

A Stackelberg Game Model between Manufacturer and Wholesaler in a Food Supply Chain

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Abstract: This paper describes an application of the Stackelberg game model for the food supply chain. Specifically, the focus of this work is on the pork industry and considers a production game. Such game includes two players, manufacturer and wholesaler, who both aim to maximize profit. The role of leader is played by the manufacturer, and follower by the wholesaler. Decisions involved in the game are the level of production, quantity to be sold by the leader, and level of purchased products by the follower at each time period. This paper presents a case study, and results show that coordination between these players is seen in cost savings and improved service level.

1 INTRODUCTION

In developing countries, food demand continues growing, given rising incomes and population growth. In the context of countries with a strong and continuous economic development, it is forecasted that protein consumption will rise, along with meat consumption, resulting in an active industry. Meat production will increase by 17% in developing countries and 2% in developed countries from 2014 to 2024. Pork is the most produced and consumed red meat worldwide (FAO, 2016). In this context, several complex problems are faced by chain managers, who need to integrate stakeholder operations in order to coordinate product flow along the chain. One of the most challenging problems is related to planning and scheduling of operations for processing the carcasses (body of the animal gutted and bloodless) into pork and by-products, later to be sold to wholesalers to satisfy required demand. In this framework, the coordination and integration between two agents is critical to improve efficiency and increase supply chain productivity.

Operations Research (OR) is one of the most important disciplines that deal with advanced analytical methods for decision making. OR is applied to a wide range of problems arising in different areas, and their

fields of application involve the operations management of the agriculture and food industry. There are several works related to these topics, see (Ahumada and Villalobos, 2009) for a review of agricultural supply chains; see (Bjørndal et al., 2012) for a review of operations research applications in agriculture, fisheries, forestry and mining; see (Higgins et al., 2010) for an application of agricultural value chains using network analysis, agent-based modeling and dynamical systems modeling; (Plà et al., 2014), draw out insights for new opportunities regarding OR for the agricultural industry. Specifically, (Rodríguez et al., 2014) presents a key description of opportunities focused on the pork supply chain.

Furthermore, game theory has been deeply used to analyze the interactions between different agents in the supply chain (Hennet and Arda, 2008). Game theory is a suitable tool to support decision making where there is more than one participant (or player) (Marulanda and Delgado, 2012). The Stackelberg model was originally introduced in the context of static competition games in 1934 by the economist H. von Stackelberg (Von Stackelberg, 1952), with an important impact on economic sciences. Such a problem can be also seen as a bilevel optimization problem (Dempe, 2002). In this model, the leader announces his strategy first. Next, the follower observes the

leader's actions and reacts to them, so as to maximize profits. Interactions within the food supply chain are captured through a game between a leader and a follower. In this model, here the leader presents advantage, and consequently is who decides first about his operational decisions: location, technology, amount to processing, raw materials, and prices (Yue and You, 2014).

In many decision making processes there is a hierarchical structure among agents and decisions are taken at different levels of the hierarchy. Interactions among participants in the supply chain are captured through a game between a leader with a follower who follows the structure of a Stackelberg game. When we see this model as a two level decision problem, it is called bilevel optimization program. In this case, we have a leader (associated with the top level) and a follower (associated with the lower level). If both the leader's and follower's constraints are linear, it is a bilevel linear programming problem (BLPP) (Migdalas et al., 2013). Here, decisions of a player on one level can affect decision-making behavior on other levels, even though the leader does not completely control the actions of the followers.

A vast majority of research on bilevel programming has centered on the linear version of this problem, alternatively known as the linear Stackelberg game. The BLPP can be written as follows (Bard, 2013):

For $x \in X \subset R^n, y \in Y \subset R^m, F : X \times Y \rightarrow R^1$, and $f : X \times Y \rightarrow R^1$.

$$\min_{x \in X} F(x,y) = c_1x + d_1y \tag{1}$$

$$\text{s.t } A_1x + B_1y \leq b_1 \tag{2}$$

$$\min_{y \in Y} f(x,y) = c_2x + d_2y \tag{3}$$

$$\text{s.t } A_2x + B_2y \leq b_2 \tag{4}$$

where $c_1, c_2 \in R^n, d_1, d_2 \in R^m, b_1 \in R^p, b_2 \in R^q, A_1 \in R^{p \times n}, B_1 \in R^{p \times m}, A_2 \in R^{q \times n}, B_2 \in R^{q \times m}$. Sets X and Y place additional constraints on the variables, such as upper and lower bounds. In this model, once the leader selects an x , the first term in the follower's objective function becomes a constant; and the same is valid for the follower's constraints.

