Robust Statistical Prior Knowledge for Active Contours
Prior Knowledge for Active Contours

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Abstract: We propose in this paper a new method of active contours with statistical shape prior. The presented approach is able to manage situations where the prior knowledge on shape is unknown in advance and we have to construct it from the available training data. Given a set of several shape clusters, we use a set of complete, stable and invariants shape descriptors to represent shape. A Linear Discriminant Analysis (LDA), based on Patrick-Fischer criterion, is then applied to form a distinct clusters in a low dimensional feature subspace. Feature distribution is estimated using an Estimation-Maximization (EM) algorithm. Having a currently detected front, a Bayesian classifier is used to assign it to the most probable shape cluster. Prior knowledge is then constructed based on it's statistical properties. The shape prior is then incorporated into a level set based active contours to have satisfactory segmentation results in presence of partial occlusion, low contrast and noise.

1 INTRODUCTION

Active contour methods have been introduced in 1988 (M. Kass and Terzopolous, 1988). The principle consists in moving a curve iteratively minimizing energy functional. The minimum is reached at object boundaries. Active contour methods can be classified into two families: parametric and geometric active contours. The first family, called also snakes, uses an explicit representation of the contours and depends only on image gradient to detect objects. The second one uses an implicit representation of the contours by level set approach to handle topological changes of the front. A number of active contour models based on level set theory have been then proposed which can be divided into two categories : The boundary-based approach which depends on an edge stopping function to detect objects (Malladi and Vemuri, 1995; V. Caselles and Sapiro, 1997) and the region-based approach which is based on minimizing an energy functionnal to segment objects in the image (T.F.Chan and L.A.Vese, 2001). Experiments show that region-based models can detect objects with smooth boundaries and noise since the whole region is explored. However, there is still no way to characterize the global shape of an object. Especially in presence of occlusions and clutter, all the previous models converge to the wrong contours.

To solve the above mentioned problems, different attempts include shape prior information into the active contour models. Many works have been proposed which can be classified into statistical or geometric shape prior. (M. Leventon and Faugeras, 2000), associated a statistical shape model to the geodesic active contours (V. Caselles and Sapiro, 1997). A set of training shapes is used to define a Gaussian distribution over shapes. At each step of the surface evolution, the maximum a posteriori position and shape are estimated and used to move globally the surface while local evolution is based on image gradient and curvature. (Chen and Geiser, 2001) defined an energy functional based on the quadratic distance between the evolving curve and the average shapes of the target object after alignment. This term is then incorporated into the geodesic active contours.

(Fang and Chan, 2007) introduced a statistical shape prior into geodesic active contour to detect partially occluded object. PCA is computed on level set functions used as training data and the set of points in subspace is approximated by a Gaussian function to construct the shape prior model. To speed up the algorithm, an explicit alignment of shape prior model and the current evolving curve is done to calculate pose parameters unlike (M. Leventon and Faugeras, 2000) where a MAP of pose is performed.

(M.A. Charmi and Ghorbel, 2008) introduced a

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new geometric shape prior into the snake model. A set of complete and locally stable invariants to Euclidean transformations (Ghorbel, 1998) is used to define new force which makes the snake overcome some well-known problems. In (M-A. Mezghich, 2013), a new geometric shape prior for a region-based active contour (T.F.Chan and L.A.Vese, 2001) was defined which is based on shape registration by phase correlation using Fourier Transform. The method presented encouraging segmentation results in presence of partial occlusion, cluder and noise under rigid transformation. Similar work was presented in (M-A. Mezghich and F.Ghorbel, 2014) for an edge based active contours (Malladi and Vemuri, 1995) to help the curve reach the true contours of the object of interest.

For all the above presented approaches, the shape or the model of reference is known in advance. To generalize the idea of shape prior to more complex situations where many models of reference are available and we have to choose to most suitable one, some works have been presented.

In (Fang and Chan, 2006) a statistical shape prior model is presented to give more robustness to object detection. This shape prior is able to manage different states of the same object, thus a Gaussian Mixture Model (GMM) and a Bayesian classifier framework are used. Using the level set functions for representing shape, this model suffer from the curse of dimensionality. Hence PCA was used to perform dimensionality reduction.

