

A Prospective Fuzzy Approach for the Development of Integral Seismic Risk Scenarios for Barcelona, Spain

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Abstract: We create a set of synthetic seismic risk scenarios by combining stochastic seismic simulations with social fragility indicators by mean of a fuzzy Mamdani type inference nested-model. The original values of the social economic variables were modified by arbitrary increments to simulate either constrains or improvement in their reported levels, and the Fuzzy Seismic Risk Model was applied again for each of these variations to produce a range of final integral seismic risk levels. Even if this experiment clearly needs to be further tuned, the use of fuzzy inference in the creation of risk scenarios becomes a simpler task once suitable membership functions have been defined, since the non-linear influence of each of the variables involved can be easily quantified. The final product is capable to facilitate the prospective view needed in decision-making planning while avoiding compensability issues, commonly reflected when composite indicators are used to represent social dimensions.

1 INTRODUCTION

A seismic risk scenario comprehends a plausible representation of future impacts of seismic activity over a geographical area. However there is no single 'best' scenario construction method appropriate for all applications and in each case, the appropriate method is determined by the context and the application of the scenario. One of the most common scenario type that has been applied in impact assessments is based on an ordered variation of the variables of a seismic risk model and its posterior use to produce incremental scenarios for sensitivity studies. In this type of technique particular seismic risk components, either physical or social are changed incrementally by plausible arbitrary amounts. Also referred to as synthetic scenarios, (IPCC, 1994) the incremental scenarios facilitate the construction of response areas, which can assist in identifying critical thresholds of response to a significant seismic risk. In the same way, incremental scenarios provide information on an ordered range of variable changes and can readily be applied in a consistent and replicable way in different studies and regions, allowing for direct inter-comparison of results. However, such scenarios do not necessarily present a realistic set of changes that are physically plausi-

ble. They are usually adopted for exploring system sensitivity prior to the application of more credible, model-based scenarios.

Any seismic risk scenario though, should represent future conditions that account for both the dynamic of societies and the nature of the hazard when they come together in an urban environment. Nevertheless, different issues arise when different dimensions are supposed to be aggregated to represent a unique reality, in this case: the seismic risk. If a methodology based on composite indices is chosen, compensability or an abusive use of weights are just a few of the several inaccuracies reported in literature (Munda, 2012, Booyens, 2002). This is why we used the Integral Seismic Fuzzy Risk model (ISFR) proposed by Gonzalez *et. al*, 2014, which takes into account the simultaneous non-linear influence of the physical risk (structural exposed elements along with its vulnerabilities) and the social vulnerability components that might reduce or enhance resilience and social fragility. By using the ISFR aspects such as compensability are avoided since neither linear or geometric methods are used to aggregated the indicators, while weights are not used in this exercise.

By incrementing the reported values of the social aggravation indicators and combining them with

the outputs of an stochastic seismic risk model, we can represent plausible futures seismic risk states for the City of Barcelona, assuming such increments are achievable. In order to represent either positive and negative trends, we incremented and decreased indices values. Therefore a model's output width range can be established, that can be then used to calibrate any possible decision made out from the model's results.

2 THE INTEGRAL SEISMIC FUZZY RISK MODEL

The Integral Seismic Fuzzy Risk Model is composed of different sub-modules called respectively: Aggravation Fuzzy Model, Physical Risk Fuzzy Model, and Total Risk Fuzzy Model. Each one of them is formed by different sub-modules that are the result of the aggregation of the different indicators proposed by Carreño *et al.*, (2012). We performed indicator's aggregation by means of Fuzzy Inference System type Mamdani (FIS). Starting from the original raw values of the indicators, each of the outputs of each sub-model acts also as inputs for the next inference in the structure of the model. In this sense we guarantee that all variables once in the model remain as fuzzy sets, giving the chance to connect them through a new FIS without losing consistency, allowing model completeness. Figure 1 shows a conceptualization of the Integral Seismic Fuzzy Risk Model.

2.1 Main Functioning

In order to estimate a level of social aggravation at Barcelona, we used the reported values of social economic fragility and resilience capacity reported by Carreño *et al.*, (2012) and modified by Gonzalez *et al.*, 2014. Such indicators try to capture the state of preparedness and recovery of the city of Barcelona, and comprises reported statistical data from the city Council and values based in experts opinion. The indicators used for the estimation of aggravation levels can be seen in Table 1.

Table 1: Set of Indicators used to estimate Aggravation.

Indicator	
Fragility	-Human Health Resources (HHR) -Development Level (DL) -Emergency Operability (EO)
Resilience	-Marginal Slum (MS) -Social Disparity Index (SDI) -Population Density (PD)

For the level of physical risk, we used the values estimated from a probabilistic earthquake risk damage scenario, which was performed over the 248 small statistical areas (ZEP) defined by Barcelona government during the project ICC/CIMNE, 2004.

This scenario was grouped in classes corresponding to different damages on critical structures, as shown in Table 2

Table 2: Set of Indicators used to estimate Physical Risk.

