Color Edge Detection using Quaternion Convolution and Vector Gradient

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Abstract: In this paper, a quaternion-based method is proposed for color image edge detection. A pair of quaternion mask is used for horizontal and vertical filter since quaternion convolution is not commutative. The detection procedure consists of two steps: quaternion convolution for edge detection and gradient vector to enhance edge structures. Experimental results demonstrate its capabilities on natural color images.

1 INTRODUCTION

Color image segmentation remains a challenge area in computer vision. The problem consists in the choice of the method according to which a color image will be segmented. The techniques used to solve the problem of color images edge detection are classified into two categories: monochromatic-based techniques and vector-valued techniques (Koschan and Abidi, 2008).

The first category applies the existed gray-level segmentation methods by dealing out each color-channel (red, green, and blue) separately and then combines them to obtain a final segmentation result.

The second category defines discontinuity of the chromatic information as a vector-valued and processes the three color channels simultaneously (Pei and Cheng, 1997; Denis and al., 2007).

Recently, a new technique named quaternion has been employed to represent color images (Sangwine, 1996).

Quaternion extend the Fourier transform representation to hypercomplex numbers that can be used to encode color. Consequently, a color image will be represented as a matrix of quaternion having the same dimension and the color information described by the three components in the RGB color space will be represented in the imaginary part of the quaternion. Quaternion algebra for color image was first used in color image processing by Sangwine and Pei (Pei and Cheng, 1997), (Sangwine, 1996); then there have been several applications using quaternion in color image processing, such as color sensitive filtering (Sangwine and Ell, 2000), edge detection in color images (Sangwine, 1998), (Evans and al., 2000)., cross correlation of color images (Moxey and al., 2003), watermarking (Ma and al., 2008).

To better understand the implementation of quaternion filters for edge detection, quaternion convolution will be briefly introduced in the next section. This article presents a method of color edge detection based on quaternion convolution followed by gradient vector to enhance edge structures. This article is organized as follows: the properties of quaternion convolution are presented in Section 2; Section 3 exposes the proposed edge detection method, Experiments related to the proposed method are given in Section 4; Section 5 draws the conclusion.

2 PROPERTIES OF QUATERNION CONVOLUTION

The theory of quaternion was first introduced by Hamilton in 1843 (Hamilton, 1853). A quaternion q is an hypercomplex number and has four components: one is a real scalar number, and the other three orthogonal components. It is usually represented as the following form:

$$q = a + bi + cj + dk \tag{1}$$

where a, b, c and d are real numbers, and the elements i, j and k are three imaginary units with the rules below:

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$$i^2 = j^2 = k^2 = -1 \quad (2)$$

$$ij = -ji = k \quad (3)$$

$$jk = -kj = i \quad (4)$$

$$ki = -ik = j \quad (5)$$

The number a is referred to the scalar part of a quaternion q denoted as S[q] and bi+cj+dk is its vector or imaginary part denoted as V[q].

The conjugate of a quaternion takes the following form: $\overline{q} = a - bi - cj - dk$ and its modulus equals $|q| = \sqrt{q\overline{q}} = \sqrt{a^2 + b^2 + c^2 + d^2}$.

If $|q| = 1$ then q is a unit quaternion. If the scalar part S[q] is equal to 0, then q is called a pure quaternion. If $|q| = 1$ and S[q] = 0, then q is a unit pure quaternion.

The addition and subtraction of two quaternion q_1 and q_2 are defined as follows:

$$\begin{aligned} q_1 \pm q_2 &= (a_1 + b_1 i + c_1 j + d_1 k) \pm (a_2 + b_2 i + c_2 j + d_2 k) \\ &= (a_1 \pm a_2) + (b_1 \pm b_2) i + (c_1 \pm c_2) j + (d_1 \pm d_2) k \end{aligned} \quad (6)$$

And their multiplication is defined as follows:

$$\begin{aligned} q_1 q_2 &= a_1 a_2 - b_1 b_2 - c_1 c_2 - d_1 d_2 \\ &+ (a_1 b_2 + b_1 a_2 + c_1 d_2 - d_1 c_2) i \\ &+ (a_1 c_2 + c_1 a_2 + d_1 b_2 - b_1 d_2) j \\ &+ (a_1 d_2 + d_1 a_2 + b_1 c_2 - c_1 b_2) k \end{aligned} \quad (7)$$

The following notation can also be used to describe the quaternion number $q = q_r + q_i i + q_j j + q_k k$. In fact, a vector in R^3 can be represented as a quaternion by setting the real part q_r to zero. Therefore, a color image (R, G, B) can be shown as a quaternion with q_i= R, q_j=G and q_k=B.

