

Derivation of Real Driving Emission Cycles based on Real-world Driving Data

Using Markov Models and Threshold Accepting

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Abstract: The European Union has decided to bring the Real Driving Emissions (RDE) law into force in 2016. From this point onward, the air pollutants a vehicle emits under real driving conditions will be measured by means of a so-called Portable Emissions Measurement System (PEMS) and then used as the basis for licensing. Compared to the emission values presently determined in the New European Driving Cycle (NEDC), a significant rise can be expected. This change is on the one hand caused by a substantially more dynamic driving style prescribed by RDE regulations, and on the other hand by considerably larger variations of ambient conditions. A trend of development resulting from this conversion is the creation of test cycles conforming to RDE regulations, which enable vehicle development to adhere to the new licensing regulations. The validity of a RDE drive is gradually verified based on multiple criteria before respective emission values are determined at the end of the process. The contribution at hand presents a new approach for generating RDE substitute cycles. At first, the criterion of driving dynamics will be focussed upon. To realize this, combinatorics of a large set of real driving data will be used to generate substitute cycles, which will exhibit driving dynamics as high as possible. This specification achieves universal, vehicle independent limitation cycles featuring high emission levels. By using the described limitation cycles, a first vehicle examination concerning the fulfilment of RDE regulations is made possible.

1 INTRODUCTION

Because of the ongoing emission discrepancies between homologation measurement and real driving measurement, the European Union has decided to bring Real Driving Emission (RDE) tests for type testing into force in January of 2016. The mentioned difference in measured emission values occurs because the New European Driving Cycle (NEDC), which is used for certification in the EU, is barely representative for loading requirements when considering real vehicle drives. In addition to this, it is possible to create a specific calibration for a prescribed cycle's already known velocity curve also known as cycle beating. Since the regulations specify a more dynamic driving style and allow for significantly more variable ambient conditions compared to the NEDC, an increase in emission is to be expected (Gerstenberg et al., 2016). The requirements for a valid RDE drive are extensive as time, velocity and distance standards are an issue (see Table 1). Furthermore, driving dynamics and cumulative difference in height are ver-

ified. In the event that these standards have been met, a RDE measuring drive can initially be considered as formally valid, after which the emissions calculation can be performed based on the currently specified evaluation methods using the tools Emroad or Clear. The applied evaluation methods normalise the RDE measurement results during the post process and make them comparable to results from Worldwide Harmonized Light-Duty Vehicles Test Cycle (WLTC), which is intended to replace the NEDC as a homologation cycle according to RDE regulations (Maschmeyer et al., 2016).

The specified changes present a great challenge for the development of vehicles. There will be no more known and reproducible static cycle elements and transients. A vehicle can no longer be applied to single velocity profiles, as it was possible when working with a strictly prescribed cycle. Instead, RDE conforming substitute cycles must be found. To ensure the adherence to RDE limits of respective pollutants from a manufacturer perspective, drive cycles will have to be generated and analysed during the de-

velopment process.

In case of gasoline engines it can be assumed that the pursued worst case cycles force the engine into non-stoichiometric combustion conditions (scavenging, enriching), since especially these operating points cause increased emission (Fraidl et al., 2016). But in accordance with the gear transmission ratio, the mentioned range is of no importance regarding cycles such as the NEDC. In this light, one could question which RDE challenges can be depicted in one single substitute cycle, since multiple combinations of operating states, which would potentially increase emissions, are imaginable.

During the development phase, it has to be ensured that the vehicle passes the real driving test under any circumstances. For that reason worst case cycles are necessary, because ensuing changes after Start of Production (SOP) can lead to immense costs for the manufacturer.

There already exist some approaches to meet the mentioned challenges. Most of them like Maschmeyer or Gerstenberg presented different approaches to concatenate the RDE requirements with measurements on test benches and describe the corresponding tasks (Maschmeyer et al., 2016), (Gerstenberg et al., 2016). Steinbach illustrated a way using model-based calibrations for which emission models were utilised to adapt the calibration of control unit functions to RDE standards (Steinbach et al., 2016). He validated the used simulation tool chain with virtual calibration steps and showed the opportunity to do different calibration changes in short time and without having the physical hardware.