Indicator	
Property Damage (PD)	-Damage Area (DA) -Dead People (DP) -Injured People (INJ)
Life Line Sources (LLS)	-Telephonic Substation Affected (TSA) -Electrical Substation Affected (ESA) -Damage in Water Mains (DWM)
Network Damage (ND)	-Damage in Gas Network (DGN) -Fallen Length of Electrical Lines (FLE) -Damage in Mains Roads (DMR)

Each of the variables described in Table 1 and 2 were implemented in the form of fuzzy inference systems (FIS). Both, FIS called Aggravation and FIS called Physical Risk are characterized by 3 linguistic classes: low, medium and high for each of its input variables, while each of its FIS output were characterized by five labels: *low*, *medium-low*, *medium-high*, *high* and *very-high*. The FIS called Total Risk represents the convolution of all the previous FIS, embedded in one main structure that has as inputs the variables representing the inferred values of physical risk and social aggravation. For the Total Risk FIS, both: inputs and outputs are characterized by 5 linguistic classes (*low*, *medium-low*, *medium-high*, *high* and *very-high*), forming a Mamdani model which is composed of a set of 25 fuzzy rules to be used in the inference process.

In the general scheme proposed here an increase in aggravation, seen as a decrease in social economic and an increase in fragility urban conditions fragility urban conditions, would be reflected as an increase in the total risk values. In turn, a decrease in aggravation levels would have as a result lower total risk levels.

Gonzalez *et. al* implemented the fuzzy risk model obtaining aggravation values for the 10 administrative districts of the city of Barcelona (Figure 2) and physical and total risk values for each of its 248 small statistical areas (as defined by Barcelona government, known as ZEP latter transformed to 233 statistical basic areas) (Figure 3 and 4).

2.2 The Scenarios

We wanted to investigate the effects on future seismic risk scenarios when a city's social vulnerability condi-

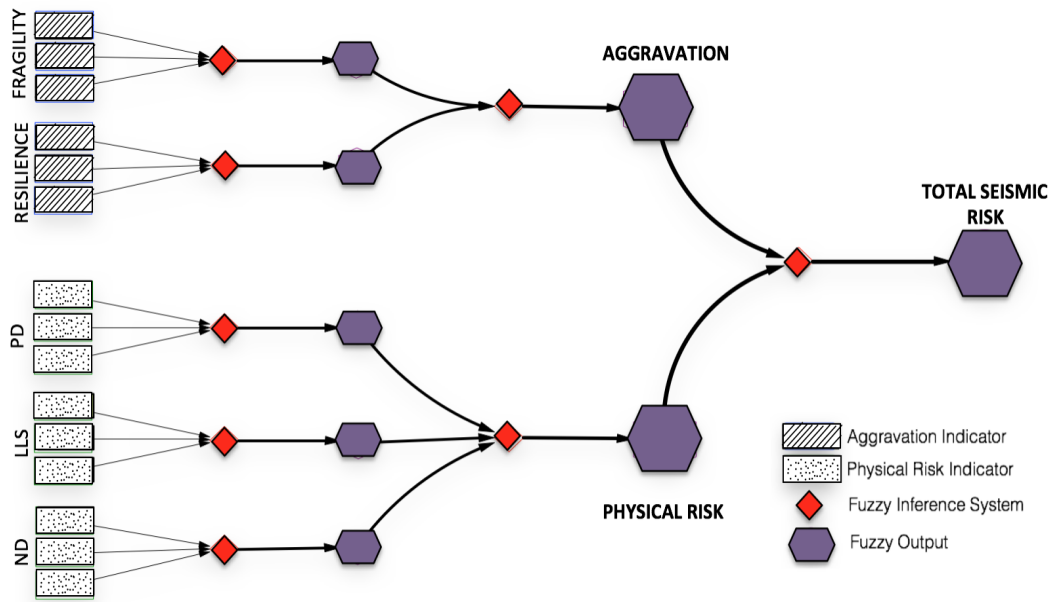


Figure 1: Conceptualization of the complete Seismic Risk Fuzzy Model. Colored rectangles represent indicators either describing social economic fragility or physical risk. Here: PD = Property Damage, LLS=Life Line Sources Damage, ND= Network Damage.

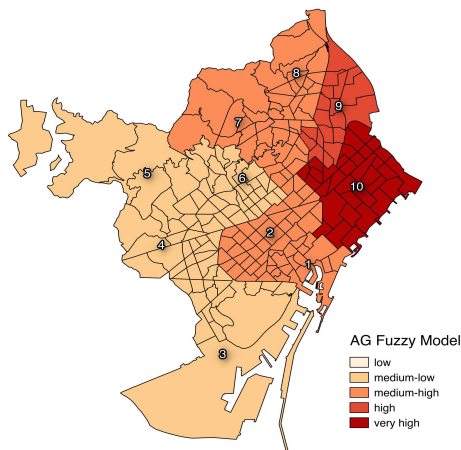


Figure 2: Aggravation values reported by the Gonzalez *et al.*, 2014: (1) Ciutat Vella, (2) Eixample, (3) Sants-Montjuic, (4) Les Corts, (5) Sarrià-Sant Gervasi, (6) Gràcia, (7) Horta-Guinardó, (8) Nou Barris, (9) Sant Andreu, (10) Sant Martí.