Bard (2013) also presents a necessary condition that (x^*, y^*) solves the linear BLPP (1)-(4) if there exist (row) vectors u^* and v^* such that (x^*, y^*, u^*, v^*) solves:

$$\min c_1x + d_1y \tag{5}$$

$$\text{s.t } A_1x + B_1y \leq b_1 \tag{6}$$

$$uB_2 - v = -d_2 \tag{7}$$

$$u(b_2 - A_2x - B_2y) + vy = 0 \tag{8}$$

$$A_2x + B_2y \leq b_2 \tag{9}$$

$$x \geq 0, y \geq 0, u \geq 0, v \geq 0 \tag{10}$$

Thus, a new nonlinear constraint (8) is generated, to represent the optimization model for the follower. However, this formulation has played a key role in the development of algorithms. One advantage that it offers is that it allows for a more robust model to be solved without introducing any new computational difficulties. There are several algorithms proposed for solving the linear BLPP since the field caught the attention of researchers in the mid-1970s. Many of these are of academic interest only because they are either impractical to implement or grossly inefficient. The most popular method for solving the linear BLPP is known as the "Kuhn-Tucker" approach and concentrates on (5)-(10). The fundamental idea is to use a branch and bound strategy to deal with the complementary constraint (8). Omitting or relaxing this constraint leaves a standard linear program which is easy to solve. Various methods proposed employ different techniques for assuring that the complementary constraint is ultimately satisfied (Bard, 2013). (Fortuny-Amat and McCarl, 1981), proposed a reformulation of the non-linear constraints; and (Bard and Moore, 1990) developed an algorithm that ensured global optimum with high efficiency.

In what follows, we propose a Stackelberg game between two food supply chain players involved in processing and selling activities. This is carried out assuming that there is a leader-follower relationship among the players. The primary challenge in this model is to support in the coordination and integration of activities and information among two supply chain agents. In Section 2, we detail the proposed Stackelberg game and model to solve this problem. After this, Section 3 presents a case study and provides the obtained results. Finally, in Section 4 main conclusions and future research are presented.

2 MATERIALS AND METHODS

To have a good relationship between agents, coordinate activities and share information, the chain should be aligned, improving efficiency and productivity. This section presents a detailed description of the Stackelberg game between two supply chain agents under the leader and follower scheme; and an optimization problem that models the interaction between the players.

2.1 Games Description

In the meat supply chain, a manufacturer is in charge of processing the raw material, from carcasses to meat products. This player decides based on demand and the yielding rate of each cutting pattern, the level of production and inventory for each product, and hence the number of carcasses required. The processing plant aims to maximize profits through the sale of meat products; and the wholesaler is in charge of distribution and marketing of the products. Meat products are purchased from the manufacturer, and then sold to end customers, with the aim of maximizing level of service.

Interactions between both players in the supply chain are captured through a game with a leader and a follower who follows the structure of Stackelberg game. The problem arises when the optimal quantity produced and sold by the manufacturer is not enough to supply the needs of wholesaler. Thus, the wholesaler imposes a penalty cost on the manufacturer for the unsatisfied demand. This penalty cost can be reduced through coordination and integration of activities, and information exchange between the two supply chain agents. By sharing information about consumer preferences and demand, the manufacturer avoids expired products and cooperates with the wholesaler to maximize their level of service, improving performance of the whole supply chain.

The proposed game considers the manufacturer as the leader and the wholesaler as the follower. Given the characteristics of this Stackelberg game, static and not cooperative, this problem can be modeled by a bilevel linear programming problem.

2.2 Bilevel Linear Programming Model

This model supports and assists in the coordination and integration of two food supply chain agents, under the leader and follower structure given by the Stackelberg game. The main assumptions and considerations for the formulation of this bilevel programming model are based on (Albornoz et al., 2015).

Sets, indexes and variables used in the model are described below:

Sets and Indexes:

- T : Number of periods of the planning horizon.
- J : Number of cutting patterns.
- $k \in K$: Set of sections per carcass.
- $j \in J_k$: Set of cutting patterns per section k .

