

Optimal Control of Plug-in Hybrid Electric Vehicle based on Pontryagin's Minimum Principle Considering Driver's Characteristic

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Abstract: In this study, an optimal control was investigated for a power split type plug-in hybrid electric vehicle (PHEV) considering the driver's characteristic. Using the dynamic model of the PHEV powertrain, Hamiltonian was defined and the optimal co-state was obtained for Pontryagin's minimum principle (PMP) control. The PMP control was performed for a normal driver who was selected based on extended driving style questionnaire (EDSQ), and the battery SOC behaviour and equivalent fuel economy were evaluated. It was found that the equivalent fuel economy by the PMP control is improved compared with the existing charge depleting/charge sustaining (CD/CS) control and the battery SOC decreased faster as the sportiness of the driver increased.

1 INTRODUCTION

In plug-in hybrid electric vehicle (PHEV) which uses the internal combustion engine and motors, the power distribution between the engine and motors has a great influence on the vehicle fuel economy (Zhang and Vahidi, 2012). As a PHEV management strategy, charge depleting (CD)/charge sustaining (CS) control is generally used. In CD mode, the vehicle is propelled only using the electric energy until the battery SOC reaches to the lower limit. This region is called, "All Electric Range (AER)". After AER, the vehicle is operated in CS mode using the engine and motor to sustain the SOC. The CD/CS control may reduce fuel economy because the engine has to be operated even at low efficiency to maintain the SOC (Jeong et al., 2016).

To overcome the disadvantage of the CD/CS control, two types of approach have been used: (1) rule based control and (2) optimal control.

Rule based control distributes the power by the rule obtained in advance, using the state of charge (SOC) of the battery (Sigmund et al., 2014) or wheel power demand (Pi, 2016). The rule based control has an advantage to apply to the vehicle in real time. However, it is heuristic and not optimal. For the optimal control, dynamic programming (DP) (Wang et al., 2015), equivalent consumption minimization

strategies (ECMS) (Gao et al., 2017), Pontryagin's minimum principle (PMP) (Kim, 2011) were used.

DP provides a global optimal solution (Chen et al., 2014). However, it only provides the optimal results for the given route and cannot guarantee the optimal results when driving cycle is changed. In addition, it is hardly implementable in real time (Karbowski et al., 2013). Due to these limitations, DP has been used to estimate the maximum potential of a given PHEV configuration (Peng et al., 2017). ECMS and PMP can be used in real time control since local optimization is performed at every time step. However, they cannot guarantee the global optimization when the constraints such as the final SOC are not satisfied (Kim, 2011). To implement the optimal control using ECMS or PMP, it is important to estimate the optimization variables such as equivalent factor, co-state that satisfy the constraints (Wei et al., 2016).

To obtain the appropriate optimization variables, studies to predict a velocity profile were performed using Markov chain (Du et al., 2016) and neural network (Murphey et al., 2013). However, it is very hard to predict the exact velocity profile due to uncertain disturbances (Karbowski et al., 2014). Furthermore, actual driving velocity can be varied depending on the driver's characteristic.

Furthermore, in actual driving, the fuel economy varies depending on the driver's characteristic, even if the optimal control is performed (Lee et al., 2015).

In this study, an optimal control was performed in real time using PMP. The PMP control was applied using the optimal co-state that was obtained for a driver who has normal driving style. PMP control performance was investigated for various drivers using the same optimal co-state and the battery SOC behaviour was evaluated with regard to the driving style.

2 MODEL OF PLUG IN HYBRID ELECTRIC VEHICLE

2.1 Vehicle Model

In Figure 1, the target PHEV is shown. In this study, Toyota Prius III were selected as a target PHEV. The target PHEV consists of one engine, two motor/generators (MGs) and two planetary gears. The engine is connected to the carrier of the planetary gear 1. The engine operation is controlled by MG1, which is connected to the sun gear of the planetary gear 1. The PHEV can provide two operating modes: (1) EV and (2) HEV.

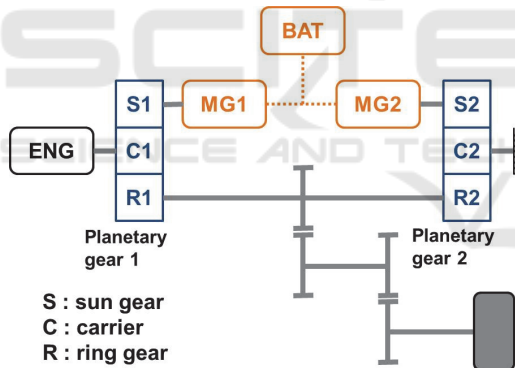


Figure 1: Target PHEV configuration.