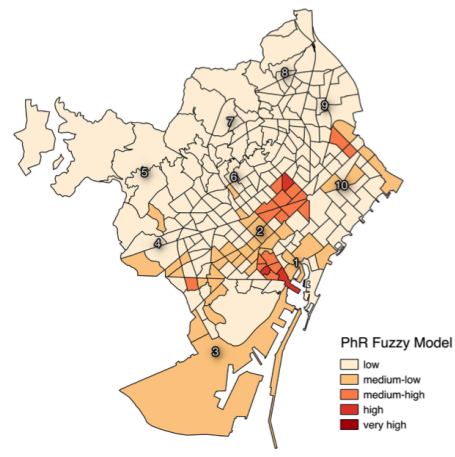


Figure 3: Physical Risk values reported by the Gonzalez *et al.*, 2014: (1) Ciutat Vella, (2) Eixample, (3) Sants-Montjuic, (4) Les Corts, (5) Sarrià-Sant Gervasi, (6) Gràcia, (7) Horta-Guinardó, (8) Nou Barris, (9) Sant Andreu, (10) Sant Martí.

tion has been altered. Therefore we modified the values of those variables representing social economic aspects of the city of Barcelona, Spain. Selected variables representing resilience and social fragility features were adjusted following an arbitrary scheme, and were then used as inputs for the FIS Aggravation. These values were then implemented in the Integral Seismic Fuzzy Risk Model to obtain a set of Total Risk values.

In order to represent plausible scenarios as results of changes in Barcelona's resilience capacity and social fragility conditions we selected different control

variables, whose further modification might modify the rest of the variables belonging to a particular dimension. For example, changes in human health resources would be reflected in changes over the emergency operability value, but no changes would be noted in the development level of the city. On the contrary, a change in human resources would be likely noted as a reduction of the social disparity index. We can differentiate then between two possible changes: a positive change, when the adjustment in control variables reduces the Aggravation level, and a negative change: when changes in control variables in-

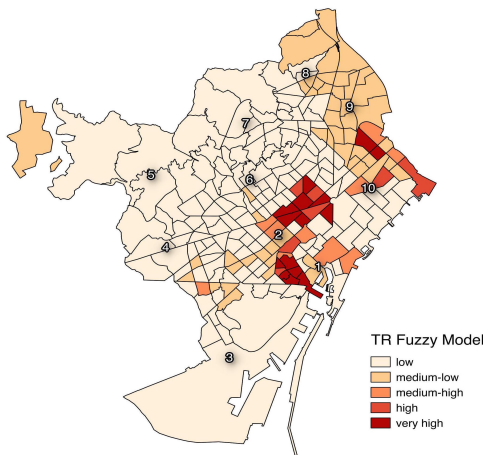


Figure 4: Total Risk values reported by the Gonzalez *et. al*, 2014: (1) Ciutat Vella, (2) Eixample, (3) Sants-Montjuic, (4) Les Corts, (5) Sarrià-Sant Gervasi, (6) Gràcia, (7) Horta-Guinardó, (8) Nou Barris, (9) Sant Andreu, (10) Sant Martí.

crease Aggravation. For this paper two different scenarios were designed, which can be seen in Tables 3 and 4.

3 RESULTS AND COMPARISON

To implement both scenarios in the Fuzzy Integral Model, we first adopt an arbitrary percentage range of increase over control variables within FIS called Aggravation. All the variables belonging to this category were incremented simultaneously following this scheme. For example, for the scenario 1, the original reported values of all three control variables were modified (HHR, EO were incremented and SDI was decremented) as 30%, 50% and 75% each time, whilst the rest of the scenario's variables remained unaltered. Each group of modified aggravation values were then used as input to the principal fuzzy model. In this way we were able to model spatial values of total risk according to the corresponding increment in the aggravation module. As we mentioned before, we implemented the so called percentage range of increase in two senses: a positive one, meaning that the aggravation was decreased following those percentages (and therefore better total risk levels should be expected) and a negative one, meaning that the aggravation level was instead, augmented or increased (leading towards worst total risk levels). We must emphasize that the original values corresponding to the natural hazard influence (an earthquake in this case) were not adjusted nor incremented in any case.

Because of space limitations, we are only showing the negative increment of aggravation levels, rep-

resenting how the aggravation levels over Barcelona City gets deteriorated. Figures from 5 to 8 show a sample of the obtained results. In these, original values either of aggravation or total risk are depicted as a dotted line. These are the estimated values by mean of the Integral Fuzzy Model as they were reported by Gonzalez *et. al*, 2014. Such values have not been altered whatsoever and were used as initial values for further comparisons. In order to have a quantitative measure of the influence of each scenario in the corresponding outcome, we determined aggravation and total risk anomalies as well. Each anomaly was calculated following a single valued-based method and subsequently expressed as a percentage. Therefore each anomaly value represent the change either positive or negative, for every given original value.

Figure 5 (upper row) shows Barcelona's spatial patterns for the estimated aggravation level corresponding to scenario 1. It is clearly seen that assuming a decline in the human health resources' dimension (HHR) and its related variables (a decrease in the emergency operability level (EO) and therefore an increment in the social development level (SDI)) can lead towards much more unfavorable aggravation's surroundings. Considering an increase of just 30% in the aggravation original level, a general increment of aggravation for the northeast of the city can be quantified.

Figure 6 (a) shows that those areas whose aggravation's values were originally significantly low (such as (3) Sants-Montjuic, (4) Les Corts, (5) Sarrià-Sant Gervasi) were almost not influenced for changes in scenario 1 while Districts (1) Ciutat Vella, (2) Eixample where within a range of increment of 20-30%. The model estimates a significant increment for district (7) Horta-Guinardó, which reaches a positive anomaly of the order of almost 67% in the case of an increment in aggravation of 75%. Districts: (1) Ciutat Vella, (2) Eixample, (8) Nou Barris, (9) Sant Andreu and (10) Sant Martí where within a range of 10-30% of change.

