Strategic Capacity Expansion of a Multi-item Process with Technology Mixture under Demand Uncertainty: An Aggregate Robust MILP Approach

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Abstract: This paper analyzes the optimal capacity expansion strategy in terms of machine requirement, labor force, and work shifts when the demand is deterministic and uncertain in the planning horizon. The use of machines of different technologies are considered in the capacity expansion strategy to satisfy the demand in each period. Previous work that considered the work shift as a decision variable presented an intractable nonlinear mix-integer problem. In this paper we reformulate the problem as a MILP and propose a robust approach when demand is uncertain, arriving at a tractable formulation. Computational results show that our deterministic model can find the optimal solution in reasonable computational times, and for the uncertain model we obtain good quality solutions within a maximum optimal gap of 10^{-4}. For the tested instances, when the robust model is applied with a confidence level of 99%, the upper limit of the total cost is, on average, 1.5 times the total cost of the deterministic model.

1 INTRODUCTION

When a manufacturing industry faces a scenario of increasing demand in the long term and its facilities are close to maximum capacity, the how to expand its production capacity is a key decision. Strategic capacity expansion should determine the level of different production factors over time, such as the number of machines and workers needed to satisfy the production requirement. The objective of this paper is to determine the optimal capacity expansion strategy that minimizes the machinery investment cost, the labor cost, the production cost, and the idle capacity cost over a defined planning horizon, when the future demand is uncertain and there is no knowledge of its distribution. The decision variables are (i) the production requirements needed to satisfy the demand in each period, (ii) the number of machines of each technology needed, (iii) the work shifts necessary to cover the production requirement, and (iv) the workers needed to perform the number of shifts. This considers simultaneously the *machine requirement planning* (MRP) and the *strategic capacity planning* under uncertainty.

In this paper, we developed a multi-item, and multi-period model with technology mixture that determines the optimal expansion strategy considering the machine numbers of each technology, labor, and work shifts needed to satisfy the demand. We considered two different cases: when the demand is deterministic and when it is uncertain. For the first case, we formulated a mixed integer linear program (MILP), which can be solved efficiently with a mixed-integer solver. Then, we incorporated the demand uncertainty into the deterministic model, following a robust approach that considers the best worst case and a non-anticipativity constraint. In this formulation, flexibility is provided to the model via a box-type uncertainty set obtaining a robust model with adjustable robustness; the non-anticipativity constraints, to make the problem tractable, are represented by affine decision rules. The model with demand uncertainty is also an MILP and can be solved with a mixed-integer solver. We used both models to evaluate the impact of a technology mixture in the capacity expansion strategy. We considered three types of technology that differ in terms of production rate, worker requirements, investment cost and production cost.

The main contributions of this work are: (i) An efficient formulation of a strategic capacity expansion model that considers the work shifts as a decision variable; and (ii) the inclusion of the uncertain nature of the demand in the model under a robust approach.

The rest of this paper is structured as follows. In Section 2, a brief literature review is presented, and then, in Section 3 we present the model formulation for the two cases, when the demand is deterministic and when the demand is uncertain. In Section 4, we

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report our computational results, and finally, in Section 5 we conclude and present future extensions to this work.

2 RELATED WORK

A *strategic capacity expansion* problem consists of defining the expansion sizes and expansion timing in order to meet the incremental demands within a long-term planning horizon. The objective is to minimize the total costs with respect to the expansion process (Luss, 1982). On the other hand, *machine requirement planning* (MRP) can be defined as the specification of the number of each type of machine needed in each period for a productive process (Miller and Davis, 1977).

A comprehensive review of the strategic capacity expansion problem can be found in Luss (1982), Van Mieghem (2003), Wu et al. (2005), Julka et al. (2007), and Geng et al. (2009) and a more recent review in Martínez-Costa et al. (2014). In particular, Martínez-Costa et al. (2014) described the major decisions and conditioning factors involved in strategic capacity planning. They classified the strategic capacity expansion models according to the *number of sites* involved in the expansion process (a single or multiple sites), the *type of the capacity expansion* considered (expansion by investing/purchasing, outsourcing/subcontracting, reduction and replacement), whether the *uncertainty of the parameters* is considered in the problem formulation, and finally, the *type of mathematical programming model* and its solution procedure.

Our model corresponds to a single-site and multi-item capacity expansion problem under uncertain demand. We consider that capacity expansion can be achieved through machine acquisition and / or by using a flexible workforce in terms of increasing or decreasing the number of shifts. In this sense, the traditional structure of the strategic capacity expansion problem does not consider the relationship between the workforce planning and capacity acquisition decisions. This is related to the natural separation between strategic and tactical decision making. However, when these decisions are addressed separately, sub-optimal solutions are frequently the result. Since workforce flexibility and capacity acquisition can represent substitutable magnitudes, flexible workforce options could be also considered as a means of increasing capacity. In particular, for capital intensive companies, the implementation of one or more shift is an additional tool that managers can use to increment capacity continuously, avoiding the huge investment cost related to equipment acquisition.