Most of the approaches didn't describe precise power demands for their tests. But to guarantee to meet the RDE requirements driving cycles for the mentioned test scenarios are necessary. This contribution to the topic at hand examines an approach intended to generate RDE worst case cycles for the test measurement. For this purpose, cycles maximizing the criteria of driving dynamics va_{pos} from (European Commission, 2016) will be developed. For the realization of this project, real driving data has been used. The data will be reassembled by means of combinatorics to create cycles featuring the maximum driving dynamics (va_{pos} limitation value), and yet also depicting realistic velocity profiles.

2 METHOD

As has been illustrated before, one of the central problems regarding RDE is finding a worst-case cycle, which will give the manufacturer the guarantee that

the respective vehicle will meet RDE standards under any circumstance. However, a cycle such as this can only be generated by using a complex algorithm because of different criteria.

Real measured drives of an arbitrary number of drivers using the type of vehicle which has to be calibrated serve as the basis for the generation of replacement cycles. The data volume should be chosen as large as possible.

2.1 Splitting into Subproblems

The authors' idea is based on breaking the cycle generation down into separate problems. The flow chart in Figure 1 illustrates the different steps. At first, only cycles conforming to general boundary conditions (see Table 1) will be created, which will feature the maximisation of vehicle dynamics in compliance with the va_{pos_95} criterion. The focus in the paper at hand will be the combinatorics approach employed for this method of cycle generation.

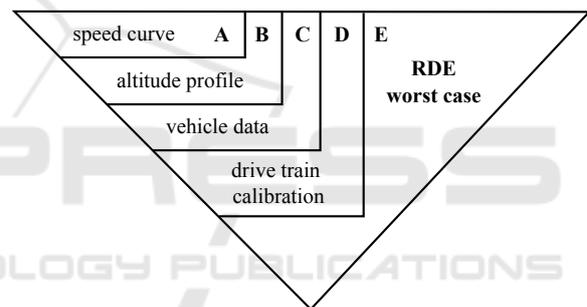


Figure 1: Subproblems for generating RDE replacement cycles.

During the subsequent generation phase, an elevation profile will be added to the already generated driving cycles. Afterwards, the cycles will be adapted to the vehicle's performance capability. It can be assumed that the vehicle construction and the used power train influence the searched worst-case cycle immensely. During the last step, the cycles will be modified according to critical emission scenarios regarding the respective engine.

In the following, the single steps for generating real drive emission cycles will be introduced. The flow chart in Figure 2 illustrates the different steps.

2.2 Stochastic Modelling

During the method's first phase, the data basis is modelled. In this context, Markov models, which are stochastic process models describing the states and transitions of unknown systems, have proved to be especially suitable. The Markov model is based on sim-

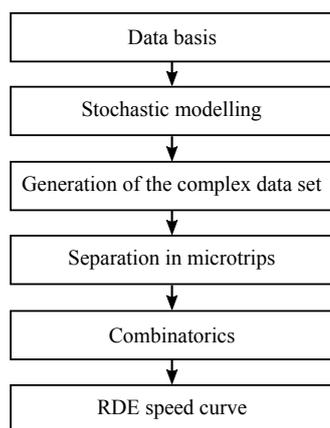


Figure 2: Schedule for generating replacement cycles.

plication by means of a so-called Markov assumption (memorylessness). Due to this, a system's subsequent state is only dependent on the current state, which causes preceding states to have no effect at all on the modification of states. This correlation is expressed in equation 1. x_n represents the current state and x_{n+1} represents the subsequent one. Preceding states as well as their impact (in equation x_1, x_2) are neglected (Ließner et al., 2017). For further information on the functionality of Markov models, see (Stroock, 2013).

$$P(X_{n+1} = x_{n+1} | X_1 = x_1, X_2 = x_2, \dots, X_n = x_n) = P(X_{n+1} = x_{n+1} | X_n = x_n) \quad (1)$$

Where,

$P()$ = probability function,

X_n = Markov variable,

x_n = current state,

X_{n+1} = Markov variable,

x_{n+1} = following state

A state x_n in this proposed application contains a discrete value of velocity and acceleration as seen in equation 2.

$$x_k = \begin{cases} v_k \\ a_k \end{cases} \quad (2)$$

Where,

x_k = state,

v_k = velocity value,

a_k = acceleration value

In order to transfer the data basis to the Markov model, it is sufficient to perform an elementary iterative transfer of the respective state transitions from one point of time to the next. Further processing is

based on the assumption that the saved state transitions in the Markov model hold all relevant information concerning this relation. This is because, after all, the Markov model contains all recorded drive data in a condensed and anonymized form. However, choosing and combining appropriate elements of the complex data set in such a way as to allow the derivation of representative replacement cycles with similar properties regarding the consumption is the actual challenge.