Figure 5 (lower row) shows total risk Barcelona's spatial patterns after the aforementioned worsening in aggravation levels. As expected, the total risk value was generally increased, being more sensible to such changes those areas were important aggravation levels were already present which is again, the northeast area of Barcelona. It can be seen how risk levels in these areas get more and more deteriorated as the changes in aggravation become more significant, drawing a clear spatial pattern contained in these particular areas, already known because of their lack of favorable social aggravation characteristics (Busquets i Grau, 2005). Figure 6 (a) shows how this deterior-

Table 3: Scenario 1. Changes on Human Health Resources. HHR=Human Health Resources, DL=Development Level, EO=Emergency Operability, MS=Marginal Slums, SDI=Social Disparity Index, PD=Population Density.

Indicator	Resilience	Positive Change				Negative Change			
HHR	change	Increases	30%	50%	75%	Decreases	30%	50%	75%
DL	no change								
EO	change	Increases	30%	50%	75%	Decreases	30%	50%	75%
	Fragility	Positive Change				Negative Change			
MS	no change								
SDI	change	Decreases	30%	50%	75%	Increases	30%	50%	75%
PD	no change								

Table 4: Scenario 2. Changes on Demography.HHR=Human Health Resources, DL=Development Level, EO=Emergency Operability, MS=Marginal Slums, SDI=Social Disparity Index, PD=Population Density.

Indicator	Resilience	Positive Change				Negative Change			
HHR	no change								
DL	no change								
EO	no change								
	Fragility	Positive Change				Negative Change			
MS	change	Decreases	30%	50%	75%	Increases	30%	50%	75%
SDI	change	Decreases	30%	50%	75%	Increases	30%	50%	75%
PD	change	Decreases	30%	50%	75%	Increases	30%	50%	75%

ration on risk levels can reach very important levels along these areas, reaching in some cases increments of almost 200% of the original risk value.

Figure 7 (upper row) shows Barcelona’s spatial patterns for the estimated aggravation level corresponding to scenario 2. Although the most sensitive geographic area is once again the northeast part of Barcelona, it can be seen how scenario 2 leads towards a more spread spatial pattern compared to scenario 1. Starting from an initial increment of 30%, areas that were not influenced in scenario 1 present now a shift in their values changing in turn from linguistic classes of *low* to *medium-low* and *high* in some cases. This can be seen in central and northern areas. Noticeably, increments of 50% and 75% present a similar pattern, with only slight differences in the south part of the city.

Figure 8 (a) shows up to what point the generalization of the aggravation level over almost all the city is not homogeneous. District (1) Ciutat Vella jumps from an anomaly of 13% for an increment of 30% up to a 33% and even 40% anomalies for the rest of increments. A similar behavior can be seen for district (2) l’Eixample. It is interesting how districts (3) Sants-Montjuic, (4) Les Corts, (5) Sarrià-Sant Gervasi, untouched by the conditions stated in scenario 1, now responds accordingly to scenario 2 and a sort of response threshold seems to appear; in districts 3 and 5 for the initial increment of 30% and even 50% the reactions are contained. But for an controlled incre-

ment of 75% those districts reach anomalies of around 40%. The same with district (6) Gràcia greatly influenced by this scenario by rapidly reaching anomalies of 40% and 55%. Districts (7) Horta-Guinardó and (9) Sant Andreu shows a more contained increments within 20%-40% while districts (8) Nou Barris and (10) Sant Martí present anomalies only within the range of 10-22% for all increments, almost not affected by scenario 2 changes.

Figure 7 (lower row) shows total risk Barcelona’s spatial patterns after an arbitrary worsening in aggravation levels. According to the conditions stated in Scenario 2, an increment of 30% leads to an aggravation spatial pattern at the northeast area of the city with values ranging in the linguistic classes of *medium-low* to *medium-high*. Although increments in aggravation values of 50% and 75% do not affect new geographical areas, such increments enhance significant risk values already present, changing them from *medium-low* and *high* to *very high*.

3.1 Discussion and Future Work

From the previous results we can get some interesting insights regarding the Fuzzy Seismic Risk Model as a tool to create risk scenarios. It is important to keep in mind that this particular exercise was not intended to simulate realistic changes in the variables belonging to the social aggravation dimension, but to evaluate the model’s performance when it needs to be