To the best of our knowledge, only Fleischmann et al. (2006), Bihlmaier et al. (2009), and Escalona and Ramírez (2012) considered the workforce in their strategic capacity planning. Fleischmann et al. (2006) studied a multi-site and multi-item strategic capacity model with machine replacement and overtime as a means to meeting demand. They considered the same average cost for any overtime, such as prolongation of a shift, weekend shifts, night shifts, or regular third shifts. The model is formulated as an MILP and solved directly using CPLEX. Bihlmaier et al. (2009) analyzed a multi-site and multi-item strategic capacity model without machine replacement that integrates tactical workforce planning via shift work implementation. They consider a detailed set of shifts, such as a late shift, night shift, Saturday shift, and Saturday late shift. They presented a two-stage stochastic MILP for strategic capacity planning under uncertain demand that is solved by Benders decomposition. Escalona and Ramírez (2012) studied the optimal expansion strategy of a process, in terms of machinery, labor, and work shifts, through an aggregated model without machine replacement. They considered that in the process one, two, or three shifts can be worked per time period. The main difficulty related to their model is that shifts are not linear with the number of machines and workers needed to meet the demand. The model is formulated as a mixed integer nonlinear problem and solved by complete enumeration by fixing the shifts during the planning horizon.

Our strategic capacity model also considers uncertain demand. Four primary approaches to considering uncertainty exist (Sahinidis, 2004), which basically comprise (i) stochastic programming, where the uncertain parameters are considered random variables with known probability distributions; (ii) fuzzy programming, where some variables are considered as fuzzy numbers; (iii) stochastic dynamic programming, where random variables are combined with dynamic programming; and (iv) robust optimization, where the uncertainty of the parameters do not follow a known probability distribution, and the solutions are robust, i.e., they perform best in the worst case.

In the literature, two-stage stochastic programming is a dominant approach to handling stochastic capacity planning under various uncertainties (Swaminathan, 2000), (Hood et al., 2003), (Barahona et al., 2005), (Christie and Wu, 2002), (Karabuk and Wu, 2003), (Geng et al., 2009), (Rastogi et al., 2011), (Levis and Papageorgiou, 2004), and (Bihlmaier et al., 2009). A major shortcoming of two-stage stochastic programming is that it generates only a static capacity expansion plan and neglects the dynamic adjustments based on new information when the demand is revealed in each period. Several studies have adapted stochastic dynamic programming to overcome this issue (Rajagopalan et al., 1998), (Asl and Ulsoy, 2003), (Cheng et al., 2004), (Li et al., 2009), (Stephan et al., 2010), (Wu and Chuang, 2010), (Pratikakis et al., 2010), (Chien et al., 2012), (Pimentel et al., 2013), and (Lin et al., 2014).

In summary, few previous research studies on strategic capacity planning considered the workforce as a tool for increasing or decreasing capacity, and to the best of our knowledge, no robust optimization approach exists for dealing with demand uncertainty in strategic capacity planning.

3 MODEL FORMULATION

Consider a capacity expansion problem of a process over a planning horizon of T $(t = 1,...,\tau)$ periods. In this process, I $(i = 1,...,n)$ items are produced, and J $(j = 1,...,m)$ technologies are available for their production. The demand for item i in the period t is d_{it}. Without loss of generality, we assume that the aggregate demand $(\sum_{i\in I} d_{it})$ will increase on the long term, i.e., the aggregate demand follows a positive trend.

Let r_{ij} be the production rate of item i produced with technology type j, and let $\bar{\mu}_j$ be the maximum utilization for each machine of type j that will be considered by the design requirement. In each period it is possible to work K shifts $(k = 1,2,3)$; in each shift the available working time is limited by H_1. The number of shifts that will be worked in the period t is determined by the binary variable W_{kt}, which will be 1 if k shifts are used in period t, else it will be 0. The number of machines of type j needed in each shift to satisfy the demand in period t working k turns is denoted by Y_{jkt}; the number of machines of type j that will be acquired in period t is denoted by V_{jt}; and the number of machines available at the beginning of the planning horizon $(t = 0)$ is B_j.

To meet the demand in each period of the planning horizon, the capacity expansion could happen by: (i) acquiring new machines (expansion by investment) or (ii) modifying the number of shifts (expansion by operational cost). Therefore, in each period it is possible to hire or fire workers. Let Uh_{jt} be the number of workers hired at the beginning of the period t to operate machines of type j, and let Uf_{jt} be the number of workers fired at the beginning of the period t that operated machines of type j. The workers available to operate machines of type j in the period t is denoted by O_{jt}, with $O_{j0} = A_j$ representing the workers available at $t = 0$. The number of workers needed to operate one machine of technology type j is represented by $\bar{O}_j$. Finally, the quantity of item i produced with technology type j in each period t is represented by the variable X_{ijt}.

For this problem the costs that will be considered are: (i) the investment cost of acquiring a machine of type j in the period t (CI_{jt}); (ii) the unitary labor cost in period t (CL_t); (iii) the production cost for one item i produced in a machine of type j in the period t (Cp_{ijt}); (iv) the opportunity cost incurred by idle capacity of technology type j (Cop_{jt}); and, finally, (v) the unitary cost of hiring and firing, denoted by C_h and C_f respectively. It will be assumed that all the mentioned costs are properly brought to present value. A glossary of the terms used in the following sections can be found in appendix A. For this work we are going to consider the cost of opening or closing a shift as negligible, even if in reality they are not cost-free.

3.1 Deterministic Formulation

When demand is deterministic we propose the following capacity expansion problem, denoted by (**P0**).