In (A.Foulonneau and Heitz, 2009) a multi-references shape prior is presented for a region-based active contours. Prior knowledge is defined as a distance between shape descriptors based on the Legendre moments of the characteristic function of many available shapes.

In this paper, we focus on extending the work presented in (I. Sakly and F.Ghorbel, 2016) that construct a statistical shape prior from a given single cluster of similar shapes according to the object to be detected. Inspired from the paper of (Tsai A, 2005), in which the EM algorithm was used for shape classification into different clusters based on level set representation, we propose to represent the available training data by an invariant set of complete shape descriptors. Then a dimensionality reduction will be performed based on LDA to have a separated shape clusters that respect to Patrick-Fischer criterion. For each cluster, we computed a statistical map to be used as shape prior. In the reduced subspace, the EM algorithm will be applied to estimated data distribution. For the current evolving curve, we use a Bayesian classifier to assign it to the most probable cluster. The improved model can retain all the advantages of level set based model and have the additional ability of being able to handle the case of multi-reference shape knowledge in presence of partial occlusions.

The remainder of this paper is organized as follows : In Section 2, we recall the principle of level set based active contour models. Then, the construction of a multi-reference shape prior constraint will be presented in Section 3. The incorporation of shape prior and the evolving schema will be presented in Section 4. Some experimental results are presented in Section 5. Finally, we conclude the work and highlight some possible perspectives in Section 6.

2 LEVEL SET BASED ACTIVE CONTOURS

The basic idea of the Level Set approach (Malladi and Vemuri, 1995) is to consider the initial contour as the zero level set of a higher dimension function called level set function and following the evolution of this embedding function, we deduce the contour evolution by seeking its zero level set at each iteration. Several models have been proposed in literature that we can classify into edge-based or region-based active contours. In (Malladi and Vemuri, 1995), the authors proposed the basic level set model which is based on an edge stopping function g. The evolutions equation of the level set function ϕ is

$$\phi^{n+1}(x,y) = \phi^n(x,y) + \Delta t g(x,y) F(x,y) |\nabla \phi^n(x,y)|, \tag{1}$$

F is a speed function of the form $F = F_0 + F_1(K)$ where F_0 is a constant advection term equals to (± 1) depends of the object inside or outside the initial contour. The second term is of the form $-\varepsilon K$ where K is the curvature at any point and $\varepsilon > 0$.

$$g(x,y) = \frac{1}{1+|\nabla G_\sigma * f(x,y)|}, \tag{2}$$

where f is the image and G_σ is a Gaussian filter with a deviation equals to σ. This stopping function has values that are closer to zero in regions of high image gradient and values that are closer to unity in regions with relatively constant intensity.

It's obvious that for this model, the evolution is based on the image gradient. That's why, this model leads to unsatisfactory results in presence of occlusions, low contrast and even noise.

3 SHAPE PRIOR FORMULATION

We will devote this section to present our multi-refrences shape prior constraint to be added to a level set based active contours. Our algorithm is composed of two steps:

1− An off-line step which consists in representing the training data by an invariants set of shape descriptors instead of using level set to avoid data alignment which is a hard task to perform for all the data. Then, the estimation of data distribution over different clusters in a reduced subspace using the EM algorithm after performing the LDA method.

2− An on-line step which consists in assigning the evolving front to the most probable cluster based on a Bayesian classifier.

3.1 Shape Description using Invariant Descriptors

Given a training data, we perform a level set segmentation of each object to determine the curve that represents the shape of the object of interest. Then we compute an invariants set of shape description introduced in (Ghorbel, 1998) as follows:

$$\begin{cases} I_{k_0}(\gamma) = |C_{k_0}(\gamma)|, \quad C_{k_0}(\gamma) \neq 0 \\ I_{k_1}(\gamma) = |C_{k_1}(\gamma)|, \quad C_{k_1}(\gamma) \neq 0, \quad k_1 \neq k_0 \\ I_k(\gamma) = \frac{C_k(\gamma)^{k_0-k_1} C_{k_0}(\gamma)^{k-k_1} C_{k_1}(\gamma)^{k_0-k}}{I_{k_0}^{k-k_1-p}(\gamma) I_{k_1}^{k_0-k-q}(\gamma)}, k \neq k_0, k_1 \;; \\ p, q > 0, \end{cases}$$

This set is complete, stable and invariant to rigid transformations.The stability criterion expresses the fact that a small distortion of the shape does not induce a noticeable divergence. This property makes invariant descriptors robust under small shape variations. To compare shapes, we used the following distance:

$$d(\gamma_1, \gamma_2) = \sum_k (|I_k(\gamma_1) - I_k(\gamma_2)|^{\frac{1}{2}})^2 \tag{3}$$

After this step, we obtain an invariants representation of the training data.