Sangwine (Sangwine, 1996) and S.C. Pei (Pei and Cheng, 1997) proposed to implement the quaternion to encode color images. Thus a color information represented in the RGB color space by three components will be described by the imaginary part of the quaternion. Therefore, each pixel of coordinates (m, n) of a color image will be encoded as follows:

$$f[m,n] = f_r[m,n] i + f_g[m,n] j + f_b[m,n] k \quad (8)$$

with $f_r[m,n]$, $f_g[m,n]$ and $f_b[m,n]$ represent respectively the red, green and blue components of the coordinate pixel (m, n).

3 THE PROPOSED METHOD FOR COLOR EDGE DETECTION

Our approach for color edge detection can be summarized in quaternion filtering and enhancement of edges by gradient calculation.

The image filtering can be performed by convolving a quaternion filter with an image. Since quaternion multiplication is not commutative, a pair of quaternion masks is required to define the following filters:

$$q_h = \begin{bmatrix} R & 0 & R^* \end{bmatrix} f_q(x,y) \begin{bmatrix} R^* & 0 & R \end{bmatrix} \quad (9)$$

$$q_v = \begin{bmatrix} R \\ 0 \\ R^* \end{bmatrix} f_q(x,y) \begin{bmatrix} R^* \\ 0 \\ R \end{bmatrix} \quad (10)$$

where $f_q(x,y)$ is the original image encoded by quaternion, the value of R is given by, $R = e^{\mu\theta}$ and μ is a three-dimensional unit vector represented by a pure unit quaternion. This pair of quaternion masks is used for both of horizontal and vertical filters q_h and q_v to detect color edges in the two directions.

We consider, in this work, $\mu = \frac{i+j+k}{\sqrt{3}}$ and $\theta = \frac{\pi}{2}$ (Sangwine and Ell, 2000), (Jin and Li, 2007).

This color edge detection filter uses a rotation in the color space around an axis named the "gray line" of the RGB space such that r = g = b and which combines all achromatic pixel values (Jin and Li, 2007). Any rotation around this axis traverses one color pixel value to another with the same luminance but with a different hue.

The color produced by this filter at an outline between two colors is halfway between the colors in the direction of the hue. Reversing the direction of the filter by interchanging R and R * in the masks changes the directions of color rotation (Sangwine, 1998). The modulus used in our algorithm in the two directions are defined as (Xu and al., 2010):

$$q_1 f_q(x,y) = \sqrt{q_{hi}^2 f_q(x,y) + q_{hj}^2 f_q(x,y) + q_{hk}^2 f_q(x,y)} \quad (11)$$

$$q_2 f_q(x,y) = \sqrt{q_{vi}^2 f_q(x,y) + q_{vj}^2 f_q(x,y) + q_{vk}^2 f_q(x,y)} \quad (12)$$

and the modulus is proportional to:

$$Mf_q(x,y)=\sqrt{q_1^2 f_q(x,y)+q_2^2 f_q(x,y)} \qquad (13)$$

In order to obtain the edge points of the color images we opted to extract them using a vector gradient. The use of the gradient serves to enhance the edge points present in an image in order to extract the most relevant information in the image.

The use of the gradient for edge enhancement is firstly based on the calculation of the gradient of the image in two orthogonal directions and subsequently the modulus of the gradient. Then, the most marked pixels obtained following the first operation are selected to identify the points having the strongest contrast by a suitable thresholding. The gradient is defined as a vector equivalent to a two-dimensional first derivative.

$$G\left[f(x,y)\right]=\begin{bmatrix}G_x\\G_y\end{bmatrix}=\begin{bmatrix}\frac{\partial f}{\partial x}\\\frac{\partial f}{\partial y}\end{bmatrix} \qquad (14)$$

The gradient modulus is given by:

$$G\left[f(x,y)\right]=\sqrt{G_x^2+G_y^2} \qquad (15)$$

and its direction is defined by:

$$\alpha(x,y)=\tan^{-1}(\frac{G_y}{G_x}) \qquad (16)$$

We applied this operation to the image resulting of quaternion filtering operation. We finally obtain the edge which will be initially in gray level, and after an automatic thresholding and thinning phase we obtain a binary representation of the final edge.