2.3 Generation Complex Data Set

The following strategy is suitable for deriving recently mixed velocity progressions from the Markov model. Based on a velocity and acceleration combination which has been set initially, a generation can be performed by means of a query concerning the saved state transition according to the Markov model. Using a weighted draw in proportion to probabilities, the subsequent velocity and acceleration combination is chosen. The described procedure is repeated until the desired scope of the complex data set has been attained. In this manner, an arbitrarily large data set is generated which contains recently mixed stochastically weighted progressions. But afterwards, identifying the elements in the complex data set which ultimately best represent RDE worst case scenarios during the substitute cycle is very challenging. To put this into practice, it is expedient to separate the complex data set into smaller units, which can then be processed separately.

2.4 Separation Into Microtrips

For disassembling the complex data set, a fragmentation into so-called 'microtrips' has proven to be useful. A microtrip is defined as a drive starting at one vehicle stop and finishing at the next (Fotouhi and Montazeri-Gh, 2013). Figure 3 depicts a microtrip segmentation into eight sections by using the WLTC as an example. When such a segmentation is applied to the complex data set, a very high number of microtrips is created. Finally, the major difficulty can be seen in choosing and combining appropriate parts. Approaches such as Lee pursue the aim of creating vehicle-specific substitute cycles by analyzing the energy level at the periphery of the wheels and then producing a selection of statistical key figures (Lee and Filipi, 2010). In contrast to this, this contribution aims for a vehicle-independent combinatorics at first. This combinatorics would contain properties occurring during the substitute cycle which are relevant for driving dynamics: distribution (velocity, acceleration, velocity multiplied by acceleration and stop) and

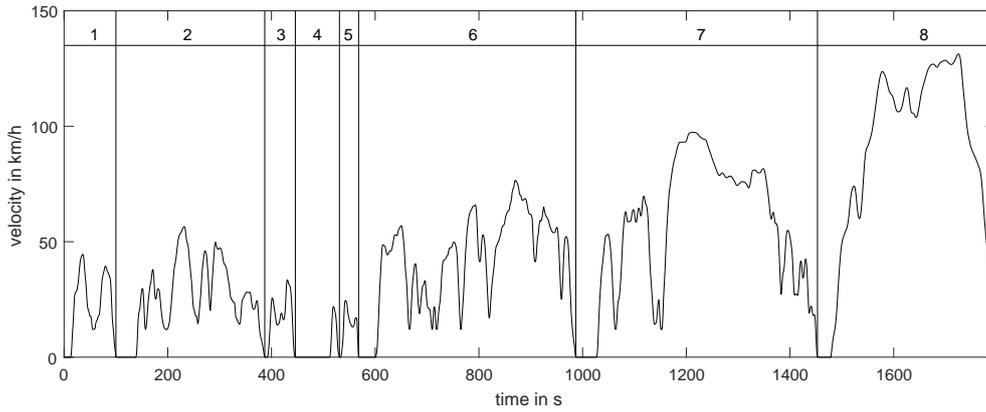


Figure 3: Separation of a drive in microtrips at the example of the WLTC.

the relative positive acceleration (RPA) index. Single microtrips have to be combined into one single substitute cycle in order to have high dynamic RDE conformable cycle as closely as possible. To achieve this, the following procedure can be employed.

2.5 Combinatorics

The main challenge in generating RDE substitute cycles is combining the previously created microtrips in such a manner as to form a substitute cycle, which will then conform to the relevant RDE criteria.