3.2 Dimensionality Reduction using Linear Discriminant Analysis

The second step of our approach consists on performing dimensionality reduction of data represented by the previous invariants features. We were based on the Fisher Linear Discriminant Analysis (FLDA). This method intends to reduce the dimension, so that in the new space, the between class distances are maximized while the within class distances are minimizing. To that purpose, FLDA considers searching for orthogonal linear projection matrix w that maximizes the following so called Fischer optimization criterion see (Fukunaga, 1990) and (Ghorbel and la Tocnaye, 1990):

$$J(w) = \frac{tr(w^T S_b w)}{tr(w^T S_w w)} \tag{4}$$

S_w is the within class scatter matrix and S_b is the between class scatter one. They are given by

$$S_w = \sum_{k=1}^{c} \pi_k E_k[(X-\mu_k)(X-\mu_k)^T] \tag{5}$$

$$S_b = \sum_{k=1}^{c} \pi_k (\mu_k - \mu)(\mu_k - \mu)^T \tag{6}$$

where μ_k=$E_k[X]$ is the conditional expectation of the multidimensional random vector X given the class k, μ corresponds to the mean vector over all classes and π_k denote the prior probability of the k^{th} class.

Because its not practical to find an analytical solution w that verify the J criteria, one possible suboptimal solution is to choose w formed by the d first eigen vectors of $S_w^{-1} S_b$ those correspond to the d largest eigen values. After computation of w, the FLDA method proceeds to the projection of the original data into the reduced space spanned by the vectors of w unlike the work of (Fang and Chan, 2006), where Principal Component Analysis (PCA) was used to perform data projection.

3.3 The EM Algorithm for Data Distribution Estimation

The EM algorithm proposed by (Dempster, 1977) is a powerful iterative technique suited for calculating the maximum-likelihood (ML) estimates in problems where parts of the data are missing. The missing data in our EM formulation is the class labels K. If the class labels for the different shapes within the database are known, then we can determine for which cluster belongs the current evolving contour. The method opts for estimating the probability densities of any mixture by a usual law while approaching as closely as possible the actual distribution of the initial mixture. In other words, the EM algorithm opts to maximize the likelihood between the probability density and the histogram of the initial mixture.

The observed data in our EM formulation is X that corresponds to the collection of data obtained after FLDA reduction. Finally, Y is the class label of each

feature to be estimated in our formulation. Let:
$\pi_k = P[Y = k]$ the prior probability,
$\pi_k^x = P[Y = k/X = x]$ the posterior probability,
$f_k(x) = P[X = x/Y = k]$ the conditional probability,
$f_X(x) = \sum_{k=1}^{K} f_k(x)$.
After estimating the value of π_k and f_k using the *EM* algorithm, we can deduce the class of each element based on Bayes rule :

$$K(x) = \arg(\max_k(\pi_k^x)) = \arg(\max_k(\pi_k f_k(x))) \quad (7)$$

4 ACTIVE CONTOURS WITH SHAPE PRIOR

In (M-A. Mezghich and F.Ghorbel, 2014) a new way to introduce shape prior to the presented level set model in section 2. The idea is to define a new stopping function that update the evolving level set function in the region of variability between the active contour and the reference until convergence is obtained.