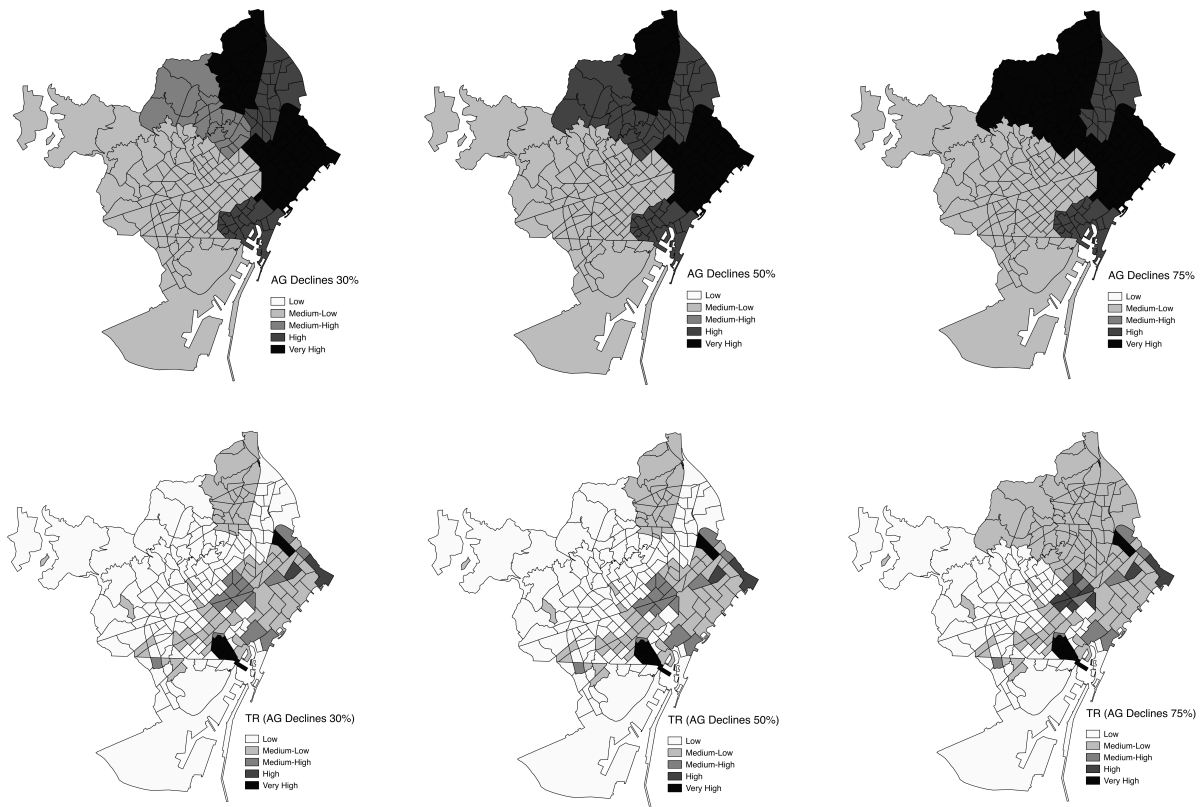


Figure 5: Scenario 1. Spatial patterns of Aggravation (AG, upper row) and Total Risk (TR, lower row) after a controlled decline in aggravation original values.

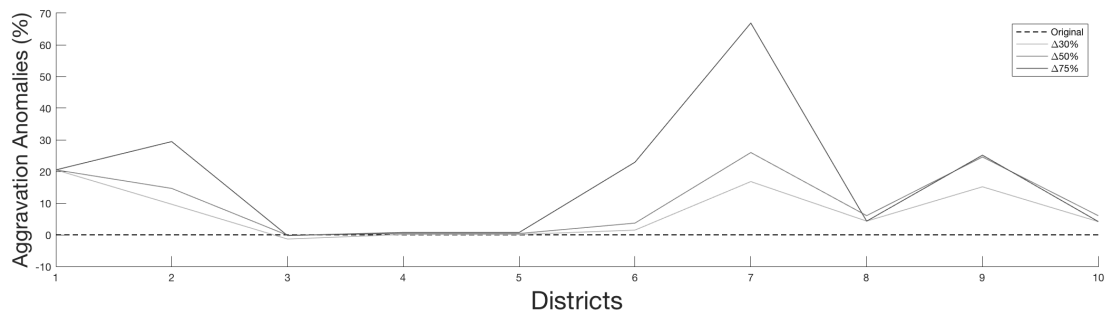
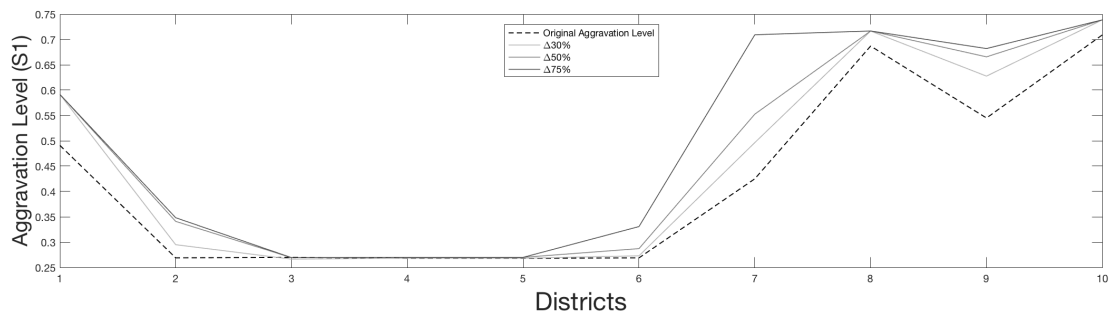
used to assess and quantify the non-linear influences in the total risk level of an urban environment after a particular modification in the social aggravation input values. Therefore the implementation of such massive percentage rate of changes in the original values of aggravation were an effort to force the model to react in order to obtain a quantitative measure of its sensitivity and capacity to be used for these goals, before future implementations of more credible range of changes can be made. Following this spirit, we decline to draw conclusions or to attempt to explain the results of this work (in its initial phase), by trying to reflect them as if they were portraits of real situations. This is why we are not yet in the position to suggest management policies to improve or try to soft the impact of the proposed scenarios, if those scenarios were to be used in a scheme of decision-making in a first place. Nevertheless, a major objective to be reached later is of course, a full risk reduction and sustainability scenario-based management scheme.