Theoretically, it would be possible to try out all variants (brute force method). Yet it becomes clear that in a complex data set with K microtrips and for the replacement cycle E necessary ones, K^E combinations emerge. For example, $K = 1000$ available and ordinary $E = 20$ for the RDE replacement cycle necessary microtrips as well as the assumption that an evaluation takes 0.1 seconds, a duration of $2.7 \cdot 10^{15}$ hours for the generation of the replacement cycle would be set. Thus a brute force approach for combination is not suitable (Ließner et al., 2017).

The RDE combinatorics outlined here is based on the combinatorics for generating average substitute cycles, as presented in (Ließner et al., 2017). In the mentioned paper, a method is presented which enables the combination of microtrips to form a substitute cycle by employing a so-called Threshold Accepting combinatorics algorithm. This procedure ensures that vehicle dynamic will be maximized.

Using an average substitute cycle as a basis for RDE combinatorics has multiple advantages. RDE certification has the fundamental aim of defining boundary conditions which conform as closely as possible to real driving situations. Thus, it is made possible that an average cycle, which must feature the respective duration and driving mode share, nearly or completely meets RDE standards from the beginning

on. The following methodology uses the average substitute cycle and modifies included shares until all criteria are met. The modification is performed in accordance with the Threshold Accepting Method first introduced in (Ließner et al., 2017). This is an algorithm presented by Prof. Dueck to solve combinatorial problems (Dueck and Scheuer, 1990).

Besides meeting RDE standards, one additional aim is to maximize vehicle dynamics. According to RDE guidelines, the vehicle dynamics limit is assessed by determining va_{pos_95} values for the respective area (urban, rural, motorway). These values represent, in dependence of average velocities, threshold values \bar{v}_k which must not be exceeded. The following equation illustrates the exemplary conditions for the urban area.

$$(v \cdot a_{pos})_{urban,ref_95} \leq (0.136 \cdot \bar{v}_{urban} + 14.44) \quad (3)$$

Where,

$(v \cdot a_{pos})_{urban,ref_95}$ = 95th percentile va_{pos} value,
 \bar{v}_{urban} = average urban speed

The respective intermediate steps and the algorithm for calculating va_{pos_95} can be found in (European Commission, 2016). Hence, minimizing the difference between the actual va_{pos_95} values and respective set values can be seen as a possible approach for the maximization of vehicle dynamics. The following function is a measure of quality for optimising va_{pos_95} values:

$$J = \sum_k w_k \cdot ((v \cdot a_{pos})_{k,ref_95} - (v \cdot a_{pos})_{k,cur_95}) \quad (4)$$

Where,

$k = \{\text{urban, rural, motorway}\}$,

$(v \cdot a_{pos})_{k,ref_95}$ = set value of 95th percentile,

$(v \cdot a_{pos})_{k,cur_95}$ = actual value of 95th percentile,

w_k = number of elements with $a_k > 0.1\text{m/s}^2$

The sampling of all courses, which have been used in the context of RDE, is generally 1 Hz. Since the route that is to be completed has the same length of at least 16 km in all speed ranges, it can be derived that the urban area exhibits the largest number of measured values and hence, also the largest number of va_{pos_95} values. This consideration helps to determine the weighting (equation 4). In this manner, it can be ensured that the optimization preferably covers the urban speed range. The weighting corresponds logically to emission layers as well, since frequently occurring partial loads, engine operation in the scavenging range and unfavourable regeneration conditions cause increased emissions in urban areas. The va_{pos_95} value is only one scalar that characterise the vehicle dynamic of the whole RDE cycle. It can be used to check the validity of a RDE cycle. If it is the aim to get the maximum possible vehicle dynamic in a cycle all the sampling points around va_{pos_95} need a high dynamic too. In the RDE context it means that all sampling points under the va_{pos_95} boundary, which is equivalent to 95 % of the elements with $a_k > 0.1$ in the respective area, should attain va_{pos} values as close as possible to the va_{pos_95} value. The other sampling points (5 %) should achieve the maximum possible dynamic of the respective vehicle.

Algorithm 1: RDE replacement cycle generation.