$$g_{shape}(x,y) = \begin{cases} 0, \ if \phi_{prod}(x,y) >= 0, \\ sign(\phi_{ref}(x,y)), else, \end{cases} \quad (8)$$

where $\phi_{prod}(x,y) = \phi(x,y) \cdot \phi_{ref}(x,y)$, ϕ is the level set function associated to the evolving contour, while ϕ_{ref} is the level set function associated to the shape of reference after alignment. As it can be seen, the new proposed stopping function only allows for updating the level set function in the regions of variability between shapes. In these regions g_{shape} is either 1 or -1 because in the case of partial occlusions, the function is equals to 1 in order to push the edge inward (deflate) and in case of missing parts, this function is equals to -1 to push the contour towards the outside (inflate). In our work, the direction of evolution is handled automatically based on the sign of ϕ_{ref}. The total discrete evolution's equation that we propose is as follows

$$\frac{\phi^{n+1}(i,j)-\phi^{n}(i,j)}{\Delta t} = ((1-w)\, g(i,j) + w\, g_{shape}(i,j))F(i,j)|\nabla\phi^n(i,j)|, \quad (9)$$

w is a constant weighting factor between the image-based force and knowledge-driven force, generally chosen > 0.5 to promote the evolution towards the reference shape.
We generalized the approach in the recent work (I. Sakly and F.Ghorbel, 2016) for the case of several references of the same shape. In fact, in some fields like in medical context, generally the prior information on shape is obtained from a training data. For this reason, we propose a dynamic weighting term w which takes into account the statistical properties of the given cluster of shapes. Hence, the evolving contour will converge to the most probable front.

For each pixel $x(i,j)$ of the image, we will count its degree of belonging to the target object, i.e. the number of times over all the training set, the pixel belongs to the target shape. Then we will name it $w(i,j)$.

Hence, to incorporate a prior knowledge based on a set of similar shapes, the proposed model is:

$$\frac{\phi^{n+1}(i,j)-\phi^{n}(i,j)}{\Delta t} = ((1-w(i,j))\, g(i,j) + w(i,j)\, g_{shape}(i,j))F(i,j)|\nabla\phi^n(i,j)|, \quad (10)$$

5 EXPERIMENTAL RESULTS

We will start this section by describing our algorithm, then we will comment the obtained results on simulated and real data.

5.1 The Approach's Algorithm

The proposed approach is composed of two steps as follows:

Algorithm 1: Multi-references shape prior for active contour.

1: **Off-Line step**
2: For each cluster C_k of similar shapes (represented by level set function ϕ), we compute it's statistical map w_k.
3: We compute the invariants set of shape descriptors for all the training data as presented in section 3.1.
4: We perform the FLDA method on the obtained invariants description of shapes.
5: We estimate the data distribution over different clusters in a reduced subspace using the EM algorithm.
6: **On-Line step**
7: Apply the classic level set based active contour until the evolving contour became stable (equation 1).
8: Assign the detected front to the most similar cluster C_k based on Bayes classifier (equation 7).
9: Continue the evolution based on both the statistical map w_k of the selected cluster that represent the shape prior and the image data represented by the gradient(equation 10).

5.2 Application to Simulated Data

We consider a mixture of 3 clusters generated by a multivariate normal distribution $N_k(\mu_k, \sigma_k)$; $k \in \{1,2,3\}$. Each cluster C_k is composed of 1000 items and each item has 10 dimensions. The figure below shows data distribution in the reduced space formed by the two largest eigenvectors.

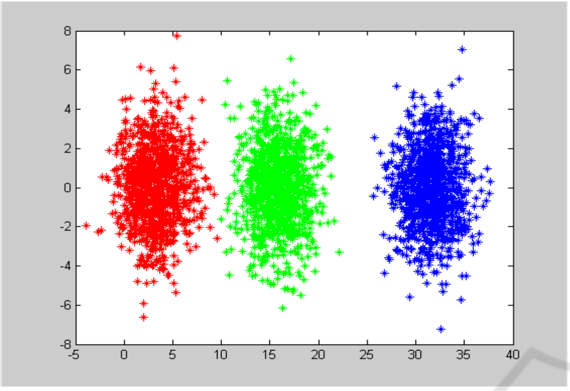

Figure 1: Data distribution in the reduced features space.

As it can be seen, the clusters are well separated in the reduced subspace. Performing an EM classification of this data, we obtain 1 misclassified element from the available 3000 items. The errors is about 1/3000

5.3 Application to the Segmentation Problem

For the experiment below, we consider 3 different clusters of shapes (triangle, plus and circle). Given that the invariant descriptors presented in section 3.1 are complex numbers, i.e each descriptor $I_k = r_k e^{i\theta_k}$, we assign for each shape a characteristic vector $U = [r_1..r_n\theta_1..\theta n]^T$. The FLDA will be applied on the N characteristic vectors U.