The model maintain a suitable performance when representing spatial patterns of social aggravation in those geographical areas were desfavorable social vulnerability conditions has been previously reported. The results of the experiments shows as well a coher-

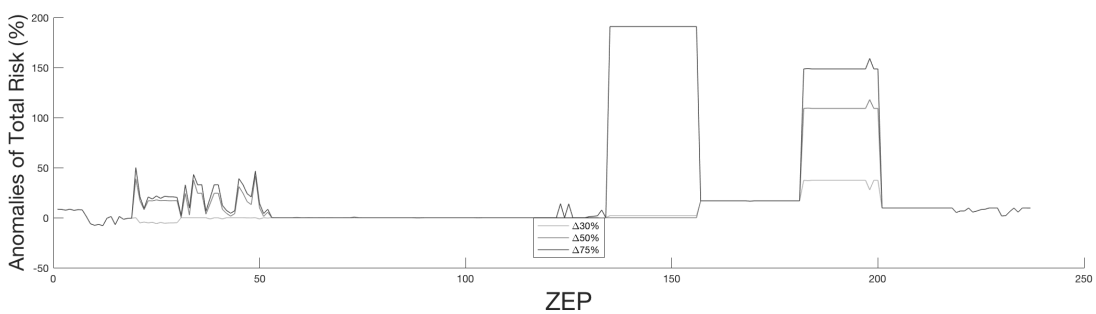
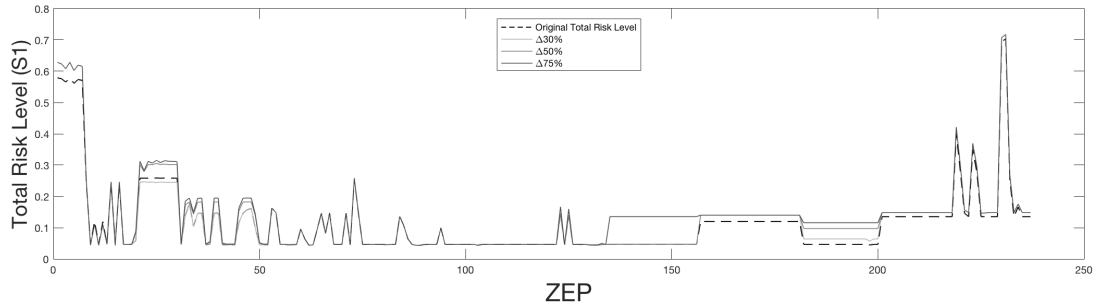
ence with those results reported by Cardenas *et. al* for Barcelona city. For example, an unexpected result could have been the estimation of high social aggravation levels in those areas where no such levels have ever been reported. However the model shows a suitable representation of the non homogeneous changes in risk values when the social aggravation is, precisely, aggravated.

The main differences between scenario 1 and scenario 2 can be seen in a first place, in terms of the increased number of areas affected by an increment on the aggravation original levels. Scenario 2 tend to generate impacts over more geographical areas than scenario 1, being more affected the central part of Barcelona which comprehends larges areas of districts 2, 4 and even regions of districts 6. In the case of scenario 1, the affected areas are contained along districts 1, 8, 9 and 10.

Another difference relies in the magnitud on which the different Districts are getting affected by the arbitrary increment in aggravation levels and the anomalies produced because of these changes, as can be seen in Table 5 and Table 6. According to these results, it is clear that it is scenario 2 (a change in the demography levels of the city) the responsible of



(a) Aggravation levels for Scenario 1. (up) Aggravation values (low) Aggravation anomalies expressed as percentages. (1) Ciutat Vella, (2) Eixample, (3) Sants-Montjuic, (4) Les Corts, (5) Sarrià-Sant Gervasi, (6) Gràcia, (7) Horta-Guinardó, (8) Nou Barris, (9) Sant Andreu, (10) Sant Martí



(b) Total Risk levels for Scenario 1. (up) Total Risk values. (low) Anomalies of Total Risk expressed as percentages

Figure 6: Aggravation (a) and total risk (b) levels after a controlled decline on the original aggravation values.

sustained changes over all Barcelona area, even over those districts presenting low and steady levels of either aggravation and total risk. This clearly represent the weight in any future risk scenario of the presence of large amounts of persons living in an area where

aggravation levels gets incremented. Although given the current model's develop stage the given estimations are not realistic, the fuzzy quantification of such results glimpses as a valuable source of information for decision-makers once the model gets improved.