- $r \in R$: Set to represent the different types of carcasses.
- $i \in P$: Set of Products.

Parameters:

- H : Carcasses available to process during the whole planning horizon.
- α_r : Proportion of carcasses of type r .
- Ψ_{ijr} : Yield of product i using cutting pattern j on carcasses of type r .
- p_i : Selling price per product i .
- c_j : Operational cost of pattern j .
- c_j^e : Operational cost of pattern j in overtime.
- h : Holding cost of product per period for the leader.
- f_i : Cost for unsatisfied-demand of product i for the leader.
- W : Warehouse capacity (in kg.) for the leader.
- t_j : Operation time for cutting pattern j .
- T_w : Available hours in regular time.
- T_w^e : Available hours in overtime.
- δ : Auxiliary parameter for better control the available carcasses.
- P : Purchase cost of carcasses.
- d_{it} : Demand of product i at each period t .
- G : Warehouse capacity (in kg.) for the follower.

Decision Variables:

- v_{it} : Quantity of product i to be sold in t .
- x_{it} : Total quantity of product i to have in period t .
- s_i : Total quantity of excess product i at the end of planning horizon.
- H_t : Number of carcasses to be processed at each period t .
- z_{jt} : Number of times to perform the cutting pattern j in period t in normal work hours.
- z_{jt}^e : Number of times to perform the cutting pattern j in period t , in overtime.
- I_{it} : Quantity of product i to hold for the leader in t .
- u_{it} : Unsatisfied-demand of product i in t .
- I_{it}^r : Quantity of product i to hold for the follower in t .

$$\max \sum_{i \in P} \sum_{t=1}^T p_i v_{it} - \sum_{t=1}^T P H_t - \sum_{t=1}^T \sum_{j \in J} \sum_{r \in R} (c_j z_{jrt} + c_j^e z_{jrt}^e)$$

$$-\sum_{i \in P} \sum_{t=1}^T hI_{it} - \sum_{i \in P} \sum_{t=1}^T f_i u_{it} \quad (11)$$

s.t

$$\alpha_r H_t - \sum_{j \in J_k} (z_{jrt} + z_{jrt}^e) = 0; t \in T, k \in K, r \in R \quad (12)$$

$$x_{it} - \sum_{r \in R} \sum_{j \in J_k} \psi_{ijr} (z_{jrt} + z_{jrt}^e) = 0; i \in P, t \in T, k \in K \quad (13)$$

$$\sum_{r \in R} \sum_{j \in J_k} t_j z_{jrt} \leq T_w; t \in T \quad (14)$$

$$\sum_{r \in R} \sum_{j \in J_k} t_j z_{jrt}^e \leq T_w^e; t \in T \quad (15)$$

$$v_{it} - x_{it} - I_{it} + I_{i,t+1} = 0; i \in P, t = 1, \dots, T-1 \quad (16)$$

$$v_{iT} - x_{iT} + s_i - I_{iT} = 0; i \in P \quad (17)$$

$$\sum_{t=1}^T H_t \leq H \quad (18)$$

$$-\sum_{t=1}^T H_t \leq -\delta H \quad (19)$$

$$\sum_{i \in P} I_{it} \leq W; t \in T \quad (20)$$

$$v_{it} \geq 0, x_{it} \geq 0, s_i \geq 0, I_{it} \geq 0, H_t \text{ integer}, \\ z_{jrt} \text{ integer}, z_{jrt}^e \text{ integer} \quad (21)$$

$$\min \sum_{i \in P} \sum_{t=1}^T u_{it} \quad (22)$$

s.t

$$v_{it} + u_{it} + I_{it}^r - I_{i,t+1}^r = d_{it}; i \in P, t = 1, \dots, T-1 \quad (23)$$

$$v_{iT} + u_{iT} + I_{iT}^r = d_{iT}; i \in P \quad (24)$$

$$\sum_{i \in P} I_{it}^r \leq G; t \in T \quad (25)$$

$$I_{i,T}^r = 0; i \in P \quad (26)$$

$$u_{it} \geq 0; I_{it}^r \geq 0 \quad (27)$$

Where $v_{it}, s_i, H_t, z_{jrt}, z_{jrt}^e, I_{it}$ are decision variables of the leader and u_{it}, I_{it}^r are decision variables of the follower. The manufacturer's objective (16) is to maximize profits. Profits are understood as the difference between total revenues from selling the products and the following costs: inventory, production, purchases of carcasses and unsatisfied-demand penalties. Conversely, the follower (22) is simply trying to maximize his service level.

