0.4_3* have been selected from the set of considered graph instances. Figure 1-(a-c) respectively depict the emergence of average solution quality (average value (*Avg*) based on 10 runs) over successive generations. The curves in Figure 1-(a-c) clearly demonstrate that the proposed three variants of ACO approach for this problem converge rapidly towards the high quality solution.

5 CONCLUSIONS

This paper concerns the *tree t-spanner problem* for connected, undirected and weighted graph and proposes three variants of ACO approach. All components of each variant of ACO approach are the same except the updating rule regarding the pheromone trails component. All three variants of ACO approach have been tested on a set of randomly generated graph instances. Experimental results demonstrate the effectiveness of solution quality obtained by all three variants of ACO approach on each instance. It was also found empirically that the performance of all three variants of ACO approach are close to each other. Convergence analysis on some instances show that all three variants of ACO approach converge rapidly towards the high quality solutions over successive generations for such instances.

As a future work, other metaheuristic techniques will be developed for this problem, as this problem is under-studied in the domain of metaheuristic techniques.

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