Table 1: Specifications of target vehicle.

| Power split type | | Specifications |
|------------------|----------------|----------------|
| Engine | Max power(kW) | 73 |
| | Max torque(Nm) | 142 |
| MG1 | Max power(kW) | 42 |
| | Max torque(Nm) | 153.4 |
| MG2 | Max power(kW) | 60 |
| | Max torque(Nm) | 207 |
| Battery | Max power(kW) | 27 |
| | Capacity(kWh) | 4.4 |
| Vehicle | Mass(kg) | 1600 |
| | Tire radius(m) | 0.317 |

In Table 1, the vehicle specifications are shown.

From the lever analysis of the PHEV configuration in Figure 1, the torque and speed equations were derived as follows:

$$\begin{bmatrix} T_{MG1} \\ T_{MG2} \end{bmatrix} = \frac{Z_{S1}}{Z_{R1} + Z_{S1}} \begin{bmatrix} 0 & 1 \\ (Z_{R1} + Z_{S1}) \cdot \frac{Z_{S2}}{Z_{R2}} & \frac{Z_{R1}}{Z_{S1}} \cdot \frac{Z_{S2}}{Z_{R2}} \end{bmatrix} \begin{bmatrix} \frac{1}{N_{FGR}} T_{req} \\ T_{eng} \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} \omega_{MG1} \\ \omega_{MG2} \end{bmatrix} = \begin{bmatrix} -\frac{Z_{R1}}{Z_{S1}} & (\frac{Z_{R1}}{Z_{S1}} + 1) \\ 0 & \frac{Z_{R2}}{Z_{S2}} \end{bmatrix} \begin{bmatrix} N_{FGR} \cdot \omega_{req} \\ \omega_{eng} \end{bmatrix} \quad (2)$$

where T is the torque, ω is the speed, Z is the gear teeth number and N is the gear ratio. The subscripts $MG1$, $MG2$, $S1$, $R1$, $S2$, $R2$, FGR , req , and eng represent $MG1$, $MG2$, sun gear1, ring gear1, sun gear2, ring gear2, final reduction gear, required, and engine, respectively. The required battery power can be calculated as follows:

$$P_{bat} = \eta_1^k \cdot T_{MG1} \cdot \omega_{MG1} + \eta_2^k \cdot T_{MG2} \cdot \omega_{MG2} \quad (3)$$

where η_1 is the efficiency of $MG1$, η_2 is the efficiency of $MG2$. And k is defined as follows:

$$k = \begin{cases} 1 & \text{(generating)} \\ -1 & \text{(motoring)} \end{cases} \quad (4)$$

Finally, the required battery power is considered as a function of the required torque, required speed, the engine torque, and engine speed.

$$P_{bat} = (T_{req}, \omega_{req}, T_{eng}, \omega_{eng}) \quad (5)$$

3 APPLICATION OF OPTIMAL CONTROL

3.1 Pontryagin's Minimum Principle (PMP)

In this study, an optimal control based on Pontryagin's minimum principle (PMP) was used. PMP is a control method to minimize Hamiltonian at each time step. Hamiltonian was defined as,

$$H = \dot{m}(P_{bat}(t)) + \lambda \cdot \dot{SOC}(SOC, P_{bat}(t)) \quad (6)$$

where \dot{m} is the cost function which is the rate of fuel consumption, \dot{SOC} is the state function which is the rate of SOC, P_{bat} is the control variable which is the required power of the battery, and λ is co-state of PMP.

The rate of fuel consumption can be obtained from the fuel consumption map for the given engine torque and speed as,

$$\dot{m} = f(T_{eng}, \omega_{eng}) \quad (7)$$

From Equation (5), the required battery power can be calculated using the torque and speed of the engine.

$$P_{bat} = (T_{eng}, \omega_{eng}) \quad (8)$$

Hence, we can determine the rate of fuel consumption, \dot{m} as the function of the battery power.

$$\dot{m} = f(P_{bat}) \quad (9)$$

In this study, Coulomb counting method was used to estimate the battery SOC (Chang, 2013),

$$SOC(t) = SOC_{init} - \frac{1}{Q_{bat}} \int_{t_0}^t I dt \quad (10)$$

where SOC is the battery SOC, Q is the capacity, I is the current. The subscripts $init$, bat , t , and t_0 represent the initial, battery, current time, initial time, respectively.