Problem (**P0**):

$$\min_{\mathbf{X},\mathbf{V},\mathbf{Y},\mathbf{Uh},\mathbf{Uf},\mathbf{W}} TC = \Bigg\{ \sum_{ijt} X_{ijt}\, Cp_{ijt} + \sum_{jt} Cop_{jt} \left(B_j + \sum_{l=1..t} V_{jl} - \sum_{k} Y_{jkt} \right) + \sum_{jt} V_{jt} CI_{jt} + \left(\bar{O}_j \sum_{k} k\, Y_{jkt} \right) CL_t + \sum_{jt} Uh_{jt}\, C_h + \sum_{jt} Uf_{jt}\, C_f \Bigg\} \tag{1}$$

s.t:

$$\sum_{j\in J} X_{ijt} \geq d_{it} \qquad \forall i,t \tag{2}$$

$$\sum_{i\in I} \frac{X_{ijt}}{r_{ijt}} \leq \bar{\mu}_j\, H_1 \sum_{k\in K} k\, Y_{jkt} \qquad \forall j,t \tag{3}$$

$$\sum_{k\in K} Y_{jkt} \leq B_j + \sum_{l=1..t} V_{jl} \qquad \forall j,t \tag{4}$$

$$\sum_{k\in K} W_{kt} = 1 \qquad \forall t \tag{5}$$

$$Y_{jtk} \leq M\, W_{kt} \qquad \forall j,t,k \tag{6}$$

$$\sum_{k\in K} k \left(Y_{jkt} - Y_{j,k,t-1} \right) = \frac{Uh_{jt} - Uf_{jt}}{\bar{O}_j} \qquad \forall j,t \tag{7}$$

$$\sum_{k\in K} k \left(Y_{jk0} \right) = \frac{A_j}{\bar{O}_j} \qquad \forall j \tag{8}$$

$$\mathbf{X} \geq 0 \tag{9}$$

$$\mathbf{Y}, \mathbf{Uh}, \mathbf{Uf}, \mathbf{V} \in \mathbb{Z}^+ \quad (10)$$

$$\mathbf{W} \in \{0,1\} \quad (11)$$

The objective is to minimize the total cost (TC), considering the production cost, opportunity cost, investment cost, labor cost, and the cost of firing or hiring workers. The satisfaction of demand is ensured by (2). Constraint (3) restricts the total demanded time by the total available time; (4) ensures that the number of machines of technology j available in the period t is greater than the number of machines needed to satisfy the demand assigned in that period to the technology j. Constraints (5) and (6) ensure that the number of work shifts used in period t are of only one type, i.e., $k = 1$ or $k = 2$ or $k = 3$. Constraint (7) represents the continuity and requirement for workers in each period t, and constraint (8) represents an initial condition.

It is easy to show that the problem (**P0**) is equivalent to the problem presented by Escalona and Ramírez (2012) by incorporating the constraints (4) and (5) and considering the following equivalences of variables:

$$w_t NN_{jt} = \sum_{k \in K} kY_{jkt} \quad (12)$$

$$V_{jt} = ND_{jt} - ND_{j,t-1} \quad (13)$$

$$ND_{jt} = B_j + \sum_{l=1..t} V_{jl} \quad (14)$$

$$NN_{jt} = \sum_{k \in K} Y_{jkt} \quad (15)$$

where w_t is the decision variable that determines the number of shifts needed in period t, NN_{jt} is the number of machines of type j needed in the period t to satisfy the demand, and ND_{jt} is the number of machines of type j available in the period t.

3.2 Uncertain Formulation

In this paper we use a robust approach, where the uncertainty set is defined as a box, which is a particular case of the polyhedral set (Bertsimas and Sim, 2004),(Bertsimas and Thiele, 2006), (Guigues, 2009), and the problem is formulated as an affine multi-stage robust model with a simplified affine policy (Lorca et al., 2016).

Given the uncertain nature of the demand, it cannot be predicted with exactitude. In the best scenario, the estimate of the demand will be close to the actual value, but this will not happen frequently. The most frequent outcome is to have a variation (delta) between the estimate and actual demand. This delta tends to increase as the period analyzed is further in the future. When the variation is positive, the company over-produces, incurring a cost for not selling the excessive units produced; this cost can be estimated using the production cost or an opportunity cost. On the other hand, if this delta is negative, the demand cannot be completely satisfied; in this case the cost incurred by the company is one of lost sales. The lost-sale cost is often discussed since it can result in a loss of profit, a loss of future clients, or the loss of clients whose demand could not be satisfied, causing a bad reputation and loss of confidence in the company. Taking this into consideration, a negative delta is highly undesirable, and therefore it is fundamental to determine how to address the uncertain nature of the demand such that this delta is non-negative (or even 0) most of the time.

Let $D_{it} = \left\{ d_{it} \mid d_{it} \in \left[\bar{d}_{it} - \Gamma \hat{d}_{it} \ , \ \bar{d}_{it} + \Gamma \hat{d}_{it} \right] \right\}$ be the uncertainty set of d_{it}, where $\bar{d}_{it}$ is the nominal value of the demand, and let Γ represent the conservativeness of the model that can be associated with the risk factor of the companies. Denote by D the aggregate uncertainty set, i.e., $D = \bigcup_{it} D_{it}$.