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1: Load average cycle
2: Choose initial THRESHOLD  $T > 0$ 
3: As long as improvement occurs
4:   Modify driving cycle slightly (change one microtrip)
5:   Calc.  $\Delta E := \text{quality}(\text{old conf.}) - \text{quality}(\text{new conf.})$ 
6:   If  $\Delta E > T$  & RDE-criteria are satisfied
7:     THEN old conf. := new conf.
8:     If for too many iterations no improvement
9:       THEN lower THRESHOLD  $T$ 
10:    If no further improvements are made
11:      THEN stop
12: End
calc. = calculate, config. = configuration

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As can be understood through pseudocode above, the combination process begins with an average substitute cycle featuring a length of 6300 seconds ± 14 % (line 1) which met the RDE trip duration between 90 and 120 minutes. Additionally, an initial threshold value $T > 0$ controlling the optimization process has been set for the optimization (line 2). In (Dueck and Scheuer, 1990), Prof. Dueck indicates that the threshold parameter does not react very sensitively to the solution quality and hence, does not have to be elaborately optimized. Consequently, the iterative modification of the initially assembled substitute cycle commences (line 3-12). During each respective loop run, the substitute cycle is slightly modified (line

4). This modification is achieved by randomly changing one of the E microtrips. Instead, a microtrip in the complex data set is randomly chosen and then put in. A subsequent evaluation of the quality assesses this modification. The modified substitute cycle is only adapted as a new reference, if the quality has improved for more than the given threshold value T , if the resulting cycle meets all RDE standards and the resulting cycle length corresponds to the predefined interval (line 5-7). By means of two further inner loops, an adaptation of the optimization process is achieved. On the one hand, the threshold value T is reduced after a certain number of inexpedient modifications (line 8-9), which incrementally reduces the subsequent cycle's demanded improvement as well. These consequences lead to the fact that only solutions performing substantially better can be adapted as a new reference solution. The demanded improvement is mitigated by the reduction of threshold values during the optimization process. But on the other hand, the optimization process will be terminated if after a large number of modifications, no further improvement has been achieved (line 10-11). This practice ensures that the optimization is only performed in correspondence to the achievement of improvements. Taking sample solutions during the optimization process is made possible by the iterative approach (Ließner et al., 2017).

3 RESULTS

This chapter illustrates various results of the generation of RDE replacement cycles which will be discussed in the following subsections. For the presentation of individual aspects two examples based on different data sets were prepared. The velocity curves are shown in Figure 4 and 5. Figure 4 represents real driving data. The cycle in Figure 5 was generated with NEDC velocity curves. With the help of the replacement cycle based on the NEDC data it can be demonstrated very well that the presented algorithm provide the desired results. The evaluation of the cycles from Figure 4 and 5 concerning to the RDE criteria is shown in Table 1. It compares the different characteristic values of the replacement cycles with the given values of the RDE legislation.

3.1 Fulfillment of the RDE Requirements

As is demonstrated in Table 1, all RDE standards concerning vehicle dynamics can be met. This is not only

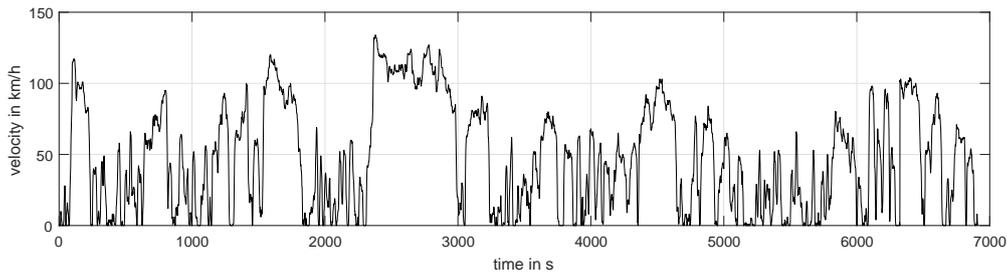


Figure 4: RDE replacement cycle based on real driving data.