From Equation (10), \dot{SOC} can be obtained as follows:

$$\dot{SOC}(t) = -\frac{I(t)}{Q_{bat}} \quad (11)$$

The battery current can be expressed as,

$$I = \frac{P_{bat}}{V_{bat}} \quad (12)$$

where P is the power, V is the voltage.

The battery voltage can be calculated using the open circuit voltage and internal resistance as,

$$V_{bat} = V_{oc} - R_{int}I \quad (13)$$

where R is the resistance. The subscripts oc , int represent the open circuit, internal, respectively.

From Equation (11), (12), (13), \dot{SOC} can be represented as follows:

$$\dot{SOC} = -\frac{1}{Q_{bat}} \frac{V_{oc} - \sqrt{V_{oc}^2 - 4R_{int}P_{bat}}}{2R_{int}} \quad (14)$$

Assuming that the open circuit voltage, V_{oc} and internal resistance, R_{int} of the battery are the function of SOC, time derivative of SOC is represented as

$$\dot{SOC} = g(SOC, P_{bat}(t)) \quad (15)$$

In battery model, since V_{oc} and R_{int} do not change much in usable SOC range, state function, \dot{SOC} can be obtained as only a function of P_{bat} .

$$\dot{SOC} = g(SOC, P_{bat}) \cong g(P_{bat}) \quad (16)$$

From Equation (16), \dot{SOC} is a function of P_{bat} , which is independent of SOC. Therefore, time derivative of co-state is zero.

$$\dot{\lambda} = -\lambda \frac{\delta g}{\delta(SOC)} = 0 \quad (17)$$

From Equation (17), it is seen that co-state, λ is constant.

4 DRIVING DATA COLLECECTION

4.1 Driver Selection

Through extended driving style questionnaire (EDSQ) (Lajunen, 2004), various drivers who have different driving style were selected. In Table 2, drivers' EDSQ score are shown. Based on the EDSQ score of the selected driver, driving style was defined as Sporty, Normal and Eco.

Table 2: EDSQ score of drivers.

| Driver# | EDSQ Score | Driving Style |
|----------|------------|---------------|
| Driver 1 | 58 | Sporty |
| Driver 2 | 49 | Sporty |
| Driver 3 | 35 | Normal |
| Driver 4 | 32 | Eco |
| Driver 5 | 20 | Eco |

4.2 Route Selection

We chose a route which includes various road styles such as city, highway and slope ways. Total distance of the route is 12km and it takes about 20~30 minutes. To reflect more accurate driving styles of each driver, the driving data were collected by GPS at 10~12 in the morning which can avoid other disturbances such as high traffic congestion.



Figure 2: Selected route (12km).

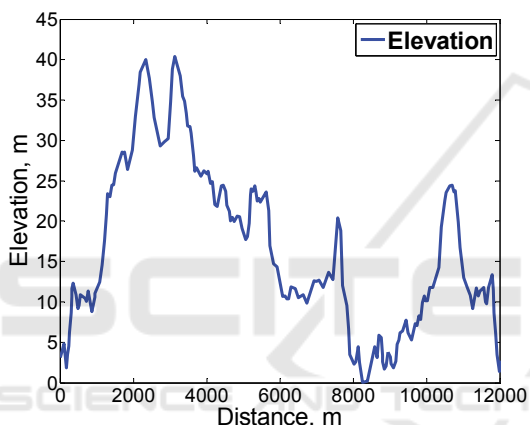


Figure 3: Elevation of the route.

5 SIMULATION RESULTS

5.1 Comparison of PMP and CD/CS

First, simulation was performed for Driver 3 who has ‘Normal’ driving style based on EDSQ. The optimal co-state was obtained for Driver 3 using the shooting method.

Since all electric range (AER) of the target PHEV is 23.4km, the SOC behaviour was investigated when the vehicle drove the selected route (Figure 2) two times, which is 24km. In the simulation, the initial and final SOC were set as 0.6 and 0.3, respectively.

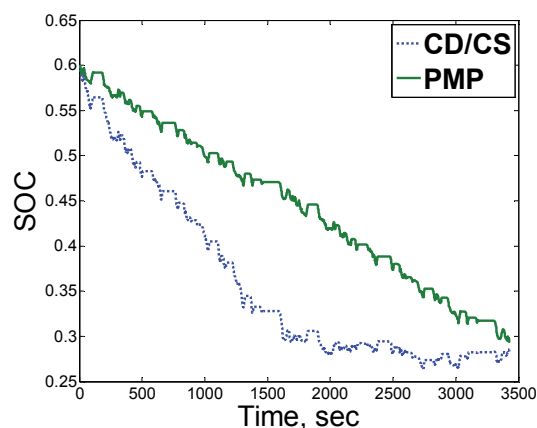


Figure 4: Battery SOC for driver 3.