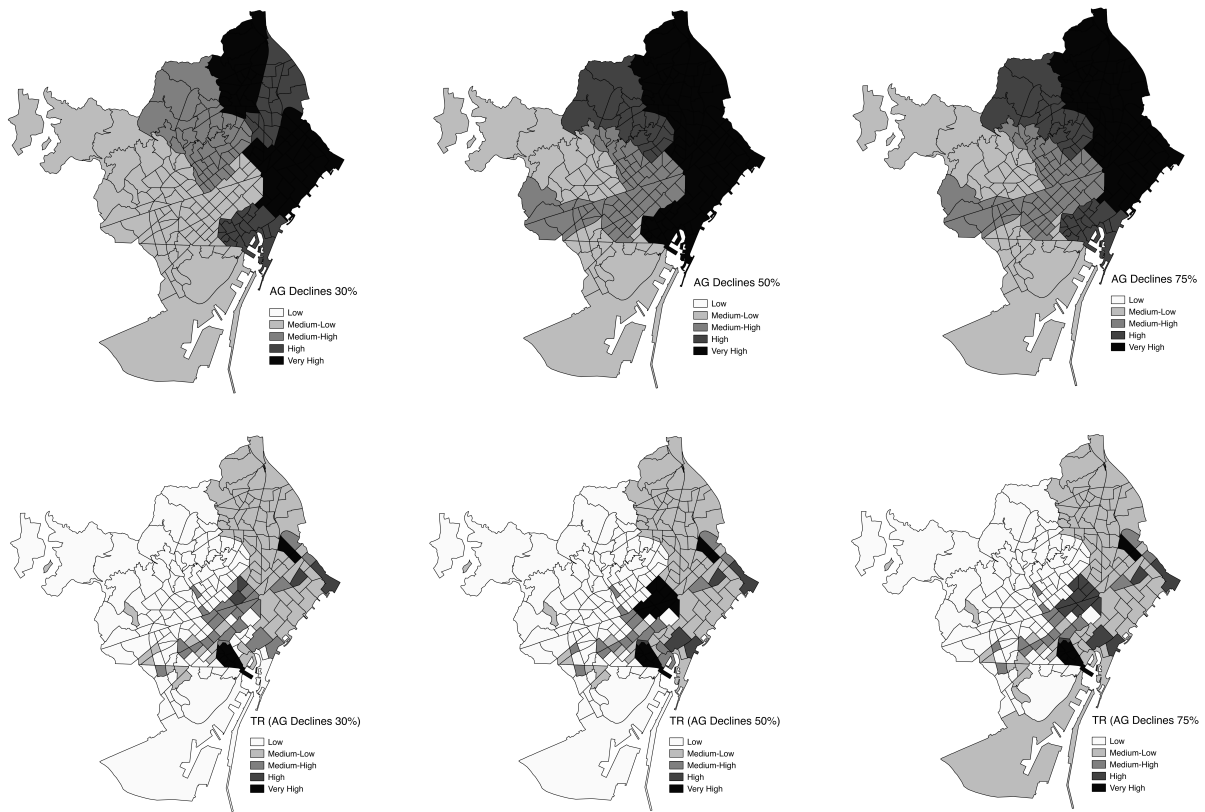


Figure 7: Scenario 2. Spatial patterns of Aggravation (AG, upper row) and Total Risk (TR, lower row) after a controlled decline in aggravation original values.

In the case of scenario 1, representing a deterioration of the human health capacity of the city (whether medics, nurses, or other trained personnel) the model estimate that the areas presenting a significant lack of resilience capacity and high levels of social fragility, will be ultimately the most affected by such social health weakness in the future. Creating a more serious and critical risk landscape to be solve before it happens.

It is interesting to note how the increment in the values of total risk levels are not longer related with

the presence of physical risk levels exclusively, as it where in the traditional composite indices methodologies (see Carreño, M.L, 2012) and the influence of social aggravation can be more clearly explored. This particular characteristic is very important in terms of the reliability and realism of risk future scenarios if fuzzy methodologies are to be selected. The possibility to include a more accurate estimation of how the complex dynamic's process of natural risk is influenced by the social dimension stressed by particular circumstances is, certainly, one of the final goals of this research.

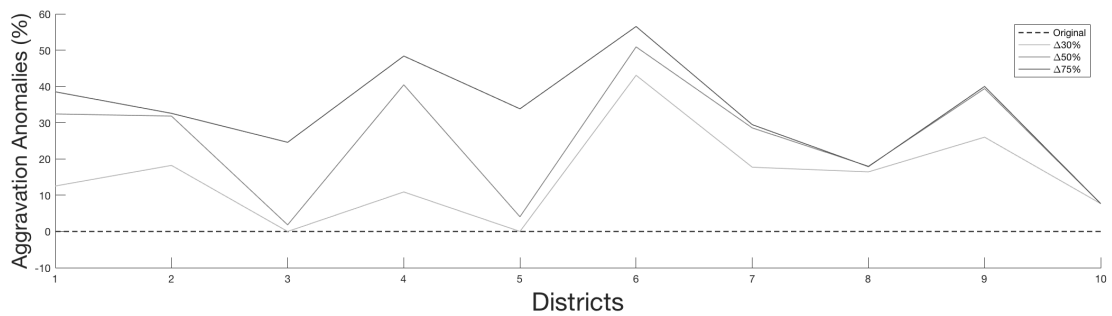
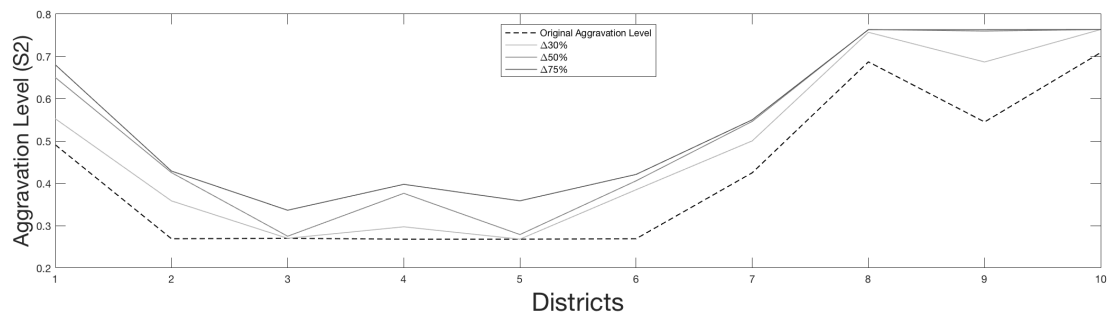
Table 5: Scenario 1. Aggravation behavior.