Since we sought a robust model that can avoid a negative delta, our worst case will be the one that consider the maximum value that d_{it} can take under the uncertainty set D_{it}; in this case this corresponds to $\bar{d}_{it} + \Gamma \hat{d}_{it}$. Therefore, the demand satisfaction constraint for the robust model can be written as

$$\sum_{j \in J} X_{ijt} \geq \bar{d}_{it} + \Gamma \hat{d}_{it} \quad \forall i,t \quad (16)$$

Replacing (2) in the problem (**P0**) by (16), we obtain a robust model denoted by (**P1**). It is easily noticed that the deterministic problem has two different types of decision variables: (i) strategic decisions, i.e., decisions that affect the productivity in the long term and cannot be modified at the moment of demand realization; and (ii) operational decisions that are made in each period and which therefore depend on the realization of the demand. In the second type we have the production quantity decision variable ($\mathbf{X}$). This variable has a clear dependence on the demand realization, dependence that will be addressed via an affine decision rule of the form

$$X(\mathbf{d})_{ijt} = \bar{\chi}_{ijt} + \lambda_{ij} \sum_{\tau=1}^{t} \left(d_{i\tau} - \bar{d}_{i\tau} \right), \quad (17)$$

where the first term ($\bar{\chi}_{ijt}$) represents the nominal value of the quantity of item i to be produced in period t with machines of type j, and λ_{ij} is the percentage of the accumulated over-demand assigned to the item i and machines of technology j. Taking the definition of the uncertain set (D) and the affine decision rules defined by (17) and incorporating them in problem (**P0**), we obtain our affine multi-stage robust model denoted by (**P2**).

Problem (**P2**):

$$\min_{\bar{\chi},\mathbf{V},\mathbf{Y},\mathbf{Uh},\mathbf{Uf},\mathbf{W},\lambda,Z} \Bigg\{ Z+\sum_{jt} V_{jt} CI_{jt} + \sum_{jt} Cop_{jt}\left(B_j+\sum_{l=1..t} V_{jl}-\sum_{k} Y_{jkt}\right) + \sum_{jt}\left(\bar{O}_j \sum_{k} k\,Y_{jkt}\right) CL_t + \sum_{jt} Uh_{jt}\, C_h + \sum_{jt} Uf_{jt}\, C_f \Bigg\} \quad (18)$$

s.t: (4), (5), (6), (7), (8), (9), (10), (11)

$$\sum_{j\in J}\left(\bar{\chi}_{ijt}+\lambda_{ij}\sum_{\tau=1}^{t}\left(d_{i\tau}-\bar{d}_{i\tau}\right)\right) \geq d_{it} \qquad \forall i,t,d \quad (19)$$

$$\sum_{i\in I}\frac{\bar{\chi}_{ijt}+\lambda_{ij}\sum_{\tau=1}^{t}\left(d_{i\tau}-\bar{d}_{i\tau}\right)}{\bar{\mu}_j H_1 r_{ijt}} \leq \sum_{k\in K} k\,Y_{jkt} \qquad \forall j,t,d \quad (20)$$

$$\sum_{ijt}\left(\bar{\chi}_{ijt}+\lambda_{ij}\sum_{k=1}^{t}\left(d_{ik}-\bar{d}_{ik}\right)\right) Cp_{ijt} \leq Z \qquad \forall d \quad (21)$$

$$\sum_{ij}\lambda_{ij}=1 \quad (22)$$

$$Z\in\Re^{+}, \lambda\in[0,1] \quad (23)$$

We created the auxiliary variable Z in constraint (21) to denote the worst-case production cost. Constraints (19) and (20) are obtained by replacing the variable $\mathbf{X}$ in constraints (2) and (3) by the affine decision rule (17).

Making some straightforward arrangements, and taking into consideration that constraints (19)-(21) are robust constraints that should hold for all $\mathbf{d}\in D$, which is equivalent to maximizing each constraint over the uncertainty set D, it is possible to replace constraints (19)–(21) with the following set of constraints:

$$\sum_{j}\bar{\chi}_{ijt} \geq \Gamma_t \hat{d}_{it}\left(1-\sum_{j}\lambda_{ij}\right) + \sum_{j}\lambda_{ij}\sum_{\tau=1}^{t-1}\Gamma_\tau \hat{d}_{i\tau}+\bar{d}_{it} \qquad \forall i,t \quad (24)$$

$$\sum_{i}\frac{\bar{\chi}_{ijt}}{r_{ijt}}+\sum_{i}\left(\frac{\lambda_{ij}}{r_{ijt}}\sum_{\tau=1}^{t}\Gamma_\tau \hat{d}_{i\tau}\right) \leq \bar{\mu}_j H_1 \sum_{k\in K} k\,Y_{jkt} \qquad \forall j,t \quad (25)$$

$$\sum_{ijt}\bar{\chi}_{ijt} Cp_{ijt}+\sum_{it}\left(\Gamma_t \hat{d}_{it}\sum_{k=t}^{T}\sum_{j} C_{ijk}\lambda_{ij}\right) \leq Z \quad (26)$$

With these replacements, we obtained a MILP that considers the uncertain nature of the demand and that can be solved by a MIP solver in reasonable computational time. Note that if $\Gamma_t=0$, the solution is equal to the problem (**P0**) (the nominal problem).

4 COMPUTATIONAL STUDY

The computational study was developed with the following objectives: (*i*) evaluate the computational performance (in terms of CPU time) of the proposed model compared with that presented in the literature; (*ii*) analyze the behavior of the total cost to changes of Γ, i.e., analyze how the total cost grows as more demand (over its expected value) is considered; and (*iii*) test the applicability of our model in an industrial-size example. Since problems (**P0**) and (**P1**) are particular cases of (**P2**), we will only analyze (**P2**).