In fig.2, the first, second and fourth columns represent the segmentation result without prior information. The third and last ones represent the segmentation result using the proposed approach. This result improves the robustness of the segmentation process in presence of missing parts and partial occlusions of the target objects. Similar experiment was performed on the $MPEG-7$ shape data base (link : http://www.cis.temple.edu/ latecki/TestData/mpeg7shapeB.tar.gz). Some shapes of this data base are presented in fig.3.

We consider two classes of shapes fig.4. For each class, we take 20 examples for learning. The descriptors distribution according to the first two axes ares presented in fig.5.

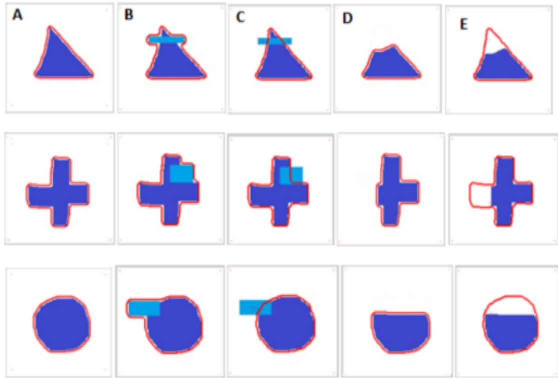

Figure 2: Comparison of the segmentation results of (A,B,D) traditional active contour without using shape prior model and (C,E) our proposed method using shape prior model for partially occluded objects.

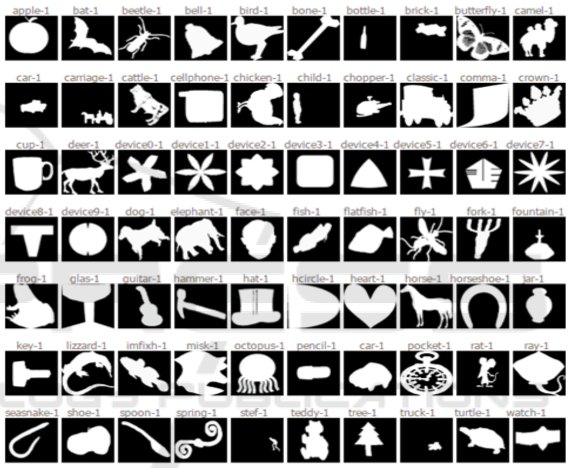

Figure 3: A selected set of MPEG7 data base.

Figure 4: A selected two shape classes (a Cup and a Jar).

Segmentation results with the proposed model are show in fig.6.

6 CONCLUSIONS

A novel method of level set based active contours with statistical shape prior is presented in this paper. An in-

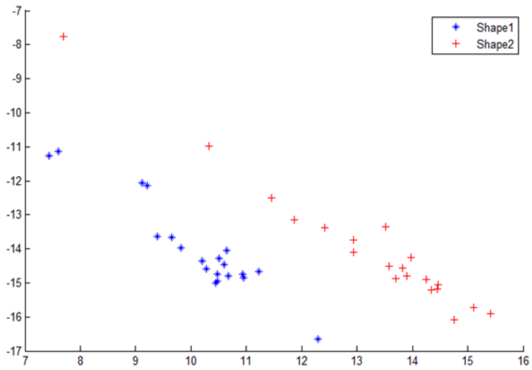

Figure 5: Features distribution in the reduced subspace.

Figure 6: Segmentation results without shape prior (first row) and with shape prior (second row).

variant and complete set of shape descriptors is used to represent the training data. Then a Linear Discriminant Analysis (LDA) is applied to form a separated shape clusters in a low dimensional feature subspace. An EM algorithm is then performed to estimate the data distribution. Given an evolving curve, we compute it's set of shape descriptors then we assign it to the most similar cluster based on a Bayesian classifier. The prior knowledge is obtained from the statistical map of that cluster. In the subsequent curve evolution, our model will depends on both data and prior knowledge to recover the true contour of the object of interest.

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