A feasible solution of the model satisfies a different set of leader's constraints. Constraint (12) ensures a balance between cutting patterns and the number of

carcasses to be processed at each time period. Equality is forced because it is not possible to leave unprocessed raw material. Constraint (13) calculates the total kilograms of each product retrieved by all the cutting patterns applied at each time period. Constraint (14) ensures that the labor time does not exceed the viable working hours of regular time. Constraints (15) ensure that the labor time does not exceed the viable working hours during overtime. Constraints (16) and (17) determine the quantity of product to be processed and held considering the excess product. This is the amount that the manufacturer sells when the wholesaler does not purchase all products at the end of the planning horizon (without revenues because it is a sunk cost). Constraints (18) and (19) impose a lower and upper limit according to the animal availability from suppliers and a given percentage δ to allow an extra flexibility in the total number of carcasses to be processed. Constraint (20) ensures that the holding capacity for products is never exceeded. Constraint (21) defines the domain of decision variables.

On the other hand, the set of follower's constraints are the following. Constraints (23) and (24) ensure that the requested level of each product is addressed, allowing the existence of unsatisfied-demand if the manufacturer does not provide enough products to satisfy the demand. Constraint (25) ensures that the capacity for holding products is never exceeded. Constraint (26) satisfies the condition to not holding products at the end of the planning horizon. Constraint (27) defines the domain of decision variables.

In this paper, we solve a linear relaxation of model (11)-(27) using the equivalent reformulation describes in (5)-(10), obtaining a nonlinear optimization problem. To solve this last model, (Fortuny-Amat and McCarl, 1981) propose an equivalent mixed-integer linear program. This formulation adds $(IT + T + I)$ binary variables and $2(IT + T + I)$ new constraints that replace nonlinear constraints of type (8). The resulting model can be solved using a mixed-integer solver only for small size instances. To solve medium and larger instances can be solved by the algorithm proposed by (Bard and Moore, 1990).

3 RESULTS

In this section, a case study is presented to illustrate the suitability and advantages of the proposed bilevel optimization model. Basic parameters (such as prices, costs and warehouse capacities) were created using market information gathered from different pork producers. Different countries use different cutting patterns for producing meat products according to their

history and gastronomic culture. The case study considers cutting patterns used by a Mexican pork firm that must plan its production over a time horizon. First, pork carcasses were split up into 5 sections, and for each section a set of cutting patterns was assigned. In total, the company operates with 17 cutting patterns, and manages 40 pork products.

The case study represents a batch of fattened pigs arriving every day to the manufacturer to be slaughtered and later processed as carcasses. It is assumed the available amount of carcasses during the whole horizon is fixed and known. The total amount of carcasses available over a time horizon was set at 5000. The yield matrix for a carcass per product, section and cutting pattern was obtained from production lines (in kg). Considering the large amount of products, we selected 15 products with the highest prices, representing 67% of total demand. Labour capacity is considered as 8 hours per day in normal time, and 3 hours per day of overtime. To perform each cutting pattern, a specific amount of labour time is required. The solved instance considered 10 planning periods. In addition, the shelf life was 10 days, not enough to overcome during the planning horizon.

Results in a bilevel linear programming model, for which reformulations are presented, becomes a mixed-integer linear problem with 700 and 300 decision variables for the leader and follower, 2825 constraints and 475 dual variables. The first instance represents the coordination and integration of the two supply chain agents. Moreover, it is also assumed that demand is given, and the available amount of carcasses is fixed and known. Table 1 summarizes the achieved results:

Table 1: Results.

Profit [US\$]	3.557.190
Service level	73,7%
Unsatisfied demand [u]	44.522
Carcasses acquired [u]	4.424
Carcasses acquired rate	88%
Excess Product [u]	38.523

Results from the case study showed a net profit of \$3.557.190 dollars for the leader a service level of 73,7% for the follower. Under this solution 88% of whole carcasses available in the planning horizon (4.424 carcasses) were used. This demonstrated that the demand was not completely satisfied. It also notes that the manufacturer did not reach its maximum capacity, and the occupancy rate carcasses during the planning horizon was less than 100%, meaning that there was still raw material to be processed.. Table 2 shows the acquisition of carcasses during the planning

horizon (10 days):

Table 2: Value of H in each period of time.