All experiments were performed with an AMD A6 2.0 GHz processor with 6 GB RAM memory, and the models were solved using CPLEX 12.6.

4.1 Industrial Size Example

We implemented our model using the information presented by a cosmetic company about a packaging process, in particular a sachet filling one, of four items that have high growth potential in the long term. For this process, three types of technology were evaluated. These technologies differ in (*i*) investment cost, (*ii*) unitary production cost, and (*iii*) workers required to operated one machine. For this implementation, we considered a time horizon of ten periods with each period being one year.

The cosmetic company provided the demand forecast for each item, i.e., the nominal value of the demand and the standard deviation of the forecast error. For each item and each period, the company treated the uncertainty through reliability intervals of the form $\hat{F}_{it}\pm\sigma_{it}Z_{1-\frac{\alpha}{2}}$, where $\hat{F}_{it}$ is the demand forecast for the item i at period t, $Z_{1-\frac{\alpha}{2}}$ is the quantile associated with a confidence level of $1-\alpha$, and σ_{it} corresponds to the standard deviation of the forecast error for item i at period t. Note that for this reliability interval, the company assumed that the forecast errors are Gaussian white noise.

Treating the demand uncertainty via reliability intervals can be easily related to the demand uncertainty set defined in Section 3.2 through the following relationships $\bar{d}_{it}=\hat{F}_{it}$, $\Gamma=Z_{1-\frac{\alpha}{2}}$, and $\hat{d}_{it}=\sigma_{it}$, $\forall i\in I,\ t\in T$. Therefore, the demand uncertainty set to be used in this illustrative example is of the form $D_{it}=\left\{d_{it}\mid d_{it}\in\left[\hat{F}_{it}-Z_{1-\frac{\alpha}{2}}\sigma_{it}\ ,\ \hat{F}_{it}+Z_{1-\frac{\alpha}{2}}\sigma_{it}\right]\right\}$, where $Z_{1-\frac{\alpha}{2}}\in\{0,\ 0.25, 0.52,\ 0.84,\ 1.28,\ 1.64,\ 1.96,\ 2.33,\ 2.58,\ 3.29\}$, which corresponds to confidence levels of 0%, 20%, 40%, 60%, 80%, 90%, 95%, 98%, 99%, and 99.9%, respectively. Each problem has 343 variables (133 continuous, 180 integer, and 30 binary variables) and 235 linear constraints.

For this computational study we considered the following set of parameters:

- Demand: The nominal value of demand and the standard deviation of the forecast error are presented in Table 2 and Table 3 in B.
- Production rate (r_{ij}): The production rate is the same for each type of technology, i.e., $r_{ij} = r_i$, $\forall j \in J$, $r_1 = 120$, $r_2 = 170$, $r_3 = 400$, and $r_4 = 600$.
- Maximum utilization (μ_j): $\mu_1 = 0.8$, $\mu_2 = 0.9$, $\mu_3 = 0.98$.
- Available time per shift (H_1): 2080[hours].
- Number of workers needed per machine of technology j ($\bar{O}_j$): $\bar{O}_1 = 3$, $\bar{O}_2 = 2$, and $\bar{O}_3 = 1$.
- Investment cost (CI_{jt}): $CI_{jt} = CI_{j1}(1.15)^{1-t}, \forall t = \{2,...,T\}$, with $CI_{1,1} = \$25000$, $CI_{2,1} = \$50000$, and $CI_{3,1} = \$75000$.
- Opportunity cost (Cop_{jt}): $Cop_{jt} = CI_{jt}f$, $\forall j \in J, \forall t \in T$, where f represents the relation between the useful life and the depreciation time of the machine, and corresponds to 0.10.
- Annual labor cost per worker (CL_t): $CL_t = CL_1(1.15)^{1-t}, \forall t = \{2,...,T\}$, with $CL_1 = \$16032$.
- Unitary production cost (Cp_{ijt}): The production cost will depend only on the type of technology, i.e., $Cp_{ijt} = Cp_{jt}$, $\forall i \in I$, $Cp_{jt} = Cp_{j1}(1.15)^{1-t}$, $\forall j \in J, \forall t = \{2,...,T\}$, with $Cp_{1,1} = \$1$, $Cp_{2,1} = \$0.75$, and $Cp_{3,1} = \$0.5$.
- Unitary firing and hiring cost: $C_h = \$500$ and $C_f = \$4500$.
- Initial conditions: The number of workers and machines available at the beginning of the planning horizon, for each technology, are 0, i.e., $B_j = A_j = 0$, $\forall j \in J$.

4.2 Results of the Industrial Size Example

The expansion route for this problem involves different shifts and the use of only one type of technology. According to the results, the technology type selected is, regardless of the value of α, the one with the highest investment cost but with the lowest production cost and fewest required workers (type 3). The nominal total cost for the expansion under deterministic demand ($1-\alpha = 0$) is \$8152716. In Figure 1 we show the proportion of the total cost over the nominal cost for each value of $1-\alpha$.

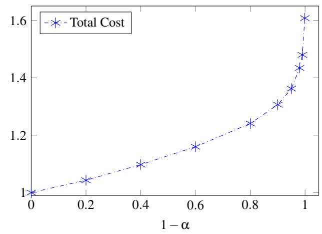


Figure 1: Total cost behavior.