4 EXPERIMENTAL RESULTS

In this section, we present the results of our approach tested on five natural color images. All the images and ground truths used in this work can be found in the Berkeley segmentation dataset and benchmark (Martin and al., 2001). Fig. 1 shows the comparison results of the quaternion filtering associated to the gradient vector with the marginal approach of Multiscale Product (MP). The MP uses the first derivative of a Gaussian function as wavelet. The scales used in this work are s_1=1/16 and s_2=1/20. The choice of the MP's approach is due to the fact that this marginal method outperformed many marginal states-of-the-art edge detectors (BenYoussef and al., 2014), (BenYoussef and al., 2016).

The results obtained with our approach show a large number of missed detections in some case such as image n°118035 and 8068; and more details detected in other cases like image n°67079 compared with MP's approach.

In order to check the validity of our approach, we propose to make experiments using two parameters: The accuracy, which uses the true and false edges (eq. 17); and the Signal-to-noise ratio (SNR) with a white Gaussian noise having a variance value of 0,01 (eq. 18).

Accuracy=((TP+TN)/(TP+TN+FP+FN))*100 (17)

Where TP, TN, FP and FN are respectively True-Positive, True-Negative, False-Positive and False-Negative edge points. These parameters are explained by the confusion matrix presented below.

The confusion matrix is commonly used to expose results for binary decision problems (Kirkwood and Sterne, 2003), (Khaire, and Thakur, 2012). By comparing the marked pixels provided by a classification method, four cases are available as shown in the following table.

Table 1: Confusion matrix for the edge detection problem.

		Reality	
		Edge	Non-Edge
Classification	Edge	TP	FP
	Non-Edge	FN	TN

The edge detector attempted to extract edges that can be classified into four categories: True Positive (TP), False Positive (FP), True Negative (TN), and False Negative (FN). The first category determinates edge pixel detected correctly as edge. The second defined non edge pixels which are extracted wrongly as edge pixel. The TN is the category of non edge pixel detected correctly as non edge pixel. Finally, the FN defined edge pixel detected wrongly as non edge pixel.

$$SNR=10\log\left[\frac{\sum_{x=0}^{N-1}\sum_{y=0}^{M-1}\left(Mf(x,y)\right)^2}{\sum_{x=0}^{N-1}\sum_{y=0}^{M-1}\left(\widehat{Mf}(x,y)-Mf(x,y)\right)^2}\right] \qquad (18)$$

Where Mf and $\widehat{Mf}$ are respectively the edges of the clean and the noisy image.