Table 3: Results of PMP and CD/CS control.

| Control | PMP | CD/CS |
|--------------------------------|---------|--------|
| Fuel consumption (kg) | 0.2711 | 0.6756 |
| Equivalent fuel economy (km/l) | 27.64 | 25.31 |
| Improvement (%) | 9.21 | |
| Co-state (λ) | -1.2484 | |

In Figure 4, the simulation result of the battery SOC by PMP control was compared with the existing CD/CS control. In CD/CS control, the vehicle was driven in EV mode using the electric energy until the battery SOC reached to 0.3. After that, the vehicle was operated using the engine and motor for the SOC balancing.

It is seen that the battery SOC by PMP control decreased slowly. On the other hand, the battery SOC by CD/CS control decreased rapidly in CD mode and was maintained by the SOC balancing in CS mode.

In Table 3, the simulation results were compared. It is seen that the equivalent fuel economy of the PMP control is 27.64km/l, which is improved by 9.21% compared with that of the existing CD/CS control.

5.2 PMP Control for Various Drivers using the Same Co-State

Now, PMP control was applied to various drivers using the same co-state ($\lambda=-1.2484$) that was obtained for Driver 3.

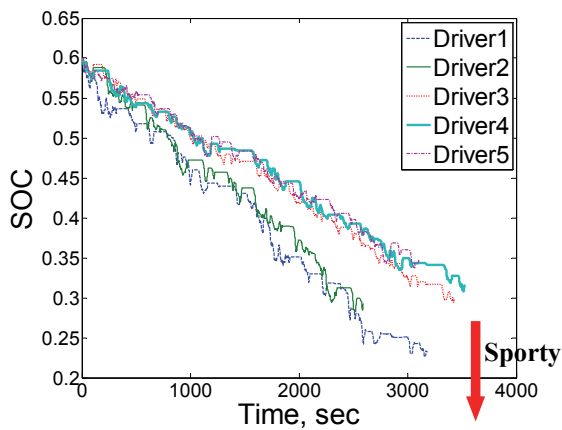


Figure 5: Battery SOC for various drivers.

Table 4: Results of PMP control for various drivers using the same co-state.

| Driver # | Final SOC | EDSQ Score | Pearson correlation |
|----------|-----------|------------|---------------------|
| Driver 1 | 0.2322 | 58 | -0.9418 |
| Driver 2 | 0.2930 | 49 | |
| Driver 3 | 0.2979 | 35 | |
| Driver 4 | 0.3156 | 32 | |
| Driver 5 | 0.3479 | 20 | |

Simulation results are shown in Figure 5. It is noted that the SOC decreased faster as the sportiness of the driver increased.

In Table 4, the simulation results are compared for five drivers. The final SOC decreased as EDSQ score increased, in other words, the sportiness of the driver increased. This can be proved by Pearson correlation value, -0.9418, which shows strong negative relationship. It is also noted that the final SOC showed some difference from the target final SOC, 0.3.

To meet the SOC constraints, when the PHEV is operated by a sporty driver, the engine needs to be turned on more often to charge the battery meanwhile the battery has to be used more often for an eco-driver. Since the co-state was obtained for Driver 3, this co-state cannot satisfy the SOC constraints when the vehicle is driven by the driver with different characteristic.

This implies that the optimal co-state needs to be determined by considering the driver's characteristic as well as the vehicle speed profile.

6 CONCLUSIONS

An optimal control was investigated for a power split type PHEV considering the driver's characteristic. To apply the optimal control, dynamic equations of the target PHEV powertrain were obtained and Hamiltonian was defined as a function of the rate of fuel consumption and the rate of the battery SOC. Representing the rate of SOC as a function of the battery power, the optimal co-state was obtained for Pontryagin's minimum principle (PMP) control.

Driving data were collected for the selected route which includes city, highway and slope ways. In addition, driving style was defined as Sporty, Normal and Eco based on EDSQ.

The PMP control was performed for the normal driver using the optimal co-state obtained. It was found from the simulation that the equivalent fuel economy by PMP control is improved by 9.21% compared with the existing CD/CS control. It was also found that the battery SOC by the PMP control decreased faster as the sportiness of the driver increased when the same co-state was applied for various drivers. It was found that the optimal co-state needs to be determined by considering the driver's characteristic as well as the vehicle speed profile.

For future works, a correlation between driver characteristic and optimal co-state will be obtained and an algorithm to find out the optimal co-state for a selected velocity profile will be investigated considering the correlation factors.

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