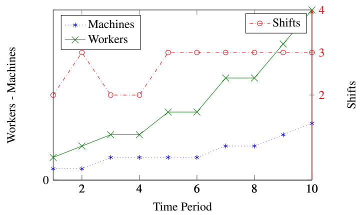
From Figure 1 we observe that the total cost increases exponentially when more variable demand is considered, and therefore each percentage increase is more expensive than the previous one; for example, if a reliability level ($1-\alpha$) of 0.9 is selected, i.e., the demand can be satisfied 95% of the time, the total cost increases to 1.3 times the nominal cost, but if the reliability level selected is 0.99 ,the total cost increases to 1.48 times the nominal cost. Figure 2 presents the expansion route in terms of shifts, total number of machines, and total number of workers needed in each period for four different instances. In all instances, we observed that the shifts changed along the planning horizon, and that an operational expansion (increasing the number of shifts) is always preferred before realizing an investment; with this is possible to determine that is more advisable to expand via shifts before purchasing more machines. Therefore, if the shifts are considered fixed, it is possible to arrive at sub-optimal solutions.

4.3 Sensitivity Analysis

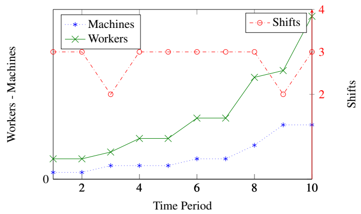
To analyze the impact of the parameters in the expansion strategy, we developed a sensitivity analysis, where the following parameters were varied: (*i*) investment cost, (*ii*) operational cost, and (*iii*) number of workers. For each one the technology mixture and the shifts structure will be analyzed considering a confidence level of 0.9, i.e., $\Gamma = Z_{0.95} = 1.64$. The shifts structure will be considered as the aggregate number of shifts along the planning horizon (ANS), i.e., $ANS = \sum_{t,k} kW_{kt}$.

Investment Cost. The investment cost of the actual selected technology type ($j = 3$) was increased until it is not selected anymore. This increase was measured with respect to technology type 1. Figure 3 shows the technology mixture versus the investment cost of technology type 3, (a) when the operational costs are different for each type of technology and (b) when they are the same.

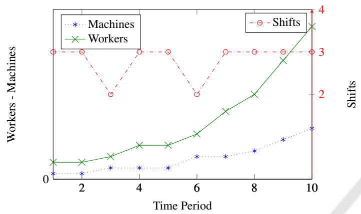
From Figure 3 is possible to observe a gradual transference from machines of technology type 3 to



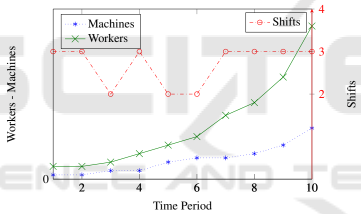
(a) $1-\alpha = 0$ $(\Gamma = 0)$



(b) $1-\alpha = 0.8$ $(\Gamma = 1.28)$

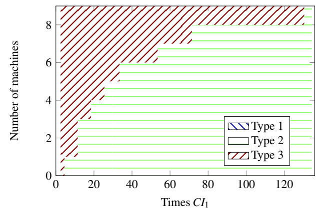


(c) $1-\alpha = 0.95$ $(\Gamma = 1.96)$

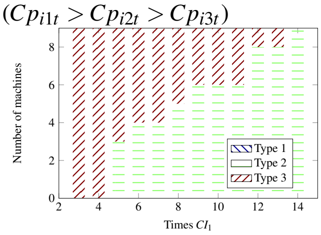


(d) $1-\alpha = 0.999$ $(\Gamma = 3.29)$

Figure 2: Expansion route by Γ.

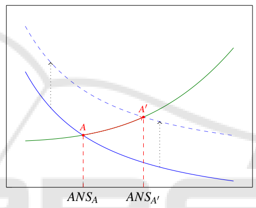


(a) Different operational cost $(Cp_{i1t} > Cp_{i2t} > Cp_{i3t})$

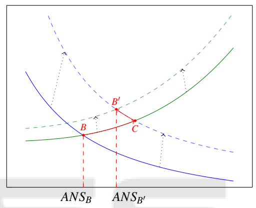


(b) Same operational cost $(Cp_{ijt} = Cp_{it} \forall j \in J)$

Figure 3: Sensitivity to investment cost.



(a) Without technology mixture



(b) Under technology mixture

Figure 4: Equilibrium dynamic under investment cost variation.

machines of technology type 2. Note that this transference happens sooner when the operational costs are the same for all the technologies. In particular, from Figure 3(a), total transference is achieved when the investment cost of technology type 3 is at least 131 times CI_1, and from Figure 3(b), total transference is achieved when the investment cost of the machines of type 3 is 14 times the investment cost of machines of type 1.

We observe that the ANS increases when the investment cost increases. This behavior can be explained for two cases, when the rise of the investment cost (i) does not induce technology mixture, and (ii) when it does induce technology mixture. Figure 4 shows the behavior of the cost equilibrium under varying investment cost for both cases.