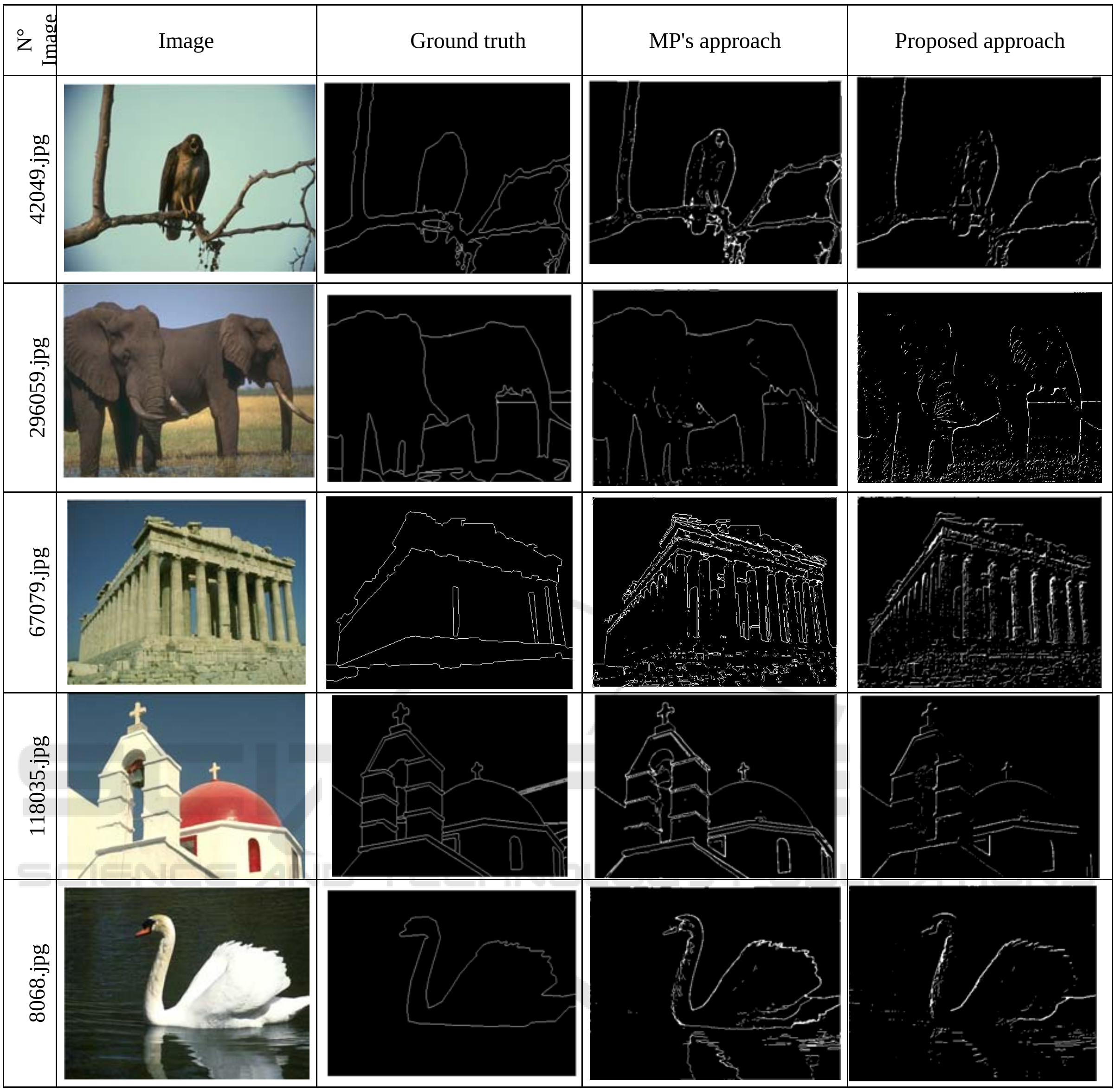

Figure 1: Comparison of edge detectors. From left to right: Image No., BSDS Image, Ground Truth, MP's approach, Proposed approach.

The tables (Table 2) and (Table 3) below show values of the considered parameters in terms accuracy and SNR concerning our approach and MP's approach.

We note that the proposed approach is characterized by the best accuracy rate of 97,65% against 96,84% for the MP's approach. However, in terms of SNR, the MP is more suitable for edge detection with an SNR rate of 17.23 against 15,91 for our proposed approach.

Table 2: Comparison of accuracy parameter of the proposed approach and MP's approach.

N° Image	MP's approach	Proposed approach
	Accuracy(%)	Accuracy (%)
42049	96.30	97.07
296059	97.54	98.33
67079	96.48	98.32
118035	96.17	96.55
8068	97.70	98.00
Average	96.84	97.65
Standard deviation	0.724	0.735

Table 3: Signal-to-noise ratio of the proposed approach and MP's approach.

N° Image	SNR MP's approach	SNR Proposed approach
42049	17.51	17.46
296059	18.74	14.97
67079	14.11	12.33
118035	17.21	17.30
8068	18.56	17.49
Average	17.23	15.91

5 CONCLUSIONS

In this paper, a vector approach for extraction of the most significant edges in color images has been presented. Our proposed method consists mainly of a quaternion filtering followed by a gradient vector to enhance the edge points. A pair of masks is employed for quaternion convolution to extract boundaries. The performance of our vector method was tested and compared with MP's edge detector which is a marginal method. Experimental results show that the proposed method gives better results on the studied images from Berkley database without noise. Indeed, its accuracy rate is higher than that of the MP's approach. In the presence of noise, the MP's approach outperforms our vector approach.

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