Increment	Most Affected District	Max Anomaly
30%	1, 7	20%, 15%
50%	1,2, 7,9	20%, 10%, 15%
75%	2,7,9	30%, 68%,22%

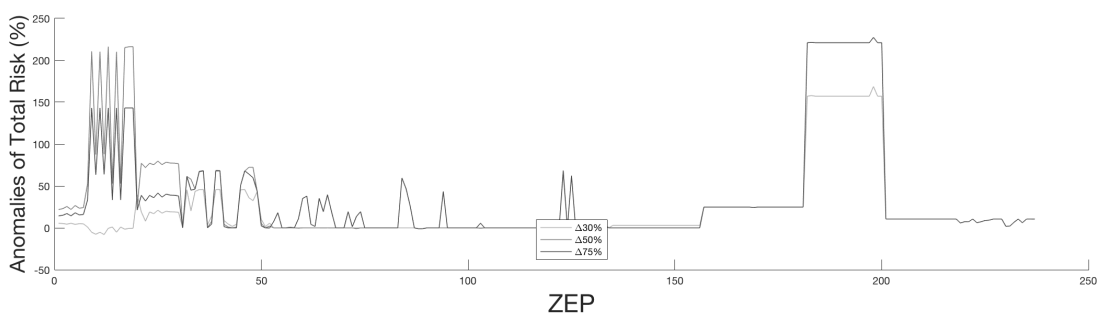
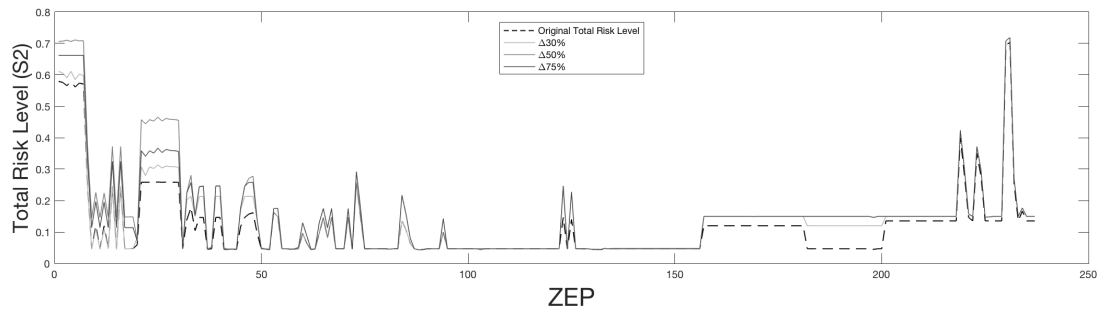
Table 6: Scenario 2.

Increment	Most Affected District	Max Anomaly
30%	6,7,8	40%, 25%, 20%
50%	1,2,6	31%, 25%, 38%, 40%
75%	1,4,5,6	40%, 50%

The modeling of synthetic scenarios considering social aspects by mean of fuzzy methods allows also a suitable representation of the fine graduality among the many different system's states, which is crucial to impose applicable and consistent limits between states without losing generality, while performing a proper identification of particular tipping response thresholds that may me leading towards one state or another. For example, the model one improved could be telling which social variable might be considered or labeled as a driven variable, either towards worst or better risk conditions, and therefore mitigation or



(a) Aggravation level for Scenario 2. (up) Aggravation values (low) Aggravation anomalies expressed as percentages. (1) Ciutat Vella, (2) Eixample, (3) Sants-Montjuic, (4) Les Corts, (5) Sarrià-Sant Gervasi, (6) Gràcia, (7) Horta-Guinardó, (8) Nou Barris, (9) Sant Andreu, (10) Sant Martí



(b) Total Risk level for Scenario 2. (up) Total Risk values. (low) Anomalies of Total Risk expressed as percentages
Figure 8: Aggravation (a) and total risk (b) levels after a controlled decline on the original aggravation values.

improvements actions might be taken.

Although the results presented in this paper are based in an arbitrary thus controlled increments on the values of the social indicators, the use of fuzzy theory

allows a simpler method to include a set of particular stressors (different of a natural hazard) over indicator's performance in time. Therefore the possible influences of economic or political constraints over the

social dimension of risk may be added.

Finally, even if the model performance is adequate, there are diverse issues still to be resolved. For example, it is important to obtain results based in realistic scenarios that may be turned into suitable management decisions. Clearly an increment in the social disparity index of 75% it is most desirable but it is not realistic in terms of actual medium-term planning. Therefore a more finer increments needs to be selected, if an incremental scenario-scheme is going to be followed. Increments in the order of 5 to 20% would be more realistic for the achievable changes to be implement along the variables regime. Following our results, the model still struggles to represent responses within this range of change. We implement a local sensitivity analysis to explore the contribution of each of the variables under a particular scenario (not shown in this paper) and we can say that the membership functions needs to be further tuned and possibly the implementation of weights might be required.

4 CONCLUSIONS

We developed synthetic scenarios to model future trends of seismic risk at urban level for the City of Barcelona, Spain by mean fuzzy methods. The scenarios try to emulate the influences of the social dimension of risk in the evolution of seismic risk under particular circumstances. By a controlled modification of social vulnerability indicators we simulated increments on Barcelona's human health resources along with its demography levels, whilst considering a set of seismic damages scenarios for the city. We obtained consistent spatial patterns of seismic risk, and a first quantification of the non-linear influence of such increments in the levels of social aggravation and total risk (considering both: social and physical elements). The initial results of this work are not meant to be realistic in terms of decision-making but to play as the the primary base for further development and investigation in this line of research to reach comprehensive management fuzzy control and management models.

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