When the increase in the investment cost does not induce technology mixture (Figure 4(a)), the aggregate investment cost curve moves upwards and the aggregate labor cost stays unchanged. This implies that the equilibrium moves from point A to point A', resulting in a higher cost and a higher ANS. In the second case, when the increase in the investment cost induces technology mixture (Figure 4(b)), the curve dynamic can be explained in two stages: (i) the aggregate investment cost increases and (ii) the aggregate labor cost also simultaneously increases; with this the equilibrium moves from point B to point C and finally to point B'. Note that, since in this case the aggregate investment cost increases significantly more than the labor cost, the new equilibrium achieved at point B' implies a higher cost and a higher ANS.

Operational Cost. The operational cost of the actual selected technology $(j = 3)$ was increased until the model stopped selecting it. This increase was measured with respect to technology type 1. Figure 5 shows the technology mixture under varying production cost.

From Figure 5, it is possible to observe a gradual transference from machines of technology type 3 to machines of technology type 2. When the operational cost of the machines of type 3 is 1.19 times that of machines of type 2, the technology transference is total.

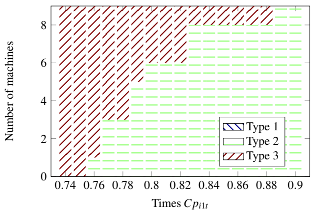
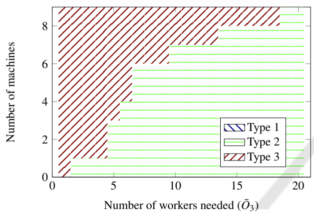


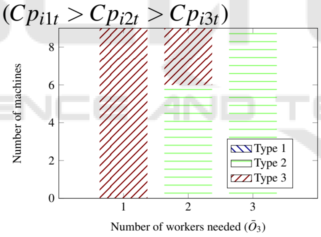
Figure 5: Sensitivity to operational cost.

We also observed that without technology mixture the *ANS* does not change.

Number of Workers. Similar to the previous analysis the number of workers required for technology type 3 was increased until the technology transference was total. The resulting technology mixture is presented in Figure 6, (a) when the operational costs are different for each type of technology and (b) when they are the same.



(a) Different operational cost ($Cp_{i1t} > Cp_{i2t} > Cp_{i3t}$)

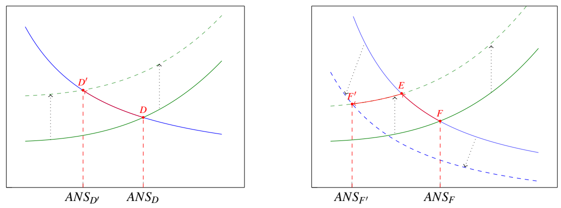


(b) Same operational cost ($Cp_{ijt} = Cp_{it} \forall j \in J$)

Figure 6: Sensitivity to required number of workers.

From Figure 6 it is possible to observe a gradual transference from machines of technology type 3 to machines of technology type 2. Note that this transference starts when both technologies require the same number of workers, and is more drastic when the operational cost is the same for all the technologies (Figure 6(b)).

In this analysis is also possible to note a relationship between the number of workers and the *ANS*. An increase in the number of workers implies a decrease in the *ANS*. Two cases can be recognized: when the rise in the number of workers (i) does not induce technology mixture, and (ii) when it does induce technology mixture. Figure 4 shows the behavior of the cost equilibrium under a variation of the workers requirement for both cases.



(a) Without technology mixture

(b) Under technology mixture

Figure 7: Equilibrium dynamic under variation in the required number of workers.

When the increase in the number of workers does not induce technology mixture (Figure 7(a)), the aggregate labor cost curve moves upwards and the aggregate investment cost stays unchanged. This implies that the equilibrium moves from point D to point D', resulting in a higher cost with a lower *ANS*. In the second case, when the increase in the number of workers induces technology mixture (Figure 7(b)), the curve dynamic can be explained in two stages: (i) the aggregate labor cost increases and (ii) the aggregate investment cost simultaneously decreases; with this the equilibrium moves from point E to point F and finally to point E'. Note that the aggregate labor cost varies significantly more than the investment cost and the new equilibrium is achieved at point E', implying a higher cost and a lower *ANS*.

For this industrial-size example, on average, 90% of the cost can be explained as operational. Therefore, when the operational costs are the same for all types of technology, the technology mixture is more sensitive under variation of the investment cost or number of workers. From the sensitivity analysis, we observed that (i) the optimal capacity expansion strategy is more sensitive to the cost that has more influence over the total cost, (ii) the required number of workers is always an important decision factor even when the labor cost represents less than 10% of the total cost, and (iii) the *ANS* has a direct relationship with the investment cost and an inverse one with the labor cost.

4.4 Computational Performance

To evaluate the computational performance of our model and to cover a wide range of data, we generated a set of 180 problems, each one randomly generated around a base case with 10 different items, 5 types of technologies, and a planning horizon of 10 periods, with each problem having 881 variables (551 continuous, 300 integer, and 30 binary variables) and 412 linear constraints. Figure 8 shows the computation times in $\log_2(sec)$ for all instances. In the abscissa the cumulative percentage of instances is presented.

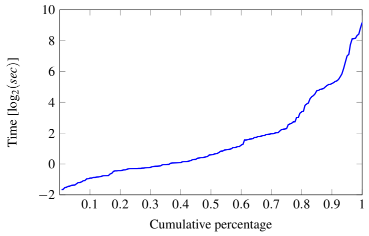


Figure 8: Computation Times.