Horizon	H_t
1	550
2	516
3	548
4	415
5	555
6	447
7	427
8	506
9	336
10	124

The quantity of carcasses acquired at each period did not show major changes during the planning horizon, except for the last day where the carcasses acquired were to process and sell just for that period, without holding products. The amount of unsatisfied demand had a total of 44.522 products during the planning horizon, and the excess product at the end of the planning horizon is 38.523. The following table shows the detail for each of the products.

Table 3: Unsatisfied demand and excess products.

i	Unsatisfied demand [u]	Excess products [u]
1	0	0
2	0	0
3	0	4.672
4	4.321	0
5	0	0
6	24.105	1.226
7	6.910	0
8	5.012	0
9	1.297	0
10	0	23.946
11	436	0
12	0	728
13	0	0
14	0	7.950
15	2.380	0

The pattern cuts used resulted in different products. Products in inventory at the end of the planning horizon are excess products, and the manufacturer sells them without revenues because it's a salvage value. This implies that the leader produces to achieve a high follower service level as long as revenues are higher than the production cost of each of product. When the cost exceeds income from sales, the leader will prefer not to produce, and pay a penalty. In this game, the agents related to two levels of the supply chain, and it was observed that decisions have an interdependent relationship. The amount of unsatisfied demand selected by the wholesaler has an effect on

the profit function of the leader. Deciding how much to produce has an effect on the quantity of unsatisfied demand by the follower.

In order to see the advantages of the bilevel programming model, these results are compared with the decision making of the leader in Table 4.

Table 4: Comparison of two models.

Models	Bilevel	Leader
Profit [US\$]	3.557.190	1.805.780
Service level	73,7%	72,5%
Carcasses acquisition rate	88%	86%
Excess Product [u]	38.523	38.886

A new instance arises when the leader and follower are not coordinated nor integrated. In this case, the leader's model optimizes his operations according to (11) - (21), adding the demand requirement from the wholesaler. Table 4 shows that the leader does not use whole carcasses available. In addition, service level is 72,5% and the production capacity of the plant are not at the maximum. Comparing both models, when the two players are coordinating and integrating information and activities, the profits and service level increases. The carcasses acquisition rate rises because the leader is producing more products so that the unsatisfied demand decreases. In relation to excess product; in the bilevel programming model, the amount at the end of the planning horizon is less than the final quantity in the one-level decision model.

In this case, we show the importance of coordinating and integrating different agents in the supply chain. The leader maximizes profits, and even if the demand is given, profits and service level are lower than when leader and follower are coordinated and integrated. The difference lies mainly in inventory cost; when two players work together, products are held in two warehouses. In this way, costs are shared, unlike when the leader works alone. Coordination and integration is not only for better profits, but also increases service level and productivity.

All instances described in the computational results were implemented and solved using CPLEX 12.6.2.0 as a solver in a Macbook Pro Retine Display, i5-5257U Broadwell and 8Gb RAM with the software optimization package IBM ILOG CPLEX.

4 CONCLUSIONS

This work presents a novel Stackelberg production game between two players in the Pork Industry. The players are manufacturer and wholesaler, representing leader and follower, respectively. This game a coor-

dinates and integrate this link of the supply chain. To represent the game, we propose a linear bilevel programming model, where the leader maximizes profits, and the follower maximizes their service level.

Our model was applied to a case study in the pork industry, concluding that coordinating and integrating both players in the supply chain is a better strategy than previously proposed solutions. In effect, this coordination obtains higher profits and better service level. Furthermore, the reformulations used to solve this bilevel model are useful in this context. They ensure global optimum with computational time required to solve the mixed linear programming problem instances less than 1 minute, proving to be efficient for dimensions resolved in this work.

Game theory in supply chain management proves to be a powerful method that allows modeling games between different players within the food industry. Future research in this area focus on resolution techniques from a bilevel linear mixed-integer programming model as well as the development of a three-level model: supplier, producer and distributor.

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