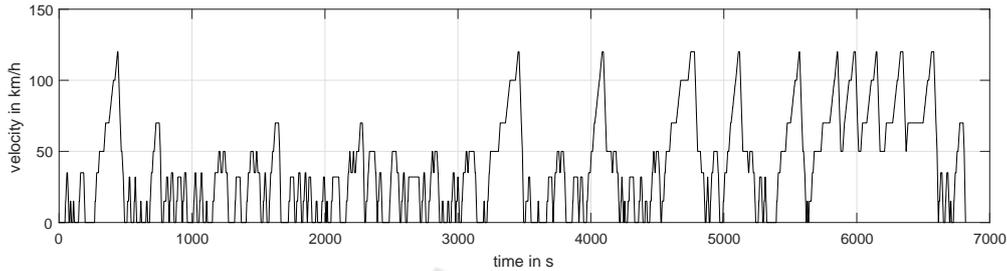


Figure 5: RDE replacement cycle based on NEDC data.

the case when applying the described cycle, but by using the outlined combinatorics, any desired number of RDE conforming cycles can be generated. The variability of usage cases is a great advantage because it presents a broad basis for the optimization and validation of RDE cycles.

3.2 Impact of the Data Basis

The requirement for a successful generation is a relevant database. Within the database, all areas (urban, rural, motorway) have to be included to a sufficient extent. When attempting to create a RDE cycle by using data sets which omits velocities above 90 km/h, this demand becomes much more transparent. In the case stated above, the missing velocity shares render a successful combinatorics impossible.

The respective route areas (urban, rural, motorway) are automatically determined by the employed combinatorics. The specific distribution is not affected in any way. The fundamental area distribution is shaped by the database's quality. In this context, the adherence to the permitted maximum velocity is not problematic. By screening microtrips with higher maximum velocities than 160 km/h out of the database beforehand, an expedient violation of this criterion can occur.

3.3 Distribution of Stoppages

Stoppages ($v = 0$) within the original data have a significant role for an expedient combinatorics. The cy-

cle parameters 'average v_{urban} ', 'stop ratio urban', as well as 'stops above 10 s' and 'stops above 180 s' underline this. These parameters will not be represented correctly in the substitute cycle if the stoppages do not have the relevant quality.

In contrast to selecting suitable shares with a velocity higher than zero, the generation of stoppages ($v = 0$) is much more simple. Since these stoppages only incorporate sequences of velocities of the value zero, additional editing is possible. If, for example, the criterion 'stop share urban' $> 10\%$ has not been met, the criterion can be fulfilled by manipulating the already existing stoppages during the post processing. However, the longest permitted driving duration must not be violated.

The calculation of the $va_{pos_{95}}$ and RPA values only consists of measurements with an acceleration of $a > 0.1m/s^2$. Nonetheless, the assessment for validating vehicle dynamics is based on the average velocities of each area. Regarding the urban area, stoppages are incorporated into the calculation as well.

3.4 Analysis of Replacement Cycles

The RDE substitute cycle, which is based on the NEDC data set (see Figure 5), evidently contains a great number of linear acceleration courses, such as can also be found in the NEDC. The maximum acceleration values are comparable to those found in the NEDC. This attribute is also very prominent in the NEDC and leads to the use of only a few engine operation points while driving the cycle. The cycle in

Table 1: Fullfillment of RDE requirements of the generated cycles.

Requirement	Unit	Set Value	RDE Cycle NEDC	RDE Cycle Real Driving
Trip duration	[min]	90-120	113.63	105.40
Urban operation	[%]	29-44	43.92	32.16
Rural operation	[%]	23-43	26.43	30.82
Motorway operation	[%]	23-43	29.65	37.01
Urban distance	[km]	≥ 16	32.00	26.20
Rural distance	[km]	≥ 16	19.25	25.11
Motorway distance	[km]	≥ 16	21.60	30.15
Maximum speed	[km/h]	145	120	134
Time $v > 100$ km/h	[min]	≥ 5	6.65	10.82
Average Speed urban	[km/h]	15-40 km/h	22.44	23.03
Stop ratio urban	[%]	≥ 10	28.44	13.06
Stop $t > 10$ s	[#]	≥ 2	51	16
va_{pos_95} urban	[m^2/s^3]	< 17.57	7.72	16.05
va_{pos_95} rural	[m^2/s^3]	< 24.51	10.03	24.23
va_{pos_95} motorway	[m^2/s^3]	< 26.87	9.10	26.39
RPA urban	[m/s^3]	> 0.14	0.14	0.28
RPA rural	[m/s^3]	> 0.06	0.11	0.15
RPA motorway	[m/s^3]	> 0.025	0.12	0.10

figure 5 ranks near the lower limit of the RPA value for urban and ranks thereby also near the lower limit of permitted vehicle dynamics. This effect highlights how much more dynamic the drives used for the RDE assessment will be compared to the ones that have been used so far.