Figure 8 shows that we observed reasonable computation times; 80% of the instances were solved in less than 10 seconds and 94% of the instances were solved in less than 100 seconds, with a geometric mean of 2.8 seconds. The nominal problem of this instances where solved with the formulation and algorithm presented in Escalona and Ramírez (2012) obtaining an average computational of 584585 seconds to arrive at the optimal solution, therefore our formulation has an average speedup of 25175*x*.

5 CONCLUSION

In this paper, we develop a capacity expansion model for multi-product, multi-machine manufacturing systems with uncertain demand. At first, a linear deterministic model is presented and later the demand uncertainty is incorporated using a robust approach formulating an affine multi-stage robust model. In contrast with most of the works presented in the literature, our model considers the shifts as a decision variable, allowing more flexibility in the type of expansion that can be used.

For instances that consider a planning horizon of 10 periods, 10 items, and with 5 types of technologies available, the computation times prove to be reasonable ones with times below 100 seconds for most of them (94%), and therefore our model performs better than the one presented in the literature, having an average speedup of 25175*x*.

From the instances that we tested, we observed the following managerial insights:

- *Fixing beforehand the number of shifts to work along the planning horizon can take us to sub-optimal solutions.*
- *The technology mixture is most sensitive to the required number of workers and to the most important cost.*
- *There exists an inverse relationship between labor cost (number of workers) and the aggregate available time and a direct relationship between investment cost and the aggregate available time.*
- *The operational cost by itself does not change the aggregate available time; in fact, if there is no change in the investment cost and number of workers, then the aggregate available time does not change, i.e. the available time, and therefore the work shifts routed along that planning horizon, depend only on the investment and labor costs.*

An interesting discussion that escaped the scope of this work is the analysis of some costs, such as the cost incurred when opening or closing a shift and the opportunity cost which can be determined following business logic instead of the accounting logic used in this work.

Possible extensions of this problem that can be considered are (*i*) the use of a scenario approach with multi-stage programming (Ben-Tal et al., 2009), (Shapiro, 2009), (*ii*) the use of $CVaR^{\Delta}$ to minimize the variability of the solution (Rockafellar and Uryasev, 2000), (Pflug, 2000), (Rockafellar et al., 2006) instead of cost minimization, and (*iii*) considering uncertainty of other parameters such as the maintenance times, production rates, costs, and/or available times.

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APPENDIX

A Glossary of Terms

Table 1: Glossary of terms.

Sets	Definition
I	Set of items indexed by i
J	Set of type of machines indexed by j
T	Set of periods indexed by t
K	Set of number of shifts indexed by k, with $k = 1,2,3$
Parameters	
Cp_{ijt}	Unitary production cost for item i produced with a machine of type j in period t
CI_{jt}	Investment cost of a machine of type j in period t
Cop_{jt}	Opportunity cost for a machine of type j in period t
CL_t	Unitary labor cost in period t
C_h	Hiring cost
C_f	Firing cost
B_j	Number of machines of type j available at the beginning of the planning horizon
A_j	Number of workers available to operate the machines of type j at the beginning of the planning horizon
d_{it}	Demand realization of item i in period t
$\bar{d}_{it}$	Nominal demand of item i in period t
$\hat{d}_{it}$	Maximum perturbation for the demand of item i in period t
r_{ijt}	Production rate of items i with machine of type j in period t
$\bar{\mu}_j$	Maximum utilization of machines type j
H_1	Available working time for each work shift
M	A big enough number
$\bar{O}_j$	Number of workers needed to operate one machine of type j
Γ	Level of conservativeness of the model
$\hat{F}_{it}$	Demand forecast of the item i at period t
σ_{it}	Standard deviation of the forecast error for item i at period t
Variables	
X_{ijt}	Number of items i produced with machines of type j in period t
V_{jt}	Number of machines of type j bought in period t
Y_{jkt}	Number of machines of type j needed in each shift to satisfy the demand in period t, working k shifts
W_{kt}	1 if k shifts are worked in period t, 0 otherwise
Uh_{jt}	Number of workers hired in period t to work machines of type j
Uf_{jt}	Number of workers fired in period t that worked machines of type j
w_t	Number of shifts to work in the period t
NN_{jt}	Number of machines of type j needed at period t to satisfy the demand
ND_{jt}	Number of machines of type j available at period t
λ_{ij}	Percentage of the accumulated over-demand assigned to the item i and machine j
$\bar{\chi}_{ijt}$	Nominal quantity of item i to be produced in period t with machines of type j
Z	Worst case production cost.

B Data for Illustrative Example

Table 2: Demand forecast of the item i at period t.

	$\hat{X}_{it}$ $[*10^3 units]$									
i \ t	1	2	3	4	5	6	7	8	9	10
1	100	111	122	138	177	236	330	411	483	605
2	328	420	549	662	788	950	1091	1479	1651	1830
3	367	470	650	762	1021	1140	1293	1736	2227	3111
4	180	226	308	391	523	686	942	1089	1452	1815

Table 3: Standard deviation of the forecast error for item i at period t.

	σ_{it} $[*10^2]$									
i \ t	1	2	3	4	5	6	7	8	9	10
1	30	44	53	70	99	179	292	416	515	858
2	83	130	210	343	532	823	1051	1771	2740	3823
3	142	243	468	747	1126	1335	1829	2590	4040	7805
4	67	96	155	236	423	742	1232	1685	3088	5054