In contrast to this, the driving cycle based on real drives (see Figure 4) represents a cycle ranked near the higher limit for driving dynamics. A specific maximum value for the va_{pos_95} values cannot be derived more easily, because the relevant reference value is calculated by using the average velocity. The discussed phenomena concerning the NEDC substitute cycle do not occur in this cycle, which is much more similar to a real drive when looking at the characteristics.

3.5 Worst Case Scenario

The worst-case cycle in connection to RDE driving cycles has already been a subject of discussion in the beginning of this paper. What is meant in this case is a set of driving cycles, which operate the assessed vehicle near the RDE validity limit. The objective has to be that passing the RDE assessment by using these cycles must ensure that all possible drives conforming to RDE will pass the assessment. It has already been discussed in section 2.1 that the paper at hand only gives a first, partial solution. Nevertheless, the emissions have to be maximized already during this partial step. One approach would be to increase the driving cycle's dynamics, which would also increase

the emissions regarding vehicles with a combustion engine.

Another determining factor for the increase of emissions during a driving cycle is the distribution of stoppages ($v = 0$). Especially the conditions in the exhaust gas aftertreatment system play an important role here. To give an example, it is well-known that a certain temperature in catalytic converters is a prerequisite for best executing the required reaction of reducing emissions. Should this not be the case, significantly higher values will occur during the cold start phase. Even extensive idle times during a drive can lead to similar effects. Such pauses should be implemented into the worst-case case in a manner which enables cooling processes to repeatedly cause comparable situations.

A worst case cycle should also contain high dynamic maneuvers in the cold start phase. The end of that cold start phase is defined by a temperature of the coolant over 70°C or after five minutes in the wording of the law. So most of the combustion engines especially in cold environments don't reach their normal operating temperature during this time.

4 SUMMARY AND OUTLOOK

In the contribution at hand, the possibility of generating RDE cycles from real driving data has been illustrated. By applying the outlined approach, specific substitute cycles for different types of vehicles

or markets, which can differentiate greatly in characteristics or velocity distribution, can be developed. The possibility of generating a large quantity of different cycles and using them during the development process makes the avoidance of phenomena such as cycle beating possible.

The main focus was not only on the generation of RDE cycles, but these cycles were also supposed to cause a larger quantity of emissions in the assessed vehicles. To meet the requirements in a first step, the maximum permitted dynamics in the contest of RDE ambient conditions were demanded. Additionally, considerations were presented which increase emissions due to the chosen sequence of operational conditions, regardless of strictly set requirements.

In the future, RDE cycles will play an important role in both the vehicle development and the assessment of vehicle emission values. At present, RDE drives are commonly selected on the basis of imprecise compilations of driving road criteria (see Table 1) and then traced on real roads. This procedure is on the one hand very time-consuming, and on the other hand does not guarantee that RDE requirements will be met, especially when unpredictable disruptions such as traffic jams occur. At some point, the recording of RDE measurement drives will predominantly take place on chassis dynamometers, since these can be adapted to be nearly identical to the chosen ambient conditions and can also follow the set velocity course precisely.

During vehicle development, the usage of simulations and procedures such as the model-based calibration will gain more importance due to RDE requirements. Even now, engineers face the challenge of constantly reducing development periods and steadily increasing numbers of vehicle variants while still using conventional development methods. Thus, the majority of calibration will be executed at the computer and at test benches of different complexity. A significant part of this practice will be various driving cycles, because they constitute the most realistic testing scenarios in vehicle development.

Further contributions can continue the gradual generation of RDE worst-case cycles (see Section 2.1). Correspondingly, an elevation profile will be overlaid in a next step, after which vehicle-specific cycles can be derived. Based on this procedure, one obtains a set of cycles representing the worst-case case concerning emissions for the respective vehicle.

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